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STRENGTH OF MATERIALS
HYDRAULICS
WATERWHEELS

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PREFACE

The volumes of the International Library of Technology are made up of Instruction Papers, or Sections, comprising the various courses of instruction for students of the International Correspondence Schools The original manuscripts are prepared by persons thoroughly qualified both technically and by experience to write with authority, and in many cases they are regularly employed elsewhere in practical work as experts The manuscripts are then carefully edited to make them suitable for correspondence instruction The Instruction Papers are written clearly and in the simplest language possible, so as to make them readily understood by all students Necessary technical expressions are clearly explained when introduced

The great majority of our students wish to prepare themselves for advancement in their vocations or to qualify for more congenial occupations. Usually they are employed and able to devote only a few hours a day to study Therefore every effort must be made to give them practical and accurate information in clear and concise form and to make this information include all of the essentials but none of the non-essentials To make the text clear, illustrations are used freely These illustrations are especially made by our own Illustrating Department in order to adapt them fully to the requirements of the text

In the table of contents that immediately follows are given the titles of the Sections included in this volume, and under each title are listed the main topics discussed

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CONTENTS

NOTE.—This volume is made up of a number of separate Sections, the page numbers of which usually begin with 1. To enable the reader to distinguish between the different Sections, each one is designated by a number preceded by a Section mark (§), which appears at the top of each page, opposite the page number. In this list of contents the Section number is given following the title of the Section, and under each title appears a full synopsis of the subjects treated. This table of contents will enable the reader to find readily any topic covered.

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§ 39

STRENGTH OF MATERIALS

(PART 1)

STRESS, DEFORMATION, ELASTICITY, AND STRENGTH

STRESS AND DEFORMATION

1. Definitions of Stress.—As explained in *Analytic Statics*, Part 1, any system of external forces acting on a body induces in the body internal forces by which the parts of the body are prevented from separating. If the body is cut by a plane anywhere, the two parts of it thus obtained exert on each other forces equal in magnitude but opposite in direction. In *Analytic Statics*, Part 1, the term *stress* was defined as denoting a pair of such equal opposing forces, that is, as a pair of forces consisting of the action exerted by any part of the body on another, and the reaction exerted by the latter part on the former, and it was there stated that the stress is measured by the magnitude of either force. It was also explained that, if a body in equilibrium is cut by a plane, the external forces acting on one of the parts thus obtained are balanced by the force that the other part exerts on the part considered.

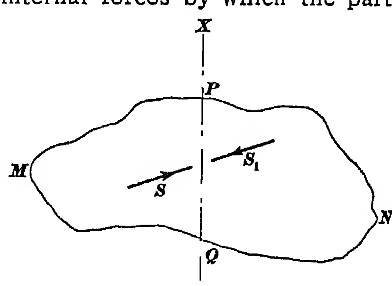

FIG. 1

If the body MN, Fig 1, is in equilibrium under the action of external forces, and it is cut anywhere by a plane XY, the part MPQ exerts on the part PNQ a force S equal and opposite to the force S_1 exerted by PNQ on MPQ The pair of forces S and S_1 constitute the stress at the section PQ S is the equilibrant of all the external forces acting on PNQ, and S_1 is the equilibrant of all the external forces acting on MPQ The part MPQ may be treated as a separate body kept in equilibrium by S_1 and the external forces acting on that part, and the part PNQ may be treated as a separate body kept in equilibrium by S and the external forces acting on that part

2. The term *stress* is also, and more generally, applied to either of the opposite internal forces acting at any section of a body Thus, with reference to Fig 1, the stress at the section PQ is either of the equal and opposite forces S, S_1 Taking the word stress in this sense, it is often defined as the internal force by which a body resists the action of external forces

3. It should be particularly noted that external forces act *on*, or are *applied to*, a body, while stresses act *in*, or are *produced* or *induced in*, the body The expression "to apply" a stress to a body is incorrect, what is really applied is one or more external forces, by which the stresses are caused, induced, or produced It is important that terms should be used in the proper sense, as looseness or inaccuracy of language leads to confusion in thinking

4. First Classification of Stress—Considering the direction in which stresses act with respect to the surface over which they are distributed, they may be either *normal* or *tangential*

A **normal stress** is a stress whose line of action is perpendicular to the surface over which it is distributed A **tangential stress** is a stress whose line of action is parallel to, or coincides with, the surface over which it is distributed. If, in Fig 1, S is perpendicular to PQ, S is a normal stress; if it has the direction PQ, it is a tangential stress.

Any inclined stress can be resolved, for purposes of analysis, into its normal and tangential components

5. When the external forces applied to a body tend to pull the parts of the body apart, the internal forces act toward each other, as represented in Fig 2 (*a*), where, for convenience, the body *A B* is shown separated into the two parts *A* and *B* cut by a plane of section P' is the action of *B* on *A*, and P'' the action of *A* on *B* This kind of stress is called **tension, tensile stress, or pull.** It is assumed that P' and P'' are perpendicular to the surface of separation Tension is, therefore, a normal stress

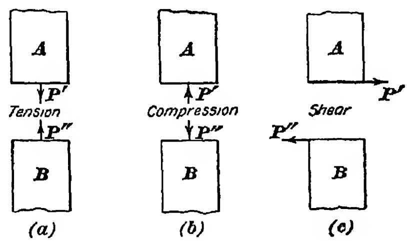

FIG 2

6. When the external forces applied to a body tend to crush the body, the internal forces act away from each other, as represented in Fig 2 (*b*) As in Fig 2 (*a*), P' is exerted by *B* on *A*, and P'' by *A* on *B* This kind of stress is called **compression, compressive stress, or thrust.** It is, like tension, a normal stress

7. When the external forces applied to a body tend to cause one part of the body to slide over the other part, the internal forces exerted by the parts on each other act along the surface of separation and prevent such sliding Thus, if the external forces are such that they tend to move *A*, Fig 2 (*c*), to the left and *B* to the right, the internal forces or stresses P' and P'' act as shown, and prevent the sliding This kind of stress, which is tangential, is called **shearing stress, or shear.**

8. Tension, compression, and shear are called **simple, or direct, stresses,** to distinguish them from bending and torsional stress, which will be defined further on

9. Second Classification of Stress.—The second classification of stress is based on the manner in which

the stress is distributed over the section separating the two parts of the body If a stress is such that all equal areas of the section of separation are under the same stress, wherever those areas are taken and whatever their extent, the stress is said to be **uniformly distributed,** or to be a **uniform stress** Otherwise, the stress is said to be **non-uniform,** or **varying.**

10. Intensity of Stress—Unit Stress.—If the magnitude of a uniform stress is divided by the area over which the stress is distributed, the result is the stress per unit of area, and is called **intensity of stress,** or **unit stress.** The latter term, although very commonly used, is misleading, for it seems to imply a special unit by which stresses are measured, whereas, stresses, being forces, are measured by the same units—such as pounds, tons, and kilograms—as other forces The expression *intensity of stress* will be here used instead of *unit stress* Intensity of stress is expressed as so many units of force per unit of area, such as pounds per square inch or tons per square foot

If P, expressed in units of force, is a stress uniformly distributed over an area A, the intensity of stress s is given by the formula

$$s = \frac{P}{A}$$

If P is in pounds and A in square inches, s will express pounds per square inch If P is in tons and A in square feet, s will express tons per square foot Similarly for any other units

EXAMPLE —A stress of 45,000 pounds is distributed uniformly over an area of $4\frac{1}{2}$ square inches What is the intensity of stress (*a*) in pounds per square inch? (*b*) in tons per square foot?

SOLUTION —(*a*) Here $P = 45{,}000$ lb , and $A = 4\frac{1}{2}$ sq in Substituting these values in the formula,

$$s = 45{,}000 \div 4\tfrac{1}{2} = 10{,}000 \text{ lb per sq in} \quad \text{Ans}$$

(*b*) In this case, P must be expressed in tons, and A in square feet P (tons) $= 45{,}000 \div 2{,}000$, A (sq ft) $= 4\frac{1}{2} \div 144$ Therefore,

$$s = \frac{45{,}000 \div 2{,}000}{4\frac{1}{2} \div 144} = \frac{45{,}000}{4\frac{1}{2}} \times \frac{144}{2{,}000} = 720 \text{ T per sq ft} \quad \text{Ans}$$

The same result could be obtained directly by multiplying the intensity per square inch, already found, by 144, and dividing the result by 2,000

EXAMPLES FOR PRACTICE

1 A uniform stress of 75,000 pounds is distributed over an area of 5 75 square inches Find the intensity of the stress (*a*) in pounds per square inch, (*b*) in tons per square foot

Ans $\begin{cases} (a) \ 13{,}043 \text{ lb per sq in} \\ (b) \ 939 \text{ T per sq ft} \end{cases}$

2 The intensity of a uniform stress distributed over an area of 12 square inches is 21,000 pounds per square inch What is the total stress P? Ans 252,000 lb

3 A uniform stress of 2,500 tons, having an intensity of 16 tons per square foot, is distributed over a plane surface What is the area of the surface? Ans 156 25 sq ft

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11. Definition of Deformation.—By the term **deformation** is meant the change of form or shape that a body undergoes when subjected to external forces The word **strain** is often used in this sense, but as it is used also in the sense of stress, it will be here dispensed with

12. Classification of Deformations —There are three kinds of deformation, corresponding to the three kinds of direct stress, namely, *tensile deformation*, *compressive deformation*, and *shearing deformation*

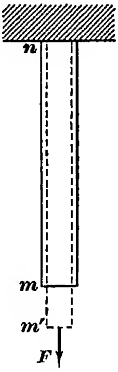


FIG 3

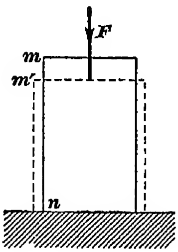


FIG 4

If a body or a part of a body is subjected to a tensile stress, it will stretch in the direction of the stress, this stretch is called a **tensile deformation.** Fig 3 represents a rod, whose natural length is $m\,n$, supported above and subjected to a pull F below If the force elongates the rod as shown by the dotted lines, the tensile deformation is $m\,m'$

If a body or part of a body is subjected to a compressive

stress, it will shorten in the direction of the stress, this shortening is called a **compressive deformation.** Fig 4 represents a block, whose natural length is $m\,n$, resting on a base and subjected to a load F. If the load shortens the block as shown by the dotted lines, the compressive deformation is $m\,m'$.

If a body or part of a body is subjected to shearing stress it will suffer a characteristic deformation Fig 5 represents a block of rubber $a\,n\,m\,b$, one face of which is firmly glued to a support and the opposite face to a small board $B\,B$ that can be pulled in the guides G If the board is pulled to the right, shearing stresses are developed at horizontal sections of the rubber block, and the face $a\,b\,m\,n$ is changed to $a\,b\,m'\,n'$.

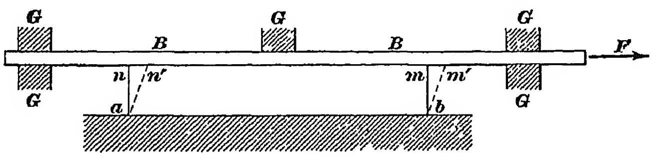


FIG 5

This change, which can be described either by the angle $m\,b\,m'$, or by the slide $m\,m'$ of the upper face with respect to the lower, is a **shearing deformation.**

13. Rate of Deformation.—In the case of tension and compression, the deformation per unit of length in a body of uniform cross-section will here be called the **rate of deformation** It is obtained by dividing the total deformation by the length of the body, the deformation and the length being expressed in the same units

Let l = natural length of a body of uniform cross-section,

K = total tensile or compressive deformation,

k = rate of deformation

Then, $$k = \frac{K}{l}$$

It should be observed that k is an abstract number, independent of the unit of length used Whether K and l are expressed in feet, inches, yards, or any other unit, the ratio k, or $\frac{K}{l}$, remains the same, provided that the same unit is used

for K as for l. If l and K are given, expressed in different units, they should be reduced to the same unit before finding the rate of deformation. If, for example, l is given in feet and K in inches, either l should be reduced to inches or K to feet.

The term **unit deformation** is frequently used to denote rate of deformation. As, however, it is misleading, it will not be used here.

EXAMPLE 1.—A rod 12 inches long is stretched $\frac{3}{8}$ inch by an external force. What is the rate of deformation?

SOLUTION.—Here $l = 12$, $K = \frac{3}{8}$, and, therefore,

$$k = \frac{\frac{3}{8}}{12} = \frac{1}{32}. \quad \text{Ans.}$$

EXAMPLE 2.—If the rate of deformation of a bar is $\frac{1}{50}$, and the original length of the bar is 15 feet, what is the length of the bar when deformed?

SOLUTION.—Since each foot of the bar is stretched $\frac{1}{50}$ ft., the whole bar will be stretched $15 \times \frac{1}{50}$, and, therefore, the length of the stretched bar is

$$15 + 15 \times \frac{1}{50} = 15.3 \text{ ft.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. The natural length of a block is 4.5 feet. If the block is compressed $\frac{9}{16}$ inch, what is the rate of deformation? Ans. $\frac{1}{96}$

2. If the original length of a rod is 9.75 feet, and a tensile force applied to it produces a rate of deformation of .016, what is the length of the stretched rod? Ans. 9.906 ft.

3. The original length of a bar is 8 inches; when the bar is under a certain tensile force, its length is 8.22 inches. What is the rate of deformation? Ans. .0275

ELASTICITY

14. Definition.—When a force is applied to a body, the body is deformed. If the force does not exceed a certain limit, which is different for different substances, the body will, on the removal of the force, regain its original form and dimensions. This property of bodies, by which they regain their form and dimensions when deforming

forces are removed, is called **elasticity**; and, to indicate that bodies possess this property, they are said to be **elastic.**

15. Elastic Limit.—For every body, there is a force, and a corresponding stress, beyond which the body ceases to be perfectly elastic, that is, beyond which the body, after the force is removed, retains all or part of the deformation caused by the force. The intensity of stress caused by this limiting force is called the **elastic limit** of the body, or of the material of which the body is composed.

16. Permanent Set.—As just stated, the deformation caused in a body when the elastic limit of the body is exceeded does not disappear entirely on the removal of the force. That part of the deformation that does not disappear is called **permanent set.**

17. Hooke's Law.—It has been found by experiment that, *within the elastic limit, the deformation caused by a force is proportional to the force.* This important principle, known as **Hooke's law,** is the foundation of the science of strength of materials.

Let F and F' be two forces, both inducing stresses smaller than the elastic limit, and causing in a body the deformations K and K', respectively. Then, according to Hooke's law,

$$K : K' = F : F'$$

Hooke's law does not apply to forces causing stresses greater than the elastic limit. Beyond this limit, the behavior of materials is irregular and imperfectly known, and it is a fundamental principle of engineering that *no part of a structure or machine should be designed for stresses greater than the elastic limit of the material of which the part is made.*

EXAMPLE.—If a load of 1,000 pounds elongates a rod $\frac{1}{2}$ inch, how great an elongation will be produced by a load of 1,200 pounds?

SOLUTION.—Let F denote the load of 1,000 lb., F', the load of 1,200 lb., K, the elongation of $\frac{1}{2}$ in., and K', the required elongation. Then, substituting in the foregoing proportion,

$$\tfrac{1}{2} : K' = 1{,}000 : 1{,}200,$$

whence $$K' = \frac{1{,}200 \times \frac{1}{2}}{1{,}000} = .6 \text{ in. Ans.}$$

18. Modulus or Coefficient of Elasticity.—Let a body of length l and uniform cross-section A be subjected to a uniform stress of magnitude S and intensity s. Let S' be another stress, acting in the same body (but not simultaneously with S), and having an intensity s'. Let the corresponding deformations be K and K', and the rates of deformation k and k'. It is assumed that s and s' are below the elastic limit. According to Hooke's law,

$$K : K' = S : S',$$

or, because $K = k\,l$ (see Art. 13), $K' = k'\,l$, $S = A\,s$, and $S' = A\,s'$,

$$k\,l : k'\,l = A\,s : A\,s',$$

or

$$k : k' = s : s',$$

whence

$$\frac{s}{k} = \frac{s'}{k'}$$

In the same manner, it may be shown that, if k'' is the rate of deformation corresponding to an intensity of stress s'', then

$$\frac{s''}{k''} = \frac{s}{k} = \frac{s'}{k'}$$

This result, which is independent of the area A, shows that, for every material, the quotient obtained by dividing the intensity of stress by the corresponding rate of deformation is constant, or has the same value for all stresses, provided only that their intensity is below the elastic limit of the material. This quotient is called the **modulus of elasticity** of the material, and is usually denoted by E. There are different moduli of elasticity, corresponding to the different kinds of stress, but those that are of greatest importance, as well as most accurately known, are the modulus of elasticity of tension and the modulus of elasticity of compression. They are often called **Young's moduli**. The modulus of elasticity is also called **coefficient of elasticity.**

The general formula for the modulus of elasticity is, therefore,

$$E = \frac{s}{k} \qquad (1)$$

In terms of the length l, the area A, the total stress P, and the total deformation K, we have, since

$$k = \frac{K}{l} \text{ and } s = \frac{P}{A},$$

$$E = \frac{\frac{P}{A}}{\frac{K}{l}} = \frac{Pl}{AK} \qquad \textbf{(2)}$$

Since k is an abstract number, formula **1** shows that E is expressed in the same units as the intensity s, that is, in units of force per unit of area. In English-speaking countries, E is usually expressed in pounds per square inch.

The value of the modulus of elasticity is determined experimentally by taking a piece of the material, as a rod or bar, measuring its length l, its area A, and the deformation K caused by an applied force P, and then substituting these values in formula **2**. It should be observed that, in this formula, l and K must be referred to the same unit of length, and A to the corresponding unit of area. Thus, if l is in inches, K must be in inches, and A in square inches.

EXAMPLE 1.—A steel rod 10 feet long and 2 square inches in cross-section is stretched .12 inch by a weight of 54,000 pounds. What is the tension modulus of elasticity of the material?

SOLUTION.—To apply formula **2**, we have the stress P = 54,000 lb., l = 10 ft. = 120 in., A = 2 sq. in., and K = .12 in. Therefore,

$$E = \frac{54,000 \times 120}{2 \times .12} = 27,000,000 \text{ lb. per sq. in.} \quad \text{Ans.}$$

EXAMPLE 2.—If the tension modulus of elasticity of a grade of steel is 28,000,000 pounds per square inch, what elongation will be caused in a bar 20 feet long and 4.25 square inches in cross-section by a force of 40 tons?

SOLUTION.—Formula **2**, solved for K, gives

$$K = \frac{Pl}{EA} \qquad (1)$$

In the present case, P = 40 T. = 80,000 lb., l = 20 ft. = 240 in., E = 28,000,000 lb. per sq. in., and A = 4.25 sq. in. Substituting these values in equation (1),

$$K = \frac{80,000 \times 240}{28,000,000 \times 4.25} = .161 \text{ in.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1 A block 9 inches long and 8 square inches in cross-section is compressed $\frac{1}{16}$ inch by a force of 60 tons What is the compression modulus of elasticity of the material? Ans 2,160,000 lb per sq in

2 The tension modulus of elasticity of a rod 15 feet long and 1 5 square inches in cross-section being 24,000,000 pounds per square inch, determine (*a*) the elongation caused by a force of 30,000 pounds, (*b*) the force necessary to cause an elongation of $\frac{1}{18}$ inch

Ans $\begin{cases} (a) & 15 \text{ in} \\ (b) & 12{,}500 \text{ lb} \end{cases}$

STRENGTH

19. Ultimate Strength —The **ultimate strength** of a given material in tension, compression, or shear is the greatest intensity of tensile, compressive, or shearing stress that the material can stand As represented in Figs 3 and 4, a specimen while being stretched or compressed changes in cross-sectional area So long as the stresses are within the elastic limit, the change in cross-section is small, but ductile materials, like wrought iron and structural steel, undergo a considerable change in cross-section just before rupture occurs It is the common practice to compute the ultimate strength, not from the maximum load and the area of the cross-section when rupture occurs, but by dividing the maximum load by the original area

20. Working Stress —The term **working stress,** or **working strength,** is applied to a part of a machine or structure to be designed, to denote the maximum intensity of stress to which that part is to be subjected If a part is to be subjected to more than one kind of stress, there are as many working stresses for it Thus, a riveted joint may be subjected to tension, compression, and shear, and, if the intensities of the tension, compression, and shear are 15,000, 12,000, and 9 000 pounds per square inch, respectively, these numbers are the working stresses

21. Factor of Safety —By **factor of safety** is meant the ratio of ultimate strength to working stress

Let f = factor of safety,
s_u = ultimate strength;
s = working strength.

Then, $$f = \frac{s_u}{s}$$

Strictly, a member has a factor of safety for each kind of stress to which it is subjected, but, if it has more than one, the least is referred to as *the* factor of safety

22. Choice of Working Stress or Factor of Safety If a working stress is selected for a material whose ultimate strength is known, the factor of safety can be computed by the formula in the preceding article, and, if the factor of safety is selected, the working stress can be computed by the same formula Hence, choosing a working stress amounts to the same thing as choosing a factor of safety

Before designing a structure, it is necessary to adopt a working stress or factor of safety, and this is a matter of great importance There are no fixed rules for the selection of a factor of safety, but the following principles should be borne in mind

1 The working stress should always be well within the elastic limit, for then the deformations are comparatively small, and not permanent

2 The working stress for a member subjected to changing stresses should be lower than that for a member subjected to a steady stress, while that for a member subjected to shocks should be still less This rule is based on the experimentally discovered fact that, if three specimens exactly alike are subjected to a steady load, a changing load, and a suddenly applied load, respectively, the first specimen will stand the greatest load, and the last, the least

Besides the uncertainty in quality of material, there are poor workmanship, deterioration of material, etc to be allowed for Then, again, it is sometimes impossible to compute the stresses to which a member is to be subjected, they must be estimated, or determined from assumptions that are only approximately true All such uncertainties

are provided for by lowering the working stress, or increasing the factor of safety Table I gives average values of the factors of safety commonly employed in American practice

TABLE I

FACTORS OF SAFETY

Material	For Steady Stress Buildings	For Changing Stress Bridges	For Shocks and Sudden Loads Machines
Timber	8	10	15
Brick and stone	15	25	30
Cast iron .	6	15	20
Wrought iron	4	6	10
Steel	5	7	15

These values will serve to give a general idea of the values used in practice All important structures and machinery are designed and constructed in accordance with specially prepared specifications, in which the working stresses or factors of safety to be used are stated In some branches of engineering construction, as in bridge work, the factor of safety is no longer used, the working stresses being given instead

SIMPLE OR DIRECT STRESSES

TENSION

23. Stresses in a Tension Piece.—Any body subjected to two equal and opposite pulls is called a **tension piece.** Tension pieces usually have the form of long bars, and the external forces are generally so applied that their lines of action coincide with a longitudinal axis passing through the center of gravity of the piece. In Fig. 6 (*a*) is represented a tension piece $B\,C$ acted on by the two equal and opposite forces F. If any section is cut by a plane per-

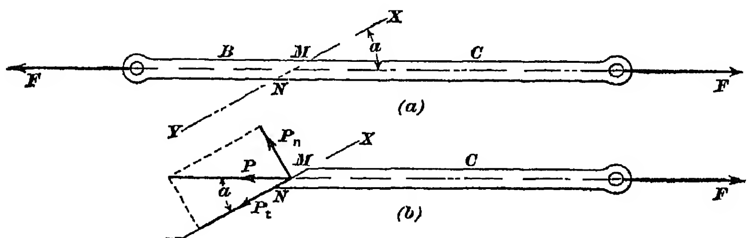


FIG. 6

pendicular to the axis of the piece, or to the common direction of the forces F, the only stress in that section will be a tension whose magnitude is F and whose intensity is $\frac{F}{A}$, denoting the area of the section by A.

If an inclined section MN is cut by a plane $X\,Y$, making an angle a with the axis of the piece, the total resultant stress P in that section will still be equal to F. This is plainly shown in Fig. 6 (*b*), where the part C is shown as a free body acted on by the forces F and P, the latter being equal to the stress at MN (see *Analytic Statics*, Part 1). In this case, however, there is both tension and shear at the section MN, for the force P may be resolved into a component P_n normal to MN and a component P_t along MN.

the former represents the tension at MN, the latter, the shear

If the area MN is denoted by A',

$$A' = \frac{A}{\sin a}$$

The intensities of tension and shear are, respectively (see Art 10),

$$s_t = \frac{P_n}{A'} = \frac{P \sin a}{\frac{A}{\sin a}} = \frac{F}{A} \sin^2 a \qquad (1)$$

$$s_s = \frac{P_t}{A'} = \frac{P \cos a}{\frac{A}{\sin a}} = \frac{P}{A} \sin a \cos a = \frac{F}{2A} \sin 2a \qquad (2)$$

The value of s_t is greatest, or a maximum, when $\sin^2 a = 1$, that is, when $a = 90°$, or when the plane XY is perpendicular to the axis of the piece For this condition,

$$\max\ s_t = \frac{F}{A} \qquad (3)$$

The value of s_s is greatest when $\sin 2a = 1$, whence $a = 45°$ For this condition,

$$\max\ s_s = \frac{F}{2A} = \tfrac{1}{2} \max\ s_t \qquad (4)$$

Since the maximum intensity of tension is twice that of shear, the tensile stress is the dangerous one, and the shearing stress is usually disregarded in tension pieces

24. Percentage of Elongation and Reduction of Area —In a tension test, the behavior of the specimen is

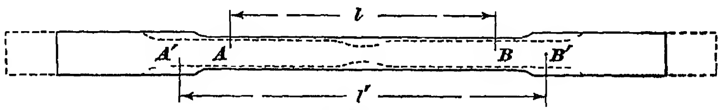


FIG 7

uniform within the elastic limit, that is, for equal increments of the load, the piece stretches equal amounts and the diminutions of cross-section are also equal But beyond the elastic limit, the elongation increases faster than the load Beyond the elastic limit, the diminution of cross-section also increases faster than the load, and ductile

materials, such as steel and wrought iron, begin to "neck down" shortly before rupture, stretching and necking continue without the load being increased

Fig 7 represents, in full lines, the original form of a tension specimen of a ductile material, and, in dotted lines, the form of the specimen at rupture If l denotes the original length between two sections A and B of the specimen, and l' the distance between the same sections, represented by A' and B', at rupture, the rate of elongation k (see Art **13**) is given by the formula

$$k = \frac{l' - l}{l} \qquad (1)$$

Denoting by k_{100} the per cent of elongation, or the elongation in 100 units of length,

$$k_{100} = 100\ k = \frac{l' - l}{l} \times 100 \qquad (2)$$

25. The **rate of reduction of area** is the ratio of the total reduction of area to the original area Denoting it by a, we have,

$$a = \frac{A - A'}{A} \qquad (1)$$

If the per cent of reduction of area is denoted by a_{100}, then

$$a_{100} = 100\ a = \frac{A - A'}{A} \times 100 \qquad (2)$$

EXAMPLE 1 —A bar whose original length was 8 inches broke when stretched to 10 32 inches What was the per cent of elongation?

SOLUTION —Here $l' = 10\ 32$ and $l = 8$, and, by formula 2 of Art **24**,

$$k_{100} = \frac{10\ 32 - 8}{8} \times 100 = 29 \text{ per cent} \quad \text{Ans}$$

EXAMPLE 2 —The original diameter of a round rod tested for tension was $\frac{5}{8}$ inch, and the diameter at rupture was 47 inch What was the rate of reduction of area and the per cent of reduction?

SOLUTION —Here $A = \frac{\pi}{4} \times \left(\frac{5}{8}\right)^2$, and $A' = \frac{\pi}{4} \times (\ 47)^2$ Therefore, by formula **1**,

$$a = \frac{\frac{\pi}{4} \times \left(\frac{5}{8}\right)^2 - \frac{\pi}{4} \times (\ 47)^2}{\frac{\pi}{4} \times \left(\frac{5}{8}\right)^2} = \frac{(\ 625)^2 - (\ 47)^2}{(\ 625)^2} = \ 435 \quad \text{Ans}$$

By formula **2**,

$$a_{100} = \ 435 \times 100 = 43\ 5 \text{ per cent} \quad \text{Ans}$$

EXAMPLES FOR PRACTICE

1 A round rod whose original diameter was $\frac{3}{4}$ inch was tested for tension until it broke The diameter at rupture was found to be 69 inch Find (*a*) the rate of reduction of area, (*b*) the per cent of reduction of area

$$\text{Ans} \begin{cases} (a) & a = 154 \\ (b) & a_{100} = 15\ 4 \text{ per cent} \end{cases}$$

2 A rod whose original length was 9 inches was found, after rupture, to measure 10 87 inches Determine the per cent of elongation

Ans $k_{100} = 20\ 78$ per cent

---

26. Constants for Materials in Tension —For use in examples, and in order that a general idea of the numerical values of the constants may be had, Table II is here given These values are rough averages, from which there are wide variations There are, for instance, many kinds of timber whose ultimate strengths differ widely from one another The same remark applies to the constants for different grades of cast iron, wrought iron, and steel

TABLE II

CONSTANTS FOR MATERIALS IN TENSION

(*Pounds per Square Inch*)

Material	Modulus of Elasticity E	Elastic Limit L_t	Ultimate Strength s_t
Timber	1,500,000		10,000
Cast iron	15,000,000		20,000
Wrought iron	25,000,000	25,000	50,000
Steel	30,000,000	40,000	65,000

EXAMPLE 1 —A round wrought-iron rod 1 inch in diameter sustains a pull of 20,000 pounds What are the intensity of tensile stress s_t and the factor of safety f?

SOLUTION —The entire stress equals the load, or 20,000 lb , and the area of the cross-section of the rod is $\frac{\pi}{4} \times 1^2 = 7854$ sq in Hence, by formula of Art **10**,

$$s_t = \frac{20,000}{7854} = 25,465 \text{ lb per sq in} \quad \text{Ans}$$

This is also the working stress, hence, by formula of Art. **21**,

$$f = \frac{50{,}000}{25{,}465} = 2, \text{ nearly} \quad \text{Ans.}$$

EXAMPLE 2.—What is the proper diameter of a round steel rod that is to sustain a steady pull of 60,000 pounds?

SOLUTION.—From Table I, the factor of safety for steel under a steady stress is 5. From Table II, $s_t = 65{,}000$. Therefore, from the formula of Art. **21**,

$$s = \frac{s_t}{f} = \frac{65{,}000}{5} = 13{,}000 \text{ lb. per sq. in.}$$

From the formula of Art. **10**,

$$A = \frac{P}{s} = \frac{60{,}000}{13{,}000} = 4.62 \text{ sq. in.}$$

$$d = \sqrt{\frac{4.62}{.7854}} = 2.4 \text{ in.} \quad \text{Ans.}$$

EXAMPLE 3.—What must be the area of a steel rod to sustain a pull of 75,000 pounds, the load being variable, as in a bridge?

SOLUTION.—Here $s_t = 65{,}000$, and $f = 7$ (Table I). Therefore, by formula of Art. **21**,

$$s = \frac{65{,}000}{7} = 9{,}286 \text{ lb. per sq. in.}$$

and

$$A = \frac{75{,}000}{9{,}286} = 8.08 \text{ sq. in.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. A steel bar 4.5 square inches in cross-section is under a stress of 78,000 pounds. What is (*a*) the factor of safety? (*b*) the working stress?

Ans. (*a*) 3.8, nearly; (*b*) 17,330 lb. per sq. in.

2. What should be the area of a steel bar in order that it may safely sustain a variable pull of 86,000 pounds? Ans. 9.26 sq. in.

3. A steel bar is to be subjected to a suddenly applied load of 40,000 pounds. What is (*a*) the working stress? (*b*) the required area?

Ans. (*a*) 4,330 lb. per sq. in.; (*b*) 9.2 sq. in.

4. How great a load can be safely sustained by a wrought-iron bar 5 square inches in cross-section, if the load is suddenly applied?

Ans. 25,000 lb.

IMPORTANT APPLICATIONS

27. Strength of Cylindrical Shells and Pipes With Thin Walls.—When a cylinder is subjected to internal pressure, the tensile stress developed in the walls or shell of the

cylinder is called **circumferential stress, or hoop tension** Fig 8 represents one-half of a cross-section of a cylinder the unmarked arrows represent the internal pressure, everywhere normal to the interior surface, while the arrows marked P represent the hoop tensions that hold down the upper half of the cylinder Evidently, these balance the upward component F of the internal pressure It can be shown by the use of advanced mathematics that the component F is the same as the resultant of a normal pressure uniformly distributed over the projection $B\,C$ of the inner cylindrical surface on the plane of section $B\,C$, that pressure having an intensity equal to the actual intensity of pressure on the walls of the cylinder

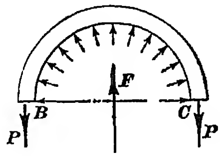


FIG 8

Let d = internal diameter of cylinder,
p = intensity of pressure on inner surface of cylinder;
l = length of cylinder considered,
t = thickness of shell

When, as here assumed, t is very small compared with d, the stress P may be treated as uniformly distributed over the surface of contact of the two halves of the cylinder, which surface consists of two rectangles of length l and width t The area of the projection represented in the figure by $B\,C$ is $B\,C \times l = d\,l$ Therefore,

$$F = p \times d\,l$$

Also, since F is balanced by $2\,P$,

$$2\,P = F = p\,d\,l \qquad (1)$$

Let s be the intensity of the tension P Then, $s = \frac{P}{l\,t}$, and $P = s\,l\,t$ These values substituted in equation (1) give

$$2\,s\,l\,t = p\,d\,l,$$

whence

$$t = \frac{p\,d}{2\,s} \qquad (1)$$

$$s = \frac{p\,d}{2\,t} \qquad (2)$$

Formula **1** serves to compute the thickness when p, d, and s (working stress) are given, formula **2** is used to compute

the intensity of stress when the intensity of pressure p and the dimensions of the cylinder are given.

EXAMPLE 1.—What is the hoop intensity of tension in a boiler shell whose inside diameter is 30 inches, plate $\frac{3}{8}$ inch thick, the steam pressure being 100 pounds per square inch?

SOLUTION.—Here $p = 100$, $t = \frac{3}{8}$, and $d = 30$. Then, formula **2**,

$$s = \frac{100 \times 30}{2 \times \frac{3}{8}} = 4{,}000 \text{ lb. per sq. in.} \quad \text{Ans.}$$

EXAMPLE 2.—As determined by hoop tension, what should be the thickness of walls of a cast iron water pipe, inside diameter 24 inches, to resist a water pressure of 200 pounds per square inch? A factor of safety of 10 is to be used.

SOLUTION.—The ultimate tensile strength of cast iron, as given in Table II, is 20,000 lb. per sq. in. As the factor of safety is 10, the working stress is 20,000 ÷ 10 = 2,000 lb. per sq. in. Substituting this and the given values in formula **1**,

$$t = \frac{200 \times 24}{2 \times 2{,}000} = 1.2 \text{ in.} \quad \text{Ans.}$$

28. The formulas in the last article relate to the radial stresses in a cylinder, that is, to the tendency of the internal pressure to break the cylinder along a plane section containing the axis. The tendency to rupture on a plane section perpendicular to the axis will now be considered. Fig. 9 represents a portion ML of the cylinder cut by a plane XY. This part is kept in equilibrium by the tension P and the internal pressure F. The tension P is distributed over a ring of width t, whose area is practically equal to $\pi d \times t$. The force F is distributed over a circular area represented in section by BC. Therefore,

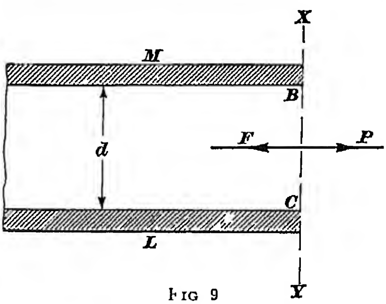


FIG. 9

$$P = \pi d t \times s, \; F = \frac{\pi d^2}{4} \times p,$$

and, since $P = F$, $\qquad \pi d t s = \frac{\pi d^2 p}{4},$

whence $\qquad s = \frac{p d}{4 t}$

Comparing this formula with formula **2** of Art **27**, it is seen that the intensity of stress necessary to prevent transverse rupture is only one-half of that necessary to prevent longitudinal rupture. The latter, therefore, is the only one that needs to be considered.

29. Temperature Stresses.—When the temperature of a body changes, the length of the body changes by a fixed fraction of itself for every degree of change in temperature. This fraction is called the **coefficient of linear expansion,** or simply the **coefficient of expansion,** and is different for different materials. Thus, if the coefficient of expansion of a substance is .000006, and the length of a bar of that substance at 40° F. is 50 feet, the length of the bar will be

at 41°, 50 (1 + .000006) feet
at 42°, 50 (1 + .000006 × 2) feet
at 49°, 50 (1 + .000006 × 9) feet
at 39°, 50 (1 − .000006) feet
at 32°, 50 (1 − .000006 × 8) feet

For nearly all substances, the coefficient of expansion is *positive*, that is, the substances expand, or increase in length, when the temperature rises, and contract, or decrease in length, when the temperature decreases.

The coefficient of expansion, per degree Fahrenheit, is usually denoted by c. The average values of this constant are given in Table III.

TABLE III

Material	Coefficient of Expansion c
Steel	.0000065
Wrought iron	.0000069
Cast iron	.0000062

30. Let c = coefficient of expansion,
l = length of rod or bar at temperature t,
l_1 = length of rod or bar at temperature t_1,
k = rate of deformation due to change $t - t_1$.

Then, since for every degree of increase, the length of the rod or bar increases *algebraically* by $c\,l$,

$$l_1 = l + c\,l(t_1 - t) \qquad \textbf{(1)}$$

It should be noticed that if t_1 is less than t, the term in marks of parenthesis becomes negative The rate of deformation k is $\frac{l_1 - l}{l}$, or, substituting the value of l_1 from formula **1**, and reducing,

$$k = c(t_1 - t) \qquad (2)$$

If the rod or bar is constrained, so that it can neither expand nor contract, the constraint must exert on it a force sufficient to prevent the deformation $l_1 - l$, or, rather, sufficient to produce the deformation $l_1 - l$, since the natural length of the bar at the temperature t_1 is l_1, and the force that keeps it at length l produces the deformation $l_1 - l$ This force, which causes a rate of deformation k, causes a corresponding stress, which is called **temperature stress.** Let s be the intensity of that stress Then, by formula **1** of Art **18**,

$$s = Ek$$

or, replacing the value of k from formula **2**,

$$s = Ec(t_1 - t) \qquad (3)$$

If $c(t_1 - t)$ is positive, the temperature stress is compressive, if negative, the temperature stress is tension

EXAMPLE 1 —A wrought-iron rod has its ends fastened to firm supports What is the intensity of temperature stress produced in it by a change of 50° in its temperature?

SOLUTION —From Table II, E = 25,000,000 For wrought iron, c = 0000069 Here $t_1 - t$ = 50° Therefore, by formula 3,

s = 25,000,000 × 0000069 × 50 = 8,625 lb per sq in Ans

EXAMPLE 2 —Suppose that, before the change of temperature described in the preceding example, the rod is under a tension of 10,000 pounds per square inch What is the stress per unit area, after the change in temperature (*a*) if the change is a fall? (*b*) if it is a rise?

SOLUTION —(*a*) The temperature stress is tension, hence, the effect of the change in temperature is to increase the already existing tension, and the final tension is

10,000 + 8,625 = 18,625 lb per sq in Ans

(*b*) The temperature stress is compression, hence, the effect of the change in temperature is to decrease the already existing tension, and the final tension is

10,000 − 8,625 = 1,375 lb per sq in Ans

31. Hoop Shrinkage.—A hoop or tire can be placed on a cylinder whose diameter is slightly larger than the internal diameter of the hoop or tire. This is done by heating the hoop or tire until its diameter is greater than that of the cylinder, when it is put around the cylinder and allowed to cool. After cooling, the hoop is in a stretched condition, and in tension.

Let D = outer diameter of cylinder,

d = inner diameter of hoop.

Then, if D is unchanged by shrinkage, the diameter of the hoop when on the cylinder is D. Hence, d is increased to D, and a deformation and the accompanying tension take place, that is, the circumference of the hoop has increased from πd to πD. The total deformation is then $\pi(D-d)$, and the rate of deformation k is

$$\frac{\pi(D-d)}{\pi d} = \frac{D-d}{d}$$

Denoting by s the intensity of stress on the hoop, we have, from formula **1** of Art. **18**,

$$s = E k = E\frac{D-d}{d}$$

EXAMPLE 1.—What should be the inner diameter of a steel tire that is to be shrunk on a wheel 20 inches in diameter, if the safe tensile stress of the tire is 20,000 pounds per square inch?

SOLUTION.—For steel, $E = 30{,}000{,}000$ (Table II); hence, substituting in the formula,

$$20{,}000 = 30{,}000{,}000\left(\frac{20-d}{d}\right),$$

whence, solving for d,

$$d = 19.987 \text{ in. Ans.}$$

EXAMPLE 2.—What is the ratio $(D-d) \div d$ for steel tires, using for working stress 20,000 pounds per square inch?

SOLUTION.—For steel, $E = 30{,}000{,}000$; hence, substituting in the formula,

$$20{,}000 = \frac{D-d}{d} \times 30{,}000{,}000,$$

whence $$\frac{D-d}{d} = \frac{20{,}000}{30{,}000{,}000} = \frac{1}{1{,}500} \text{ Ans.}$$

EXAMPLES FOR PRACTICE

1 What must be the thickness of a steel pipe 36 inches in diameter to withstand an internal pressure of 125 pounds per square inch, the tensile working stress being 15,000 pounds per square inch?

Ans 15 in

2 A steel tension piece is designed for a load of 15,000 pounds per square inch at 62° F What is the stress per square inch when the piece is loaded and the temperature is (*a*) 95°? (*b*) 5°?

Ans $\begin{cases} (a)\ 8,565 \text{ lb per sq in, tension} \\ (b)\ 26,115 \text{ lb per sq in, tension} \end{cases}$

3 A wrought-iron hoop is to be put around a cylindrical wooden pipe whose outside diameter is 40 inches If the hoop is to be under a tensile stress of 12,000 pounds per square inch, what must be its diameter?

Ans 39 98 in

4 What is the intensity of temperature stress caused in a cast-iron column by a rise of temperature of 80°?

Ans 7,440 lb per sq in, compression

COMPRESSION

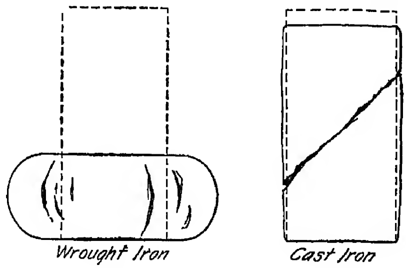
32. Compressive Strength Dependent on Length. The strength of a compression member, unlike that of a tension member, depends on the length of the member In a general way, the strength decreases as the length increases, but not uniformly Long pieces fail by bending, and are called **columns, pillars,** or **posts;** short pieces fail by crushing or shearing, and are called **short blocks.** Naturally, there is no sharp division between columns and short blocks

If l denotes the length of a compression piece, and d the least dimension of its cross-section, the division between columns and short blocks may be made approximately when $\frac{l}{d} = 10$, that is, a piece for which $\frac{l}{d}$ is less than 10 may be considered as a short block, and one for which $\frac{l}{d}$ is greater than 10 may be considered as a column At present, only short blocks will be dealt with, the subject of columns will be treated further on

33. Two Characteristic Manners of Failure of Short Blocks.—Materials in short blocks fail under compression in one of two ways

Ductile materials (structural steel, wrought iron, etc) and *wood compressed across the grain* bulge out sidewise, mash down, and crack vertically when subjected to stresses exceeding the elastic limit, as shown in Fig 10 (a) Bodies of these materials do not separate into two distinct parts, as when under tension, and fail gradually, they cannot be said to have any definite ultimate strength in compression

Brittle materials (hard steel, cast iron, stone, etc) and *wood compressed along the grain* do not mash down, but reach a definite point of failure, as in tension The brittle materials really fail by shear, the piece separating into two or more parts, the surfaces of rupture, approximately planes, make angles of about 45° with the axis of the piece, as shown in Fig 10 (b) Wood does not always separate into parts at failure, but the material adjacent to a more or less irregularly inclined section crushes down, the phenomenon as a whole resembling failure by shear



(a) Fig 10 (b)

34. Stresses in a Compression Short Block.—A short block, when tested, is usually subjected to a pressure that is distributed over the whole top and bottom surfaces of the block As the load is applied, the block shortens and its cross-sections enlarge Enlargement of the sections near the top and bottom necessitates a slipping between the bearing surfaces, which induces frictional resistance, and so there is, besides pressure on the top and bottom, some friction, whose direction is everywhere toward the center of the top and bottom faces On account of the friction, the stress at a section of a short block is more complex than at a section of a

tension piece It will be instructive to make an approximate analysis of the stress, neglecting the friction The case is then similar to that of a tension piece

Let Fig 11 (a) represent a short block, whose cross-sectional area is A, subjected to a load P Let XY be a plane inclined at an angle a with the axis of the piece, and cutting a section MN As in the tension piece considered in Art **23**, there is shear and normal stress (here compression) in the oblique section MN The upper part of the block is shown as a free body in Fig 11 (b) The notation and formulas are the same as in Art **23**, except that here the normal stress is compression instead of tension. The

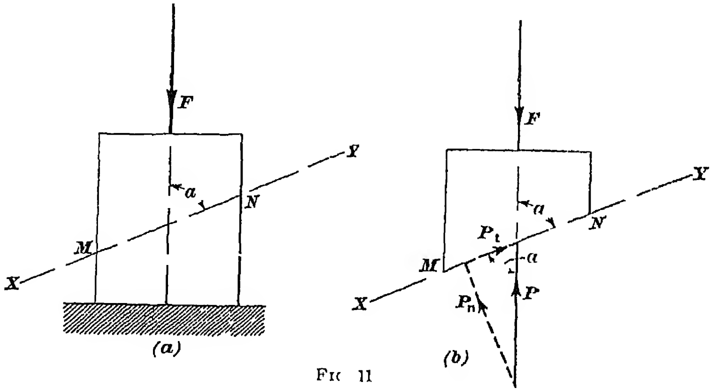


Fig 11

maximum intensity of compression occurs in sections normal to the axis, and is equal to $\frac{F}{A}$ The maximum intensity of shear occurs in sections inclined to the axis at 45°, and is equal to $\frac{F}{2A}$

Although there is shearing stress in a short block when it is compressed, the compressive stress is regarded as the important one, and, although short blocks of brittle materials, when compressed sufficiently, actually fail by shearing, they are said to have an *ultimate compressive strength*, which is computed by dividing the breaking compressive load by the original area of the cross-section of the block

35. Constants for Materials in Compression.—Table IV contains average values of the constants for the principal structural materials used in compression. These are average values from which there are wide variations.

TABLE IV

CONSTANTS FOR MATERIALS IN COMPRESSION

(*Pounds per Square Inch*)

Material	Modulus of Elasticity E_c	Elastic Limit L_c	Ultimate Strength s_c
Timber	1,500,000		8,000
Brick			2,500
Stone	6,000,000		6,000
Cast iron	15,000,000		90,000
Wrought iron	25,000,000	25,000	50,000
Steel	30,000,000	40,000	65,000

EXAMPLE 1.—How great a steady load can a 12″ × 12″ × 36″ timber block safely stand?

SOLUTION.—With a factor of safety of 8 (Table I), the working stress is, by the formula of Art. **21**,

$$8{,}000 \div 8 = 1{,}000 \text{ lb. per sq. in.}$$

The area of the cross-section is 144 sq. in.; therefore, the total pressure that the piece can stand is

$$1{,}000 \times 144 = 144{,}000 \text{ lb.} \quad \text{Ans.}$$

EXAMPLE 2.—How much would the block of example 1 shorten under the load?

SOLUTION.—From formula **2** of Art. **18**, $K = \frac{P l}{E A}$. To apply this equation, we have $P = 144{,}000$, $l = 36$, and $E = 1{,}500{,}000$ (Table IV). Therefore,

$$K = \frac{144{,}000 \times 36}{144 \times 1{,}500{,}000} = .024 \text{ in.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. With a factor of safety of 10, what load can a cast-iron cylindrical block 10 inches in diameter withstand? Ans. 706,860 lb.

2. A steel rectangular block 8 in. × 14 in., and 2 feet long, supports a weight of 1,617,300 pounds. Determine (*a*) the factor of safety f, (*b*) the shortening K caused by the weight.

$$\text{Ans.} \begin{cases} f = 4.5 \\ K = .012 \text{ in.} \end{cases}$$

3 With a factor of safety of 8, what must be the diameter of a cylindrical cast-iron block to support a weight of 75 tons? Ans 4 1 in

4 If the block in example 3 is made square and of timber, what length must the side of the square be? Ans 12 25 in , nearly

SHEAR

36. Occurrence of Shearing Stress —As explained in Arts **23** and **34**, shearing stress occurs in most sections of members under tension and of short blocks under compression, and, as will be explained further on, there are important shearing stresses in loaded beams and twisted shafts Usually, the shearing stresses at sections of structural members are accompanied by normal stresses, as in the tension piece and short block represented in Figs 6 and 11

TABLE V

CONSTANTS FOR MATERIALS IN SHEAR

(Pounds per Square Inch)

Material	Modulus of Elasticity E_r	Ultimate Strength S_r
Timber (across grain)		3,000
Timber (with grain)	400,000	600
Cast iron	6,000,000	20,000
Wrought iron	10,000,000	50,000
Steel	11,000,000	70,000

Beams can be supported and loaded so that there is only shearing stress at some given cross-section On the cross-section of a shaft supported so that there is no bending, there is shearing stress only In punching rivet holes, the principal resistance is a shearing stress distributed all around the surface of the metal that is being punched out In none of these cases is the shearing stress uniformly distributed

There are no ordinary occurrences of uncombined uniform shear from which to obtain values of the constants for materials in shear The elastic limit and modulus of elasticity (called also **rigidity modulus**) are obtained from

torsion tests. The ultimate strength is obtained from tests in which an attempt is made to rupture the material by shear alone. Such tests on metals resemble a punching operation, but the metal sheared is stayed down to a bedplate to prevent bending at the shearing surfaces.

37. Constants for Materials in Shear.—Table V gives constants for materials in shear. Like those given for tension and compression, they are rough average values. Constants for shear, being generally difficult to ascertain and of less practical value than the others, have not been so accurately determined.

EXAMPLE 1.—How great a force P, Fig. 12, can the end MN of a timber safely stand if its width is 6 inches?

SOLUTION.—The area to be sheared is $6 \times 8 = 48$ sq. in., and, since the ultimate strength is 600 lb. per sq. in., the greatest safe value for P, using a factor of safety of 10, is

$$48 \times 600 \div 10 = 2{,}880 \text{ lb.} \quad \text{Ans.}$$

EXAMPLE 2.—To test the shearing strength of a certain kind of wood, a mortise was made in a piece of it 2 in. × 2 in. in cross-section, as shown in Fig. 13, then a close-fitting steel pin inserted in the mortise was pulled downwards until the piece $a\,b\,c\,d$ was shorn out. The pull required was 4,220 pounds. What was the ultimate shearing strength of the wood?

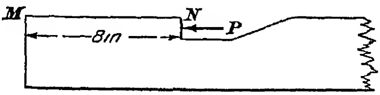


FIG. 12

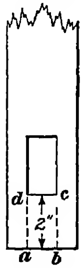


FIG. 13

SOLUTION.—There are two sheared areas, represented by $a\,d$ and $b\,c$. Each is 4 sq. in. in area, hence, the total sheared area is 8 sq. in. The shearing stress caused at rupture in the two surfaces equals the load, hence, the shearing stress per unit area—that is, the ultimate shearing strength—is

$$4{,}220 \div 8 = 527 \text{ lb. per sq. in.} \quad \text{Ans.}$$

BEAMS

38. A **beam** is any bar resting on supports and subjected to forces whose lines of action do not coincide with, but intersect, the axis of the bar. A **simple beam,** or a beam **simply supported,** is a beam resting on two supports very near its ends, the beam not being rigidly fastened to the supports. A **cantilever** is a beam with an overhanging free or unsupported end. A **restrained beam** is a beam that has both ends fixed, as a plate riveted to its supports at both ends. A **continuous beam** is a beam that rests on more than two supports.

Beams are usually horizontal, and the forces acting on them are generally vertical forces, including weights or loads and the reactions of the supports. These conditions will be assumed in what follows.

EXTERNAL SHEAR

39. Definition.—The **external shear** at any section of a loaded beam is the algebraic sum of all the forces (loads and reactions) to the right or left of the section. The external shear is sometimes called, for brevity, *shear,* but it must not be confused with the shearing stress at the section. As will be shown further on, the external shear at any section is equal to the shearing stress at the section; hence the name.

In computing the external shear, it is customary to give the positive sign to forces acting upwards, and the negative to those acting downwards. The values of the external shear at any section, as computed, respectively, from the forces on the right and from the forces on the left of the section are equal in magnitude, but of opposite signs. This follows from the fact that the external forces on the two sides of the section form a balanced system, and, since they are parallel, their algebraic sum must be equal to zero.

For example, suppose that a beam whose weight per unit of length is w is loaded and supported as shown in Fig. 14, R_1, R_2, and R_3 being the reactions of the supports. The external forces to the left of the section A are W_1, R_1, and the weight wx_1 of the part of the beam to the left of A. Hence, the external shear V at the section A, computed from the forces on the left, is $R_1 - W_1 - wx_1$. The forces to the right are W_2, W_3, R_2, R_3, and the weight wx_2 of the part of the beam to the right of A. Hence, the external shear V', computed from the forces on the right, is $R_2 + R_3 - W_2 - W_3 - wx_2$. Since V and V' form a system of parallel forces in equilibrium, it follows that $V + V' = 0$, and, therefore, $V = -V'$.

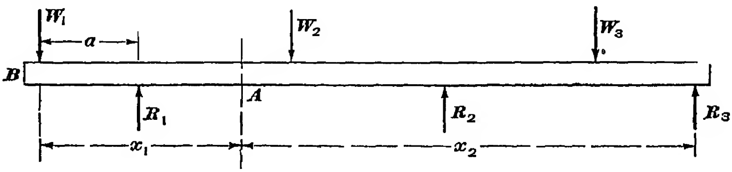


FIG. 14

It is customary to compute the external shear at any section from the forces on the left. If there are fewer forces on the right, it may be computed from the forces on the right, changing the sign of the result so as to obtain the shear as computed from the forces on the left.

40. Notation.—The letter V will be used to denote the external shear at any section of a beam, and the same letter with a subscript to denote the shear at a particular section, the subscript indicating how far the section is from the left end of the beam. Thus, V_2 denotes the external shear at a section 2 feet from the left end of the beam under consideration. The accented letters V' and V'' will be used to denote the shears just to the left and right of a section; thus, V_2' and V_2'' denote, respectively, the external shears at sections just to the left and right of a section 2 feet from the left end.

41. Shear Diagram and Shear Line.—For solving some problems on loaded beams, it is convenient in each

case to have a diagram representing the external shears at all sections of the beam Such a diagram is called a **shear diagram.** It consists of a base line or axis, equal by scale to the length of the beam, on which are marked the points of application of the loads and reactions, and another line, called the **shear line,** so drawn that the ordinate to it from

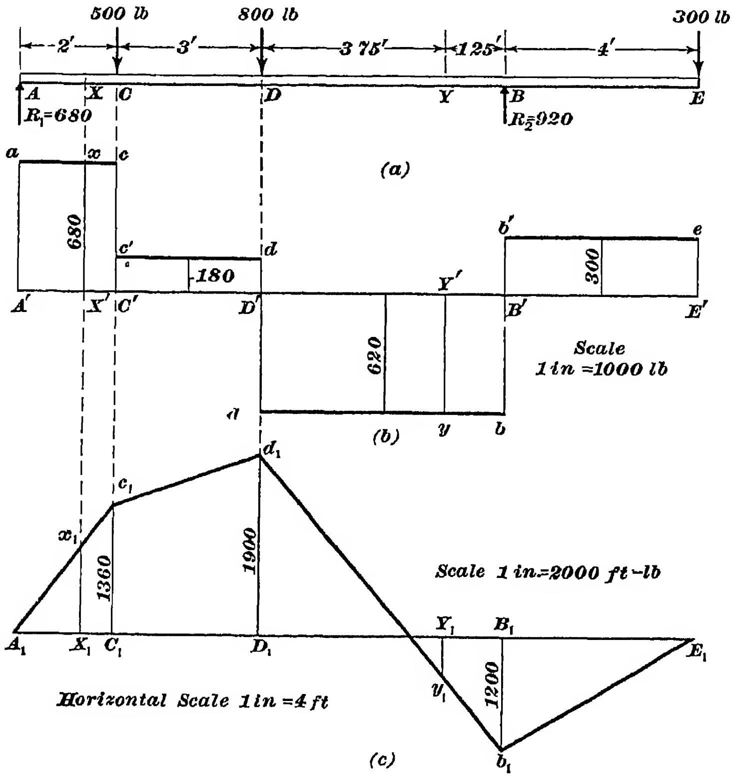


FIG 15

any point of the base represents, to any convenient scale, the external shear at the corresponding section of the beam Upward ordinates correspond to positive, and downward ordinates to negative, external shears Thus, Fig 15 (*b*) is the shear diagram for the loaded beam shown in Fig 15 (*a*), the line $A'E'$ being the base line, and the broken line

$A' a c c' d d' b b' e E'$ the shear line The ordinate $X' x$ represents the external shear at the section X The ordinate being upwards, the shear is positive At Y, the external shear is negative, as indicated by the ordinate $Y' y$

42. Construction of Shear Diagram —To draw the shear line for a loaded beam, the external shears must be computed for several sections, the number of sections depending on how many points are necessary to locate or determine the shear line As explained in Art **39**, the external shear is customarily computed from, or referred to, forces on the left of the section

EXAMPLE 1 —To construct the shear diagram for a beam supported at A and B, Fig 15 (a), and carrying loads of 500, 800, and 300 pounds at C, D, and E, respectively, as shown, the weight of the beam being neglected

SOLUTION —First, the reactions R_1 and R_2 are computed by equating to zero the moments about B and A, respectively (see *Analytic Statics*, Part 1) Thus,

$$R_1 \times 10 - 500 \times 8 - 800 \times 5 + 300 \times 4 = 0, \; R_1 = 680 \text{ lb}$$
$$500 \times 2 + 800 \times 5 - R_2 \times 10 + 300 \times 14 = 0, \; R_2 = 920 \text{ lb}$$

For any section between A and C, the shear is R_1, since R_1 is the only force on the left of C, that is,

$$V = 680 \text{ lb}$$

For any section between C and D, the shear is the algebraic sum of R_1 and -500 lb , that is,

$$V = 680 - 500 = 180 \text{ lb}$$

For any section between D and B,

$$V = 680 - 500 - 800 = -620 \text{ lb}$$

For any section between B and E,

$$V = 680 - 500 - 800 + 920 = +300 \text{ lb},$$

or, considering the forces to the right of B,

$$V = -(-300) = +300 \text{ lb}$$

To draw the shear line, a convenient scale, such as 1 in to 1,000 lb , is used to represent shears The base line $A' E'$, parallel and equal to $A E$, is drawn at any convenient distance below $A E$ Project C on $A' E'$ at C', and at C' erect an ordinate $C' c$ to represent the external shear between A and C, the shear being 680 pounds, the ordinate must be $680 \div 1,000 = .68$ in , if a scale of 1,000 lb to the inch is used Then erect a perpendicular $A' a$ at a, and draw $c a$ parallel to $A' E'$ Then $a c$ is the shear line for the part $A C$ of the beam Next, project D on $A' E'$ at D', and erect an ordinate $D' d$ to represent the external shear between C and D, which is 180 lb , the ordinate

must be 180 − 1,000 = 18 in The horizontal line $c'd$ is the shear line for CD Next project B on $A'E'$ at B, and erect the ordinate $B'b$ to represent −620 lb, this ordinate is drawn downwards, because the shear between D and B is negative From b, draw a horizontal line meeting dD' produced at d' Then, $d'b$ is the shear line for DB The shear line $b'e$ for BE is similarly drawn The broken line $acc'dd'bb'e$ is the shear line for the whole beam

NOTE —Fig 15 (c) is referred to later

EXAMPLE 2 —It is required to draw the shear diagram for a beam 10 feet long, weighing 100 pounds per foot, supported at the ends,

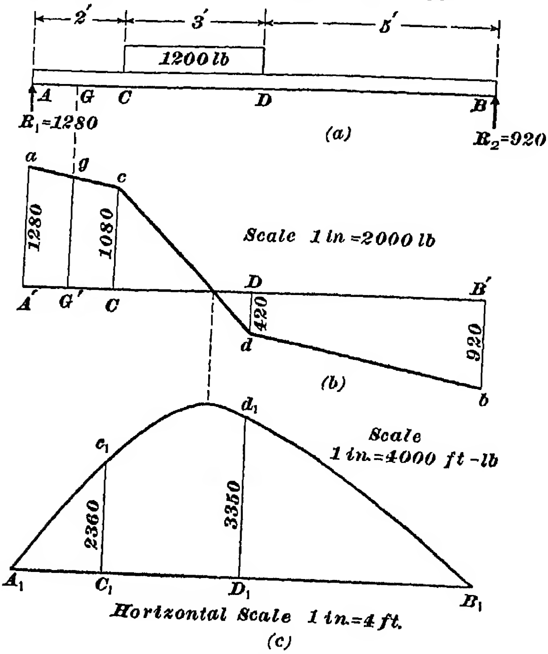


FIG 16

and sustaining a load of 1,200 pounds uniformly distributed over 3 feet of the beam, situated as shown in Fig 16 (a)

SOLUTION —To compute the reactions, the weight of the beam, which is 1,000 lb, is treated as a single load acting through the center of the beam The distributed load is treated as a single load concentrated at the center of gravity of the portion over which it is distributed, or 3 5 ft from A Applying the principles of moments to these two loads, the reactions are found to be as follows R_1 = 1,280 lb, R_2 = 920 lb

Draw the base line $A'B'$, as in the preceding example. The external shear at any section between A and C, distant x feet from A, is R_1 minus the weight of the beam between that section and A, that is, $R_1 - 100\,x$. We have, then, between A and C,

$$V = R_1 - 100\,x \qquad (1)$$

In the shear line, the shears V are ordinates, and the distances x are abscissas. As the equation (1) between V and x is of the first degree, it represents a straight line (see *Rudiments of Analytic Geometry*). To draw this line, we have for $x = 0$, $V = R_1$, and for $x = 2$ (section C),

$$V = R_1 - 100 \times 2 = 1{,}280 - 200 = 1{,}080$$

Project A and C on $A'B'$ at A' and C', respectively. Draw the ordinates $A'a$ and $C'c$, representing, to scale, 1,280 and 1,080 lb., respectively. Draw $a\,c$, which is the shear line for $A\,C$. The shear at any section G is represented by the corresponding ordinate $G'g$.

The shear at any point between C and D, distant x feet from C, is equal to R_1 minus the weight $100\,(2 + x)$ of the part of the beam between A and that section, minus the weight of the distributed load between C and that section, which is $\frac{1200}{3}\,x$, or $400\,x$. Therefore, between C and D,

$$V = R_1 - 100\,(2 + x) - 400\,x = R_1 - 200 - 500\,x = 1{,}080 - 500\,x$$

As this is an equation of the first degree, it represents a straight line. At C, $x = 0$, and $V = 1{,}080 = C'c$. At D, $x = 3$, and $V = 1{,}080 - 500 \times 3 = -420$. Project D on $A'B'$ at D', lay off the ordinate $D'd$ to represent -420 lb. (that is, draw it downwards), and draw $c\,d$, which is the shear line for CD.

Similarly, the shear line between D and B is a straight line passing through d, and, as the shear at B is $-R_2$, the other extremity b of the line is determined by drawing $B'b$ to represent $-R_2$, or -920 lb.

NOTE.—Fig. 16 (c) will be referred to further on.

EXAMPLES FOR PRACTICE

1. Draw the shear line for a beam loaded as in Fig. 17, neglecting the weight of the beam.

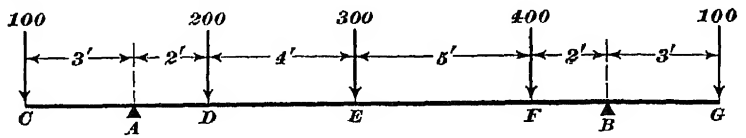


FIG. 17

2. Solve example 1, taking into account the weight of the beam, assuming it to be 50 pounds per foot.

BENDING MOMENT

43. Definition.—The **bending moment** at any section of a loaded beam is the algebraic sum of the moments of all the external forces (loads and reactions) to the right or left of the section about that section

As explained in *Analytic Statics*, Part 1, the moment of a force is considered positive or negative according as the force tends to produce clockwise or counter-clockwise rotation, respectively, about the origin of moments The value of the bending moment at any section, as computed from the forces on the right, is numerically equal to that obtained from the forces on the left, but has the opposite sign This follows from the fact that the forces acting on the beam are in equilibrium, and therefore the algebraic sum of their moments about any point is zero Therefore, if M_r is the bending moment at any section as computed from the forces on the right, and M_l is the moment as computed from the forces on the left, $M_r + M_l = 0$, and $M_l = -M_r$

As in the case of shears, it is customary to express the bending moment as computed from the forces on the left If, for convenience, the bending moment is computed from the forces on the right, its sign is changed, so that it will represent the moment of the forces on the left

As an illustration, the bending moment at the section A, Fig 14, is found as follows The forces on the left of A are W_1, R_1 and the weight $w x_1$ of the part of the beam lying on the left of A The lever arms of W_1, R_1, and $w x_1$ are, respectively, x_1, $x_1 - a$ and $\frac{x_1}{2}$, the last of which is the distance of the center of gravity of $A B$ from A The moment of R_1 is positive, while the other two moments are negative Therefore, the bending moment is

$$R_1(x_1 - a) - W_1 x_1 - w x_1 \times \frac{x_1}{2} = R_1(x_1 - a) - W_1 x_1 - \frac{w x_1^2}{2}$$

44. Notation —The letter M will be used to denote the bending moment at any section of a loaded beam, and the

letter with a subscript to denote the bending moment at a particular section, the subscript indicating the distance of the section from the left end of the beam Thus, M_2 denotes the bending moment at a section 2 feet from the left end

EXAMPLE 1 —To find the bending moment M at the section Y, Fig 15 (a), of the beam $A E$, the loads and reactions being as shown

SOLUTION —Taking moments about Y,

$M_{8\ 75} = 680 \times 8\ 75 - 500 \times 6\ 75 - 800 \times 3\ 75 = -425$ ft -lb Ans

EXAMPLE 2 —A simple beam $A B$, Fig 18, 15 feet long, and weigh-

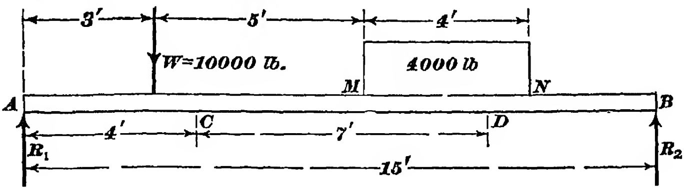


FIG 18

ing 250 pounds per foot, supports a single load of 10,000 pounds, and a uniform load of 4,000 pounds distributed over a length of 4 feet The loads being situated as shown, it is required to find the bending moments M_4 and M_{11}, at the sections C and D, respectively

SOLUTION —The forces acting at the left of the section C are R_1, W, and the weight of the part $A C$ of the beam R_1 is found by taking moments about the point B

$$R_1 \times 15 - 10{,}000 \times 12 - 4{,}000 \times 5 - \frac{250 \times 15^2}{2} = 0,$$

whence $R_1 = 11{,}208$ lb

Then, $M_4 = 11{,}208 \times 4 - 10{,}000 \times 1 - \frac{250 \times 4^2}{2} = 32{,}832$ ft -lb Ans

The forces acting at the left of D are R_1, W, the weight of the part $A D$ of the beam, and the weight of 3,000 lb , which is considered concentrated at its center of gravity, or $1\frac{1}{2}$ ft to the left of D Therefore,

$$M_{11} = 11{,}208 \times 11 - 10{,}000 \times 8 - \frac{250 \times 11^2}{2} - 3{,}000 \times 1\tfrac{1}{2}$$
$$= 23{,}663 \text{ ft -lb} \quad \text{Ans}$$

EXAMPLES FOR PRACTICE

1 A simple beam 24 feet long carries four concentrated loads of 160, 180, 240, and 120 pounds, at distances from the left support of 4, 10, 16, and 21 feet, respectively (a) What are the values of the reactions? (b) What is the bending moment, in inch-pounds, at the 180-pound load?

Ans $\begin{cases} (a)\ R_1 = 333\ 33 \text{ lb },\ R_2 = 366\ 67 \text{ lb} \\ (b)\ 28{,}480 \text{ in -lb} \end{cases}$

2 A simple beam carries a uniform load of 40 pounds per foot and supports two concentrated loads of 500 and 400 pounds at distances from the left support of 5 and 12 feet, respectively The length of the beam is 18 feet What are (*a*) the reactions, and (*b*) the bending moment at a section 8 feet from the left support?

Ans $\begin{cases} (a) \ R_1 = 854\ 44 \text{ lb} ,\ R_2 = 765\ 56 \text{ lb} \\ (b) \ 4{,}055\ 6 \text{ ft -lb} \end{cases}$

3 A beam that overhangs both supports equally carries a uniform load of 80 pounds per foot and has a load of 1,000 pounds in the middle, the length of the beam being 15 feet and the distance between the supports 8 feet What is the bending moment at a section 6 5 feet from the left end of the beam? Ans 1,610 ft -lb

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45. Moment Diagram and Moment Line—For solving some problems on beams, it is convenient to have a diagram representing the bending moments for all sections of the beam Such a diagram is called a **moment diagram.** It consists of (*a*) a base line or axis, equal by scale to the length of the beam on this axis are marked the points of application of the loads and reactions, and (*b*) another line, called the **moment line**, whose ordinates, measured from the base, represent the bending moments at the corresponding sections The moment diagram is constructed in the same general way as the shear diagram, except that the ordinates represent bending moments instead of external shears A purely graphic method of determining bending moment is given in *Graphic Statics*

EXAMPLE 1—To construct the moment diagram for the beam described in the first example of Art 42, and shown in Fig 15 (*a*)

SOLUTION—In the solution of the example in Art 42, it was shown that $R_1 = 680$ lb and $R_2 = 920$ lb First, the base $A_1 L_1$, Fig 15 (*c*), is laid off, as for the shear diagram For any section between A and C, distant x feet from A, the bending moment is $R_1 x$, that is,

$$M = R_1 x \qquad (1)$$

In the moment diagram, x represents abscissas and M ordinates Since equation (1) is of the first degree between M and x, it represents a straight line, and, since $M = 0$ when $x = 0$, that line passes through the origin A_1 At C, $x = 2$, and equation (1) gives

$$M_2 = R_1 \times 2 = 680 \times 2 = 1{,}360 \text{ ft -lb}$$

Projecting C on $A_1 E_1$ at C_1, drawing the ordinate $C_1 c_1$ equal, to any convenient scale, to 1,360 ft.-lb., and joining A_1 and c_1, $A_1 c_1$ is obtained as the moment line for AC. The bending moment at any section X, between A and C, is represented by the corresponding ordinate $X_1 x_1$.

For any section between C and D, distant x feet from C, the bending moment is $R_1(2 + x) - 500x$, that is,

$$M = R_1 \times 2 + (R_1 - 500)x = 1{,}360 + 180x$$

This equation, also, represents a straight line. When $x = 0$ (section C), $M = 1{,}360 = C_1 c_1$. When $x = 3$ (section D), $M = 1{,}900$. Projecting D at D_1, making $D_1 d_1 = 1{,}900$, and drawing $c_1 d_1$, the moment line for CD is obtained. The lines $d_1 b_1$ and $b_1 E_1$ are similarly determined. The lower ordinates indicate negative bending moments.

EXAMPLE 2.—To construct the moment diagram for the beam shown in Fig. 16 (*a*), whose weight is 100 pounds per foot.

SOLUTION.—The reactions were found in example 2 of Art. **42** to be $R_1 = 1{,}280$ lb., and $R_2 = 920$ lb. The base line $A_1 B_1$, Fig. 16 (*c*), is drawn as in previous examples. For any section between A and C, distant x from A, the bending moment is $R_1 x$ minus the moment of the weight of the beam between A and that section. That weight is $100x$, and its lever arm is $\frac{x}{2}$; its moment is, then, $100x \times \frac{x}{2} = 50x^2$. Therefore,

$$M = R_1 x - 50x^2 = 1{,}280x - 50x^2 \qquad (1)$$

When $x = 0$ (section A), $M = 0$, which shows that the moment line passes through A_1; when $x = 2$ (section C), $M = 2{,}360$. Projecting C at C_1, and making the ordinate $C_1 c_1 = 2{,}360$ ft.-lb., to any convenient scale, another point c_1 in the moment line is determined. Giving to x intermediate values, such as .5, 1, 1.5, projecting the corresponding points from AC on $A_1 C_1$, and erecting ordinates to represent the corresponding values of M as computed from equation (1), other points in the moment line are determined. Connecting these points by a smooth curve, the moment line for AC is obtained.

The remainder of the moment line is similarly constructed. An equation is first written for the moment at any section between C and D, distant x feet from C. From this equation, the curve $c_1 d_1$ is plotted. To plot the part $d_1 B_1$, an equation similar to (1) is used, using the right reaction, taking x as the distance from B, and changing the sign of the result, so that it will represent the moment computed from the forces on the left. Since, here, the moment of R_2 is negative, and that of wx positive, the resultant equation is

$$M = -(50x^2 - R_2 x) = R_2 x - 50x^2 = 920x - 50x^2 \qquad (2)$$

The distances x are laid off from B, the corresponding points are projected on $B_1 A_1$, and the ordinates are computed by equation (2). The details of the construction are left as an exercise for the student.

IMPORTANT SPECIAL CASES

46. Cantilever Supporting Load at End—Let AB, Fig 19, be a cantilever of length l, supporting a load W at its end Let X be any section at distance x from B, and let M_x be the bending moment at X The reaction at A is not readily determinable, it does not consist of a single force, since no single force passing through A can balance W, which does not pass through that point The reaction consists of a single force and a couple, as will be explained later In this case, therefore, it is simpler to determine the bending moment from the forces on the right of X The only force on the right of X is W, whose lever arm is x Therefore,

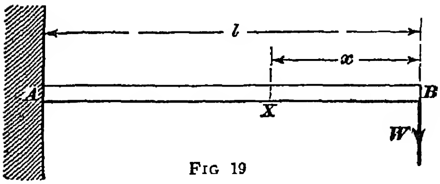


FIG 19

$$M_x = Wx \qquad (1)$$

It is evident that M_x, or Wx, is greatest when $x = l$, that is, the greatest bending moment occurs at A, and its value is given by the formula,

$$\max M = M_l = Wl \qquad (2)$$

A moment diagram could be drawn by plotting formula **1**, which is the equation of a straight line It should be noted, however, that, *in general, shear and moment diagrams are of value only in some special cases, and, as a rule, shears and bending moments are much more rapidly and accurately determined by analytic methods, that is, by general formulas, such as equation (1)*

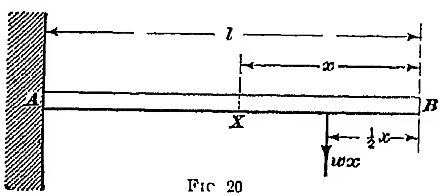


FIG 20

47. Cantilever Uniformly Loaded—Let the cantilever AB, Fig 20, carry a distributed load of w pounds per foot (lengths are supposed to be in feet) The notation being as in the preceding article, the moment at X is

determined from the forces on the right, which consist of the weight wx on XB. This weight may be treated as one force wx acting through the middle of XB, or at a distance $\frac{1}{2}x$ from B. The moment of this force about X is $wx \times \frac{1}{2}x = \frac{wx^2}{2}$. Therefore,

$$M_x = \frac{wx^2}{2} \qquad (1)$$

It is evident that M_x, or $\frac{wx^2}{2}$, is greatest when $x = l$ (section A), in which case

$$M_l = \frac{wl^2}{2} \qquad (2)$$

The total weight on the beam is wl. If this weight is denoted by W, formula 2 may be written

$$M_l = \frac{Wl}{2} \qquad (3)$$

48. Simple Beam Supporting One Concentrated Load.—Let the beam AB, Fig. 21, simply supported at A and B, carry a load W, at distance z from A. Let l be the length of the beam, and X any section, distant x from A, x being less than z. As usual, the bending moment at X will be denoted by M_x.

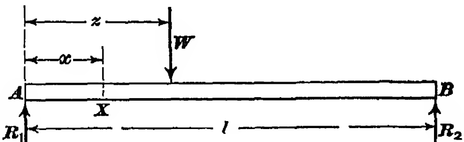


FIG. 21

Taking moments about B,

$$R_1 l - W(l - z) = 0,$$

whence

$$R_1 = W\frac{l-z}{l} \qquad (1)$$

Since the only force on the left of X is R_1, we have

$$M_x = R_1 x = W\frac{(l-z)x}{l} \qquad (2)$$

It is evident, from the last member of this formula, that, between A and W, M_x is greatest when x is greatest, that is, when $x = z$. Therefore, when a simple beam carries one concentrated load, the greatest bending moment occurs

under the load The maximum bending moment is, then,

$$M_z = W\frac{(l - z)z}{l} \qquad (3)$$

Suppose now, that, the load W being given, it is required to determine the distance z for which the moment is greatest Since the moment computed from R_1 is numerically equal to that computed from R_2, that is, $R_1 z = R_2 (l - z)$, the value of z that makes $R_1 z$ greatest must be the same as the value of $l - z$ that makes $R_2 (l - z)$ greatest Therefore, in this case,

$$l - z = z, \text{ whence } z = \frac{l}{2}$$

This shows that the greatest bending moment that a given load can produce in a simple beam occurs when the load is at the center of the beam Writing $\frac{l}{2}$ for z in formula **3**,

$$\max M = M_{\frac{l}{2}} = \frac{W l}{4} \qquad (4)$$

49. Simple Beam Uniformly Loaded —Let the beam AB, Fig 22, simply supported at A and B, carry a uniform load of w pounds per foot The total load W is $w\,l$, and, evidently,

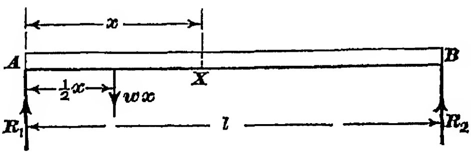


Fig 22

$$R_1 = R_2 = \frac{W}{2} = \frac{w\,l}{2} \qquad (1)$$

The bending moment at X is the moment of R_1 minus the moment of the weight $w\,x$ on AX The latter moment is $w\,x \times \frac{x}{2} = \frac{w\,x^2}{2}$ Therefore,

$$M_x = R_1 x - \frac{w\,x^2}{2} = \frac{w\,l\,x}{2} - \frac{w\,x^2}{2} = \frac{w\,x}{2}(l - x) \qquad (2)$$

Since the moment of the forces on the left is numerically equal to that of the forces on the right, the value of x that makes the former greatest is the same as the value of $l - x$ that makes the latter greatest, that is, when

$$x = l - x, \text{ or } x = \frac{l}{2}$$

This shows that the greatest moment occurs at the center of the beam Writing $\frac{l}{2}$ for x in formula **2**, we obtain,

$$\max M = M_{\frac{l}{2}} = \frac{w\,l^2}{8} = \frac{W\,l}{8} \qquad (3)$$

50. Beam Fixed at One End, Supported at the Other —The formulas for fixed or restrained beams will be given without their derivation, as they cannot be readily derived without the use of advanced mathematics

Let AB, Fig 23, be a beam of length l, fixed at A, resting on a support at B, and carrying a single load W at its center The reactions are as follows

$$R_1 = \tfrac{11}{16}\,W \qquad (1)$$

$$R_2 = \tfrac{5}{16}\,W \qquad (2)$$

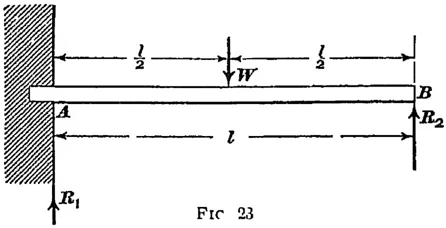


FIG 23

Here, and in the following article, R_1 really represents the shear at A, there being a couple acting at that section, in addition to R_1

The greatest bending moment occurs at A, and is given by the formula

$$M = \tfrac{3}{16}\,W\,l \qquad (3)$$

The bending moment at any other section can be computed from the reaction R_2 and the load

51. If, instead of a single load, the beam carries a uniformly distributed load of w per unit of length, the reactions are as follows

$$R_1 = \tfrac{5}{8}\,w\,l \qquad (1)$$

$$R = \tfrac{3}{8}\,w\,l \qquad (2)$$

The greatest bending moment occurs at A, and is given by the formula

$$M = \tfrac{1}{8}\,w\,l^2 \qquad (3)$$

The bending moment at any other section can be computed from the reaction R_2 and the load

52. Beam Fixed at Both Ends—Let the beam AB, Fig 24, be fixed at both ends and carry a load W in the middle The reactions are evidently each equal to $\frac{W}{2}$, that is,

$$R_1 = R_2 = \frac{W}{2} \qquad (1)$$

Here, and in the following article, R_1 and R_2 really represent the shears at A and B, there being, in addition, a couple at each of these sections The greatest bending moment occurs at A, B, and under the load, and is given by the formula

$$M = \frac{W l}{8} \qquad (2)$$

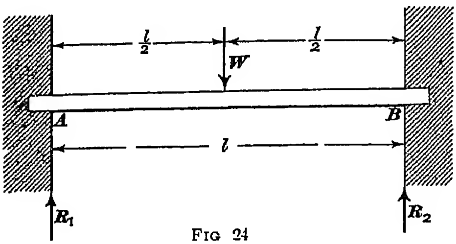


Fig 24

53. If, instead of a single load, the beam carries a uniform load of w per unit of length, then,

$$R_1 = R_2 = \frac{w l}{2} \qquad (1)$$

The greatest moment occurs at A and B, and is given by the formula

$$M = \frac{w l^2}{12} \qquad (2)$$

Example 1—A cantilever 25 feet long carries a load of 4 5 tons at its free end What is (*a*) the bending moment at a distance of 12 fee from the free end? (*b*) the maximum bending moment in the beam?

Solution—(*a*) To apply the formulas in Art 46, we have W = 4 5 T = 9,000 lb, x = 12, l = 25 Formula 1 of that article gives M_{12} = 9,000 × 12 = 108,000 ft-lb Ans

(*b*) Applying formula 2 of Art 46,

M_{25} = 9,000 × 25 = 225,000 ft-lb Ans

Example 2—Where must a load of 12,000 pounds be placed on a simple beam 20 feet long, that its bending moment shall be equal to the greatest bending moment that a weight of 6,000 pounds can produce in the beam?

Solution—Let the loads of 6,000 and 12,000 pounds be denoted by W_1 and W_2, respectively, and let z be the distance from the left (or right) support, at which W_2 causes the same bending moment as the maximum moment caused by W_1 This maximum moment occurs

when W_1 is placed at the center of the beam, and its value is by formula 4 of Art 48,

$$\frac{6,000 \times 20}{4} = 30,000 \text{ ft -lb}$$

The maximum moment caused by W', when this load is at distance z from the left support, is, by formula 3 of Art 48,

$$12,000 \times \frac{(20 - z) z}{20}$$

Equating this to the moment of W'_1,

$$12,000 \times \frac{(20 - z) z}{20} = 30,000$$

whence, transforming, reducing, and solving for z,

$$z = 17\ 1 \text{ or } 2\ 9 \text{ ft} \quad \text{Ans}$$

Therefore, W_1 may be at a distance of either 17 1 or 2 9 ft from the left support, or, what amounts to the same thing, at a distance of 2 9 ft from either support This is otherwise evident, since the bending moment is the same for any two positions of the load equidistant from the supports

EXAMPLE 3 —(*a*) What weight, uniformly distributed, will produce a maximum bending moment of 40,000 foot-pounds in a beam fixed at one end, simply supported at the other, and having a length of 20 feet? (*b*) What will be the reactions R_1 and R_2, Fig 23, for this load?

SOLUTION —(*a*) From formula 3 of Art 51,

$$w = \frac{8 M}{l^2}$$

Substituting the given values,

$$w = \frac{8 \times 40,000}{20^2} = 800 \text{ lb}$$

That is, the load is 800 lb per linear ft and the total load is $800 \times 20 = 16,000$ lb Ans

(*b*) Formulas 1 and 2 of Art 51 give

$$R_1 = \frac{5}{8} \times 16,000 = 10,000 \text{ lb} \quad \text{Ans}$$

$$R_2 = \frac{3}{8} \times 16,000 = 6,000 \text{ lb} \quad \text{Ans}$$

EXAMPLES FOR PRACTICE

1 What is the maximum bending moment in a cantilever carrying a uniformly distributed load of 180 pounds per foot, the length of the beam being 20 feet? Ans 36,000 ft -lb

2 A simple beam 24 feet long carries a load of 15,000 pounds at a distance of 8 feet from the left support What is the value of (*a*) the left reaction? (*b*) the maximum bending moment?

$$\text{Ans} \begin{cases} (a)\ 10,000 \text{ lb} \\ (b)\ 80,000 \text{ ft -lb} \end{cases}$$

3 A beam 30 feet long and fixed at both ends carries a uniformly distributed load of 400 pounds per foot Calculate (*a*) the reactions, (*b*) the maximum bending moment

Ans $\begin{cases} (a) & 6{,}000 \text{ lb} \\ (b) & 30{,}000 \text{ ft -lb} \end{cases}$

MAXIMUM SHEAR AND BENDING MOMENT

54. Fundamental Principles —The two following principles, which are established by the use of advanced mathematics, are of great importance

1 *The external shear in a beam is greatest at a section of the beam adjacent to one of the supports*

2 *The bending moment is greatest at a section of the beam where the shear changes sign*

In some cases, the shear changes sign at more than one section [see Fig 15 (*b*)], corresponding to each of these sections, there is a "peak" in the bending-moment line, as shown in Fig 15 (*c*) The greatest of the bending moments at these sections is the greatest bending moment in the beam These principles should be carefully tested in all the shear and bending-moment curves given in the foregoing articles

The *dangerous section* of a beam is usually where either the shear or the bending moment is greatest In many problems relating to beams, it is necessary to find those sections, and for that purpose the principles just stated are a great help

EXAMPLE —To determine the greatest shear and bending moment in the beam *A B*, Fig 25, simply supported at *A* and *B* and carrying

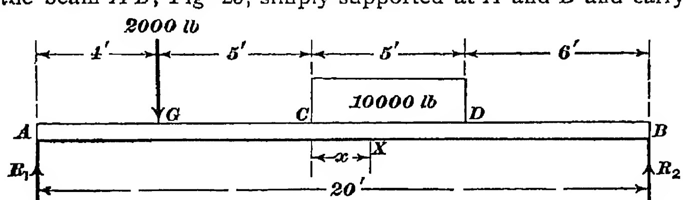


FIG 25

a single load of 2 000 pounds and a uniform load of 10,000 pounds distributed over a length of 5 feet, besides its own weight, which is 75 pounds per foot, the dimensions being as shown

SOLUTION —This problem may be solved by drawing the shear diagram, determining the sections at which the shear passes from positive to negative, computing the bending moments at those sections, and taking the greatest The analytic solution, however, is usually shorter

The weight of the beam is $75 \times 20 = 1{,}500$ lb, the lever arm of this weight is $20 - 2 = 10$ ft The center of the distributed load is $\frac{5}{2} + 6 = 8.5$ ft from B Taking moments about B,

$$R_1 \times 20 = 2{,}000 \times 16 + 1{,}500 \times 10 + 10{,}000 \times 8.5,$$

whence $$R_1 = 6{,}600 \text{ lb}$$

Also, $$R_2 = 2{,}000 + 1{,}500 + 10{,}000 - R_1 = 6{,}900 \text{ lb}$$

The greatest shear occurs just on the left of B, and is equal to R_2, or 6,900 lb Immediately on the left of G, the shear is

$$V_1' = R_1 - 75 \times 4 = 6{,}300 \text{ lb}$$

Immediately on the right of G, the shear is

$$V_1'' = V_1' - 2{,}000 = 4{,}300 \text{ lb}$$

At C,

$$V_0 = V_1'' - 75 \times 5 = 4{,}300 - 375 = 3{,}925 \text{ lb}$$

It is seen, by a simple inspection of the value of V_0 and the load between C and D, that the shear at D is negative Therefore, the shear changes sign at some point between C and D, and, as between these limits the shear decreases gradually from plus to minus, there must be a section at which it is equal to zero Let the distance of that section from C be denoted by x The expression for the shear at that section is

$$V_0 - 75 \times x - \frac{10{,}000}{5} \times x = 3{,}925 - 2{,}075\,x$$

Making this expression equal to zero, and solving for x,

$$x = \frac{3{,}925}{2{,}075} = 1.89 \text{ ft, nearly}$$

This determines the section X at which the bending moment is a maximum The value of the moment is more readily determined from the forces on the right of X, and changing the sign, so that the moment will be expressed in terms of the forces on the left (see Art 43) The value of the moment is, then,

$$-\left[-R_2 \times BX + (75 \times BX) \times \frac{BX}{2} + \left(\frac{10{,}000}{5} \times DX\right) \times \frac{DX}{2}\right]$$

$$= R_2 \times BX - \frac{75 \times \overline{BX}^2}{2} - 1{,}000 \times \overline{DX}^2$$

$$= 6{,}900 \times 9.11 - \frac{75 \times 9.11^2}{2} - 1{,}000 \times 3.11^2 = 50{,}075 \text{ ft-lb} \quad \text{Ans}$$

55. Important Rule —When, as in the preceding example, the shear changes sign between two sections including a uniformly distributed load, there must be a section, between those two, at which the shear must be zero.

The location of that section is determined as in the example just referred to. When, however, the shear has one sign immediately on one side of a single concentrated load, and the opposite sign immediately on the other side, that load marks the section at which the shear changes sign, and for which the bending moment may be greatest. (It is greatest, if there is only one such change, thus, if in Fig. 25, V_4' and V_4'' had different signs, the section of maximum bending moment would be at G, where the single load is applied.)

STRENGTH OF MATERIALS

(PART 2)

BEAMS—(Continued)

MOMENT OF INERTIA AND RADIUS OF GYRATION

INTRODUCTORY

1. The strength and stiffness of a beam, column, or shaft depend on various factors For instance, the load that a beam can bear depends on the material of the beam, on the manner in which the load is applied, and on the length and cross-section of the beam As to the area of the cross-section, the strength does not depend on that area itself, for, as every-day experience shows, a plank used as a beam will sustain a greater load when placed edgewise than when placed on its broad side It will be shown later that the strength of a beam depends on the manner in which the area is distributed or disposed with respect to a certain line of the cross-section The effect of the cross-section is measured by a quantity that depends on such disposition or distribution of area, and is called the **moment of inertia** of the cross-section with respect to the line mentioned above It should be understood at the outset that the moment of inertia of a plane figure has really nothing to do with inertia There is a certain quantity that appears in formulas relating to the rotation of a body and is called the moment of inertia of the body There is, likewise, a certain quantity that appears in the formulas for strength

and stiffness of beams, columns, and shafts, and whose mathematical expression is very similar in form to that of the moment of inertia of a rotating solid hence the name

MOMENT OF INERTIA

2. Definitions.—Let $B\,C$, Fig 1, be any plane area, and $X'\,X$ a line or axis in its plane Let the area be divided into small areas a_1, a_2, a_3, etc , distant y_1, y_2, y_3, etc from $X'\,X$ If each small area is multiplied by the square of its distance from the axis, a sum of the form $a_1 y_1^2 + a_2 y_2^2 + a_3 y_3^2 + a_4 y_4^2$ will be obtained Using the sigma notation, explained in *Plane Trigonometry*, Part 2, this sum may be represented by $\Sigma\,a\,y^2$ When the whole area is thus divided, giving to the small areas any values whatever, a certain value is found for $\Sigma\,a\,y^2$ If, now, the small areas are subdivided into smaller areas, a different value will be found for $\Sigma\,a\,y^2$ If the small areas are again subdivided, a new value will be found for $\Sigma\,a\,y^2$ As the areas are made smaller and smaller, the values of $\Sigma\,a\,y^2$ approach nearer and nearer a fixed value depending on the form of the figure $B\,C$ and on the location of the axis $X'\,X$ This fixed value is called the **rectangular moment of inertia** of the area $B\,C$ with respect to the axis $X'\,X$

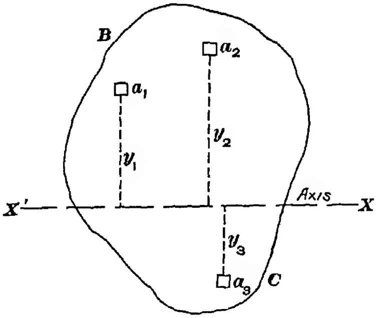


Fig 1

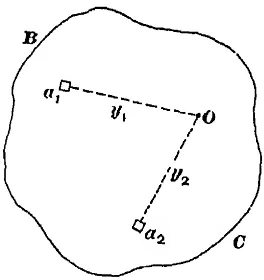


Fig 2

3. If the distances of the small areas a_1, a_2 etc , Fig 2, from a line (represented in the figure by its plan O) perpendicular to the plane of the area $B\,C$ are used, and the same operations as were described before are performed, it will be found that, as the small areas are

decreased in size, the sum $\Sigma a y^2$ approaches nearer and nearer a fixed quantity, which is called the **polar moment of inertia** of the area $B C$ with respect to the axis O

When the term *moment of inertia* is used with reference to a plane area, the rectangular moment is generally meant

4. Formulas for the moments of inertia of plane areas cannot be derived without the use of advanced mathematics A clear idea of the character of this quantity, however, may be obtained from the following illustration

Let $B C$, Fig 3, be a square whose side is d, and let $X' X$, coinciding with one of the sides of the square, be an axis about which the moment of inertia is required Let the square be divided into twenty-five equal squares, as shown The side and area of each of these small squares are, respectively, $2 d$ and $04 d^2$ For each small square, the value of y will be taken as the distance of the center of the square from the axis This distance is $1 d$ for the squares in the lower tier, $3 d$ for those in the next tier, etc The product, area × square of distance, for each little square in the lower tier is

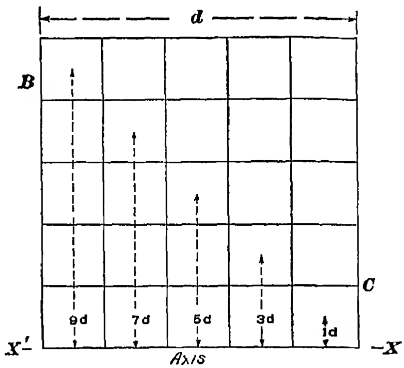


Fig 3

$$04 d^2 \times (1 d)^2 = 0004 d^4$$

For each of the squares in the successive tiers, the products are, respectively,

$$04 d^2 \times (3 d)^2 = 0036 d^4$$
$$04 d^2 \times (5 d)^2 = 0100 d^4$$
$$04 d^2 \times (7 d)^2 = 0196 d^4$$
$$04 d^2 \times (9 d)^2 = 0324 d^4$$

Since there are five squares in each tier, the sum of the products for all the little squares is,

$$\Sigma a y^2 = 5(0004 + 0036 + 0100 + 0196 + 0324) d^4 = 33 d^4$$

Now, $33\,d^4$ is not the exact value of the moment of inertia sought, but is an approximate value. A closer value will be obtained by dividing the large square into a greater number of smaller squares. If computations like the above are made for other divisions of the large square, the following results are obtained.

When the side of the small squares is:

$$\begin{aligned}
\tfrac{1}{5}d,\ \Sigma a y^2 &= .3300\,d^4\\
\tfrac{1}{6}d,\ \Sigma a y^2 &= .3310\,d^4\\
\tfrac{1}{7}d,\ \Sigma a y^2 &= .3316\,d^4\\
\tfrac{1}{8}d,\ \Sigma a y^2 &= .3320\,d^4\\
\tfrac{1}{9}d,\ \Sigma a y^2 &= .3323\,d^4\\
\tfrac{1}{10}d,\ \Sigma a y^2 &= .3325\,d^4
\end{aligned}$$

etc.

From this series of results, it is seen that the differences between successive values of $\Sigma a y^2$ become continually smaller down the column, and, although the value of $\Sigma a y^2$ increases, the increments become smaller and smaller. It is shown by means of the calculus that, however great the number of small squares may be, $\Sigma a y^2$ can never become as great as $\frac{d^4}{3}$, although, by making the number of squares sufficiently large, the value of $\Sigma a y^2$ can be made to differ from $\frac{d^4}{3}$ by as little as desired. This is expressed in mathematical language by saying that, as the number of small squares increases, *$\Sigma a y^2$ approaches $\frac{d^4}{3}$ as a limit*. This limit $\frac{d^4}{3}$ is the moment of inertia of the square with respect to the axis $X'X$.

5. Moment of Inertia of Common Areas.—Table I contains, in the column marked I_0, the rectangular moments of inertia of the figures commonly used in practice. The axis in each case passes through the center of gravity, and is designated *neutral axis* in the table for reasons that will be explained further on. The square may be regarded as a particular case of a rectangle whose base b and altitude d

TABLE I

MOMENTS OF INERTIA AND RADII OF GYRATION

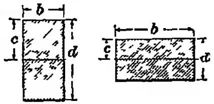
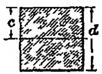
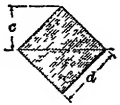
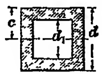
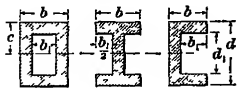
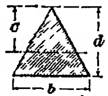
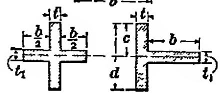
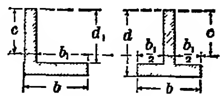
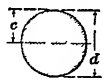
Form of Section	Dotted Line Shows Position of Neutral Axis	A	I_0	c	r_0
1 Rectangle		bd	$\frac{1}{12}bd^3$	$\frac{1}{2}d$	$\frac{d}{6}\sqrt{3}$
2 Square		d^2	$\frac{1}{12}d^4$	$\frac{1}{2}d$	$\frac{d}{6}\sqrt{3}$
3 Square		d^2	$\frac{1}{12}d^4$	$707d$	$\frac{d}{6}\sqrt{3}$
4 Hollow Square		$d^2 - d_1^2$	$\frac{1}{12}(d^4 - d_1^4)$	$\frac{1}{2}d$	$\frac{1}{6}\sqrt{3(d^2 + d_1^2)}$
5 Hollow Rect-angle, I and Channel		$bd - b_1d_1$	$\frac{1}{12}(bd^3 - b_1d_1^3)$	$\frac{1}{2}d$	

TABLE I—(Continued)

Form of Section	Dotted Line Shows Position of Neutral Axis	A	I_0	c	r_0
6 Triangle		$\frac{1}{2}bd$	$\frac{1}{36}bd^3$	$\frac{2}{3}d$	$\frac{d}{6}\sqrt{2}$
7 Cross and T		$td + t_1b$	$\frac{1}{12}(td^3 + bt_1^3)$	$\frac{1}{2}d$	
8 Angle and T		$bd - b_1d_1$	$\frac{(bd^2 - b_1d_1^2)^2 - 4bdb_1d_1(d - d_1)^2}{12(bd - b_1d_1)}$	$\frac{d}{2} + \frac{b_1d_1}{2}\left(\frac{d - d_1}{bd - b_1d_1}\right)$	
9 Circle		$\frac{\pi}{4}d^2$	$\frac{\pi d^4}{64}$	$\frac{1}{2}d$	
10 Circular Ring		$\frac{\pi}{4}(d^2 - d_1^2)$	$\frac{\pi(d^4 - d_1^4)}{64}$	$\frac{1}{2}d$	$\frac{1}{4}\sqrt{d^2 + d_1^2}$

are equal The value of the area A is given in the third column The distance c, given in the fifth column, is the distance of the most remote part of the figure from the neutral axis The radius of gyration, the character of which will be explained in a subsequent article, is given in the last column For convenience, a line passing through the center of gravity of a figure will be here called a **central line** or **central axis**

6. Reduction Formula—Let A denote the area of any figure, I_0 its moment of inertia with respect to an axis through its center of gravity, I its moment of inertia with respect to any other axis parallel to the former, and p the distance between the axes It can be proved that

$$I = I_0 + A p^2$$

That is, *the moment of inertia of a figure with respect to any axis equals the moment of inertia of the figure with respect to a parallel central axis plus the product of the area and the square of the distance between the two axes*

EXAMPLE 1—To determine the moment of inertia of a triangle with respect to its base See No 6 in Table I

SOLUTION—The distance between the base and the central line parallel to the base is $\frac{1}{3} d$, the area is $\frac{1}{2} b\,d$, and $I_0 = \frac{1}{36} b\,d^3$ Then, by the preceding principle or formula,

$$I = \tfrac{1}{36} b\,d^3 + \tfrac{1}{2} b\,d \times (\tfrac{1}{3} d)^2 = \frac{b\,d^3}{36} + \frac{b\,d^3}{18} = \frac{b\,d^3}{12} \quad \text{Ans}$$

EXAMPLE 2—To find the moment of inertia of a rectangle about its side d (No 1 in Table I)

SOLUTION—From Table I, the moment of inertia I_0 about a central axis parallel to d is $\frac{1}{12} d\,b^3$, the letters b and d being interchanged, as here the axis is parallel to d, not to b Also, $A = b\,d$, and $p = \dfrac{b}{2}$ Therefore,

$$I = \frac{d\,b^3}{12} + b\,d\left(\frac{b}{2}\right)^2 = \frac{d\,b^3}{12} + \frac{d\,b^3}{4} = \frac{d\,b^3}{3} \quad \text{Ans}$$

EXAMPLES FOR PRACTICE

1 Find the moment of inertia of a square about an axis through one corner and parallel to the diagonal that does not pass through that corner (see No 3 in Table I) Ans $I = \frac{7}{12} d^4$

2 Find the moment of inertia of a circle about a tangent Ans $I = \frac{5}{64} \pi\,d^4$

7. Least Moment of Inertia.—*The moment of inertia of a figure with respect to a central axis is less than with respect to any other line parallel to that axis* For, according to the principle or formula of the preceding article, the moment of the figure with respect to the parallel line is equal to the moment of inertia with respect to the central line plus a positive quantity

8. Principal Axes—The moments of inertia with respect to different central axes are, in general, different, and, in general, there is one central axis for which the moment of inertia is less than that for any other, and one central axis for which the moment of inertia is greater than for any other central axis These two lines are at right angles to each other, and are called **principal axes.**

The general method of finding the principal axes of a figure is comparatively complicated Many plane figures used in engineering have one or more axes of symmetry, and it is a general principle that *every axis of symmetry is a principal axis, the other principal axis being a central line perpendicular to the axis of symmetry* Thus, for a rectangle, the principal axes pass through the center of the figure and are parallel to the sides

9. Moment of Inertia of Compound Figures or Areas—Many figures may be regarded as consisting of simpler parts, they are called **compound figures** For example, a hollow square consists of a large square and a smaller square, the angle and **T** shown under No 8 in Table I consist of two slender rectangles, one horizontal and one vertical

The moment of inertia of a compound figure with respect to any axis may be found by adding, algebraically, the moments of inertia, with respect to the same axis, of the component parts of the figure

EXAMPLE 1—To derive the value of I_0 given for the hollow square, No 4 in Table I

SOLUTION—The figure may be regarded as the difference between the large outside square and the small inside square In Table I,

the side of the squares are, respectively, d and d_1. According to No. 2 in Table I, the moments of inertia of the squares are $\frac{1}{12}d^4$ and $\frac{1}{12}d_1^4$, hence, the moment of inertia of the hollow square is

$$I_0 = \tfrac{1}{12}d^4 - \tfrac{1}{12}d_1^4 = \tfrac{1}{12}(d^4 - d_1^4)$$

EXAMPLE 2.—To derive the value of I_0 given for the **T** in Table I.

SOLUTION.—The figure may be regarded as consisting of two rectangles, as shown in Fig. 4. The axis passes through the center of gravity of each rectangle, and is parallel to the base of each. Hence, according to No. 1 in Table I, the moment of inertia of the vertical rectangle is $\frac{1}{12}t\,d^3$, and the moment of inertia of the other rectangle is $\frac{1}{12}b\,t_1^3$. The moment of inertia of the entire figure is the sum of these, that is,

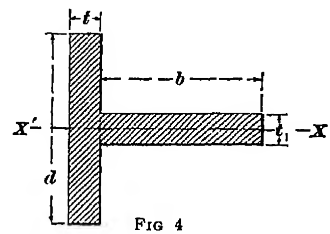


FIG. 4

$$I_0 = \tfrac{1}{12}t\,d^3 + \tfrac{1}{12}b\,t_1^3 = \tfrac{1}{12}(t\,d^3 + b\,t_1^3)$$

EXAMPLE 3.—To find the moment of inertia of the **Z** bar shown in Fig. 5, about the principal axis $X'X$, the dimensions being as shown.

SOLUTION.—The moment of inertia about $X'X$ will be denoted by I_x. The figure may be divided into the three rectangles $efgh$, $e'f'g'h'$, and $jgig'$. The moment of inertia of $efgh$ about an axis through its center of gravity and parallel to $X'X$ is $\frac{1}{12}a'\,t^3$, that of $e'f'g'h'$ about an axis through its center of gravity and parallel to $X'X$ is also $\frac{1}{12}a'\,t^3$. The distance between this axis and the axis $X'X$ is $\frac{1}{2}(b-t)$. The moment of inertia of the rectangle $efgh$ and also

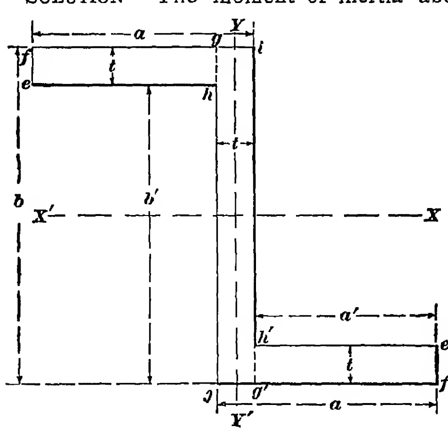


FIG. 5

of the rectangle $e'f'g'h'$ about the axis $X'X$ is then, $\frac{1}{12}a'\,t^3 + a'\,t\left[\frac{1}{2}(b-t)\right]^2$. The moment of inertia of the rectangle $jgig'$ is $\frac{1}{12}t\,b^3$. The moment of inertia of the entire figure is, therefore,

$$2\left\{\tfrac{1}{12}a'\,t^3 + a'\,t\left[\tfrac{1}{2}(b-t)\right]^2\right\} + \tfrac{1}{12}t\,b^3$$

Expanding and reducing this expression,

$$I_x = \frac{a\,b^3 - a'(b-2t)^3}{12} \quad \text{Ans.}$$

EXAMPLE FOR PRACTICE

Find an expression for the moment of inertia I_y of the **Z** bar shown in Fig 5, about the other principal axis $Y'Y$

Ans $I_y = \frac{t}{12}[2a'^3 + 6a'a^2 + bt^2]$

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10. Units Used —As will be seen from the formulas in Table I, the value of the moment of inertia always involves the product of four quantities (some of which may be equal) representing lengths Thus, the moment of inertia of a rectangle involves the product bd^3, or $b \times d \times d \times d$ When the value of a moment of inertia is given, it is necessary to specify the unit of length used, since such expressions as bd^3, d^4, etc evidently have different values according to whether b and d are expressed in inches, feet, meters, etc Generally, the dimensions of figures for which moments of inertia are computed are given in inches Some writers state the corresponding moment of inertia as so many *biquadratic inches*, for which they use the abbreviation in^4 Thus, if, when the dimensions of a figure are expressed in inches, the moment of inertia is 54, these writers call this moment 54 biquadratic inches, and write it $54\ in^4$ This notation is convenient, and will often be used in this Course Another way of expressing the same thing is to state the moment thus *54, referred to the inch*

11. Polar Moment of Inertia —For the definition of the polar moment of inertia, see Art **3.** The axis with respect to which a polar moment is taken will here be called **a polar axis.** The polar moment of inertia is computed by means of the following general principle

The polar moment of inertia of any plane figure is equal to the sum of the rectangular moments of inertia of the figure about two axes perpendicular to each other at the point where the polar axis meets the plane of the figure

Thus, the polar moment of inertia of a rectangle about an axis through its center of gravity is

$$\tfrac{1}{12} b d^3 + \tfrac{1}{12} d b^3 = \frac{bd}{12}(b^2 + d^2)$$

The polar moment of inertia of a circle about an axis through its center is

$$\frac{\pi d^4}{64} + \frac{\pi d^4}{64} = \frac{\pi d^4}{32}$$

RADIUS OF GYRATION

12. Definition.—The **radius of gyration** of a plane figure with respect to an axis is a quantity whose square multiplied by the area of the figure is equal to the moment of inertia of the figure with respect to the same axis. If r and I denote, respectively, the radius of gyration and moment of inertia of a figure, and A the area, then,

$$A r^2 = I \qquad (1)$$

whence

$$r = \sqrt{\frac{I}{A}} \qquad (2)$$

The last column of Table I gives radii of gyration corresponding to the moments of inertia given in the fourth column.

13. Computation of Radius of Gyration.—Usually, the radius of gyration of a figure is found directly from its moment of inertia by means of formula **2** of the preceding article. For example, the radii of gyration for Nos. 1 and 4, Table I, are found thus:

For No. 1, $\quad r = \sqrt{\tfrac{1}{12} b d^3 \div b d} = \frac{d}{6}\sqrt{3}$

For No. 4, $r = \sqrt{\tfrac{1}{12}(d^4 - d_1^4) \div (d^2 - d_1^2)} = \tfrac{1}{6}\sqrt{3(d^2 + d_1^2)}$

14. Reduction Formula.—Dividing both sides of the formula in Art. **6** by A, there results,

$$\frac{I}{A} = \frac{I_0}{A} + p^2$$

Now, $\frac{I_0}{A}$ is the square of the radius of gyration of the figure with respect to a central axis, and $\frac{I}{A}$ is the square of the radius of gyration with respect to an axis parallel to and distant p from that axis. Denoting these radii by r and r_0, respectively, the preceding equation becomes

$$r^2 = r_0^2 + p^2$$

That is, *the square of the radius of gyration of a figure with respect to any axis equals the square of the radius of gyration of the figure with respect to a parallel central axis plus the square of the distance between the two axes*

EXAMPLE —What is the radius of gyration of a square with respect to an axis coinciding with a side?

SOLUTION —Call the desired radius of gyration r From Table I, $r_0^2 = \frac{d^2}{12}$, also, $p = \frac{d}{2}$ Then, by the above formula,

$$r^2 = \frac{d^2}{12} + \frac{d^2}{4} = \frac{d^2}{3}, \text{ and } r = \sqrt{\frac{d^2}{3}} = \frac{d}{3}\sqrt{3} \quad \text{Ans}$$

STRESSES IN A BEAM

15. Stresses at Any Cross-Section.—Let BD, Fig. 6 (a), be a part of a beam cut by a plane PQ

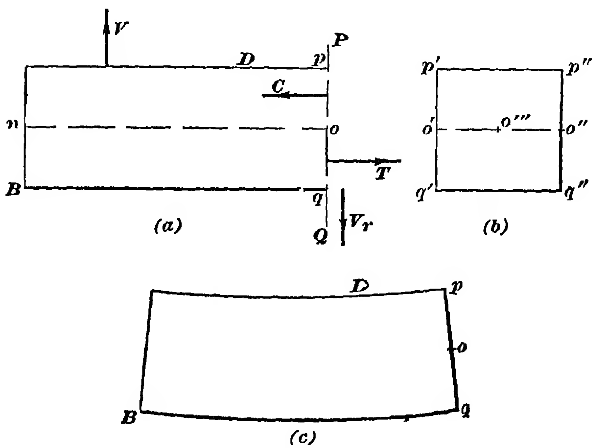


FIG 6

According to the principles of statics, the part BD may be treated as a free body kept in equilibrium by the external forces actually applied to the beam on the left of PQ, and by forces acting in the section pq, equal to the forces exerted on BD by the part of the beam on the right of PQ These last forces are the stresses existing in the beam at the section PQ Let V be the external shear at pq, that is,

the resultant of the vertical forces acting on the left of pq It is obvious that the external forces have a tendency to bend the beam, which will assume such a form as is represented, very much exaggerated, in Fig 6 (c) By this bending, the fibers (see Art 21) near q will be stretched, and those near p will be compressed Evidently, there will be a line between q and p where the fibers are neither stretched nor compressed This line, represented in Fig 6 (a) and (c) by o, and in (b) by $o' o''$, is called the **neutral axis** of the section pq, or $p' q' q'' p''$, and a plane through it and the axis of the beam is called the **neutral surface** of the beam The neutral surface is represented in Fig 6 (a) by no

In the present case, the fibers below the neutral surface are in tension, those above the neutral surface are in compression Let the resultants of these tensions and compressions be denoted by T and C, respectively, as shown in Fig 6 (a) These forces, representing normal stresses, are parallel to the axis no of the beam

16. If the end B, Fig 6 (a), is simply supported, the reaction at that end is vertical, and V represents the resultant of all the forces, including that reaction, acting on the left of the section pq This is true whether the other end of the beam is fixed or is simply supported If the end B is the free end of a cantilever, V is likewise the resultant of all the vertical forces on the left of pq In either case, the part BD of the beam is held in equilibrium by the vertical force V and the forces acting in the section pq According to the principles of statics, the algebraic sum of all the vertical forces acting on BD must be equal to zero, therefore, there must be at pq a vertical force V_r numerically equal to V, but acting in the opposite direction This force, being a tangential force exerted on BD by the part of the beam on the right of PQ, represents a shearing stress It is thus seen that there is, at every section of the beam, a shearing stress numerically equal to the external shear on the left of that section This shearing stress is called the **resisting shear.**

17. Since the only horizontal forces acting on the body BD are T and C, their algebraic sum must be equal to zero That is, the resultant tension in any section is equal to the resultant compression The tension T and the compression C form a couple, sometimes called the **stress couple.** It should be understood that T and C are resultant forces the tension and the compression are not uniformly distributed over op and oq, respectively, that is, their intensities are not the same at all points of the areas in which they act How they are distributed will be explained further on

18. Finally, the conditions of equilibrium require that the algebraic sum of the moments, about any axis, of all the forces acting on the body BD shall be zero If moments are taken about the neutral axis o, the moment of the forces on the left of pq will be the bending moment M at the section pq, the moment of the forces acting in the section pq is simply the moment of the stress couple, since the moment of V_r is zero The moment of the stress couple is called the **resisting moment** at the section pq Therefore, the resisting moment at any section is numerically equal to the bending moment at that section

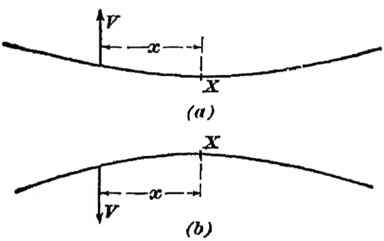


FIG 7

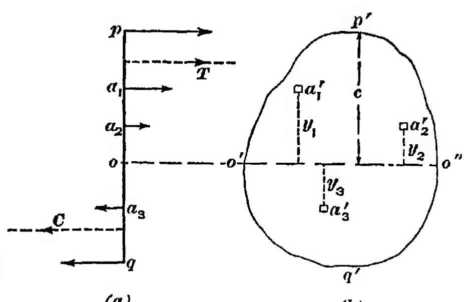
It is evident that, if the bending moment at a section, as computed from the forces on the left, is positive, the beam at that section bends downwards, or is concave upwards, as shown in Fig 7 (*a*) If the bending moment is negative, the beam bends upwards, or is concave downwards, as shown in Fig 7 (*b*) In each of these figures, the numerical value of the bending moment is Vx

19. The foregoing conditions, which follow directly from the fundamental principles of statics, are made to apply, by the use of certain assumptions that are very nearly

true, to beams fixed at both ends. A complete theory of such beams is, however, beyond the scope of this work.

20. Position of the Neutral Axis.—It can be shown by the use of advanced mathematics that *the neutral axis of any section of a beam passes through the center of gravity of the section*. Thus, in Fig. 6 (*b*), the neutral axis $o'o''$ passes through the center of gravity o''' of the cross-section $p'q'q''p''$, in Fig. 6 (*a*), the cross-section $p'q'q''p''$, and its center of gravity o''', are represented by their projections pq and o, respectively.

21. Distribution of the Normal Forces.—Let pq, Fig. 8 (*a*), be a cross-section of a beam, a side view is shown at $p'o'q'o''$, Fig. 8 (*b*). The neutral axis is o–$o'o''$. The material of a beam is imagined as consisting of threads, or **fibers**, parallel to the axis. The fiber passing through p, which is the farthest point from the neutral axis, is called the **extreme fiber**, or **most remote fiber.** The distance of this fiber from the neutral axis is denoted by c, as shown in Fig. 8 (*b*). Values of c for different forms of cross-section are given in the fifth column of Table I. The intensity of stress in any fiber is called **fiber stress.**



(*a*) FIG. 8 (*b*)

As already explained, the tension T and the compression C, which are the normal stresses at the section pq, are the resultants of non-uniform stresses. From the results of experiments, it is assumed that *the intensity of tension or compression at any point of the cross-section is proportional to the distance of that point from the neutral axis*, and that *the intensity of tension and compression at points equidistant from the neutral axis are equal*. It follows that the greatest intensity of stress occurs in the most remote fiber. Let that

intensity be denoted by f Then, since the distance of that fiber from the neutral axis is c, the intensity at unit's distance from the neutral axis is $\frac{f}{c}$, and the intensity at any distance y from the neutral axis is $\frac{f}{c}y$

22. Moment of Stress Couple, or Resisting Moment —The resisting moment, or the algebraic sum of the moments of T and C, Fig 8, about the neutral axis, will be denoted by M_r, and is determined as follows

Let a_1 be any small area distant y_1 from the axis, as shown in Fig 8 (b) According to the principle of the preceding article, the intensity of stress at the distance y_1 from the neutral axis is $\frac{f}{c}y_1$ Therefore, the total stress in a_1 is $a_1 \times \frac{f}{c}y_1$ The moment of this stress about $o' o''$ is

$$\left(a_1 \times \frac{f}{c}y_1\right) \times y_1 = \frac{f}{c} \times a_1 y_1^2$$

Similarly, the moments of such other small areas as a_2 and a_3 are

$$\frac{f}{c} \times a_2 y_2^2, \ \frac{f}{c} \times a_3 y_3^2, \text{ etc}$$

The sum of these moments is

$$\frac{f}{c}\left(a_1 y_1^2 + a_2 y_2^2 + a_3 y_3^2 + \quad . \quad . \quad . \quad\right) = \frac{f}{c} \Sigma a y^2$$

denoting by $\Sigma a y^2$ the sum in the parenthesis It has been explained that, by increasing the number of small areas a, and making them smaller and smaller, the value of $\Sigma a y^2$ finally becomes the moment of inertia I of the cross-section about the neutral axis Therefore, finally,

$$M_r = \frac{f}{c} I \qquad (1)$$

As explained in Art 18, the resisting moment is numerically equal to the bending moment M, therefore,

$$\frac{f}{c} I = M \qquad (2)$$

23. Section Modulus —The section modulus of a cross-section of a beam is the quotient obtained by dividing

the moment of inertia of the cross-section with respect to its neutral axis by the distance of the most remote fiber of the section from that axis It will be denoted by Q Then,

$$Q = \frac{I}{c} \qquad (1)$$

Since I is expressed in biquadratic units and c in length units, Q is in cubic units By this is meant that Q is expressed in terms of the product of three lengths, although the value of this quantity has nothing to do with volume Thus, if the dimensions of the cross-section are expressed in inches, and the numerical value of the section modulus is 28, it will be referred to as 28 cubic inches, and expressed as 28 in ^{3}

Formula 2 of the preceding article may now be written

$$fQ = M \qquad (2)$$

This formula is useful when a table of values of Q is available Such tables are given in many structural handbooks

EXAMPLE —What is the section modulus of a beam in which the cross-section is a rectangle, whose breadth is b and depth is d?

SOLUTION —From No 1 in Table I, $I = \frac{1}{12} b d^3$, and, since the neutral axis passes through the center of gravity of the cross-section, $c = \frac{d}{2}$, hence, by formula 1,

$$Q = \frac{\frac{1}{12} b d^3}{\frac{d}{2}} = \frac{b d^2}{6}$$

If $b = 2$ in and $d = 12$ in,

$$Q = \frac{2 \times 12^2}{6} = 48 \text{ in}^3 \quad \text{Ans}$$

24. Problem —*The loading on a beam being given, it is required to determine the maximum fiber stress in the beam.*

The fiber stress is a maximum at the most remote fiber of some section If the beam is of constant cross-section, then I and c are constant for all sections Formula 2 of Art 22 gives

$$f = \frac{Mc}{I}$$

It follows from this formula that f is greatest in the outermost fiber of the cross-section at which the bending moment is greatest Hence, to compute the maximum fiber

stress, the maximum bending moment must be determined first. Substituting the value of this moment and the values of c and I in the preceding formula, the value of f is found. In the application of this formula, the same unit of length should be used for the different quantities. Thus, if c is in *inches*, I should be in biquadratic *inches*, M in *inch*-pounds, *inch*-tons, etc., and f in pounds, tons, etc. per square *inch*.

EXAMPLE 1.—To determine the maximum fiber stress in a beam in which the maximum bending moment is 50,000 foot-pounds, assuming the beam to have a rectangular cross section 9 inches deep (vertical dimension) and 4 inches wide.

SOLUTION.—In this case, $b = 4$, $d = 9$, and, from Table I,

$$I = \frac{4 \times 9^3}{12} = 243, \quad c = \frac{9}{2} = 4.5$$

The maximum bending moment in the beam is 50,000 ft.-lb., or 600,000 in.-lb. Therefore, by the formula,

$$f = \frac{600,000 \times 4.5}{243} = 11,110 \text{ lb. per sq. in.} \quad \text{Ans.}$$

Since the bending moment is positive, the beam bends downwards at the point of maximum bending moment. The lowest extreme fiber is in tension, and the highest in compression.

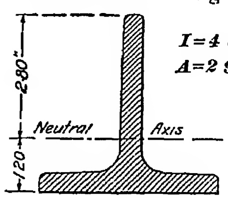


FIG. 9

EXAMPLE 2.—Assuming the section of a beam to be as represented in Fig. 9, it is required to determine the maximum tensile and compressive fiber stresses, when the maximum bending moment is 30,000 inch-pounds.

SOLUTION.—When it is required to determine separately the greatest compression and the greatest tension in the section, as in this case, two values of c must be used, one for the most remote fiber in tension and one for the most remote fiber in compression. In the present case, the lower fibers are in tension, and the upper in compression. Denoting by f_t and f_c the maximum fiber stresses for tension and compression, respectively, and substituting known values in the formula of this article, we have,

$$f_t = \frac{30,000 \times 1.2}{4.3} = 8,370 \text{ lb. per sq. in.} \quad \text{Ans.}$$

$$f_c = \frac{30,000 \times 2.8}{4.3} = 19,530 \text{ lb. per sq. in.}$$

Or, more simply,

$$f_c = \frac{2.8}{1.2} \times f_t = \frac{7}{3} \times 8,370 = 19,530 \text{ lb. per sq. in.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1 What is the maximum fiber stress in an 8-inch simple I beam 20 feet long, whose moment of inertia is 68, if the total load, including the weight of the beam, is 9,000 pounds, uniformly distributed?

Ans 15,900 lb per sq in

2 A simple T beam 10 feet long and 4 inches deep supports a load of 168 pounds per foot If the moment of inertia of the section is 4 54 in⁴ and the neutral axis is 1 12 inches from the compression flange, what is the fiber stress (*a*) in tension? (*b*) in compression?

Ans $\begin{cases} (a) \ 16{,}000 \text{ lb per sq in} \\ (b) \ 6{,}200 \text{ lb per sq in} \end{cases}$

STRENGTH OF BEAMS

25. Modulus of Rupture —The **modulus of rupture** of a material is the greatest fiber stress that a piece made of the material can stand when subjected to bending The

TABLE II

MODULI OF RUPTURE

Material	Modulus of Rupture s_b Pounds per Square Inch	Material	Modulus of Rupture s_b Pounds per Square Inch
Ash	8,000	Chestnut	4,500
Hemlock	3,500	Spruce	3,000
White oak	6,000	Stone	1,200
Brick	800	Cast iron	30,000
White pine	4,000	Wrought iron	45,000
Yellow pine	7,000	Steel	65,000

modulus of rupture is also called the **ultimate strength of flexure.** It might be inferred from the foregoing articles that the ultimate strength of a beam depends simply on the compressive and the tensile strength of the material, and that every material has two moduli of rupture, corresponding to the ultimate strengths of tension and compression Experience, however, shows that this is not the case

beam breaks when the greatest fiber stress, whethei compression or tension, has a value intermediate between the ltimate tensile and compiessive strengths of the material 'able II gives aveiage values of the modulus ot iupture s_b or diffeient mateiials The subsciipt b is used as it is the nitial lettei of "bending" The values given express pounds er squaie inch

26. Ultimate Resisting Moment.—Let s_b be the modulus of rupture of the mateiial of a beam Then, the gieatest ending moment that the beam can resist at any section is btained by wiiting s_b foi f in foimula 2 of Art 22. This oment is called the **ultimate iesisting moment** of the ection consideied, or of the beam, if the beam is of uniform ross-section By making the substitution just indicated, we iave,

$$M_r = \frac{s_b I}{c} \qquad (1)$$

In terms of the section modulus (see Art 23),

$$M_r = s_b Q \qquad (2)$$

When the beam is loaded to its utmost capacity, the bendng moment of the external forces is equal to the ultimate esisting moment, that is,

$$M = M_r = \frac{s_b I}{c} \qquad (3)$$

In terms of the section modulus,

$$M = s_b Q \qquad (4)$$

This is the formula used foi the design of beams, but, in he application of the formula, s_b is taken as the working strength of flexuie, which is the modulus of iupture divided by a suitable factor of safety

EXAMPLE 1 —A simple beam 20 feet long and having the crossection shown in Fig 9 is to suppoit a weight W (pounds) placed in he middle of the beam How heavy can the load be, the weight of he beam being neglected assuming the working flexure stiess of the naterial to be 20,000 pounds per square inch?

SOLUTION —Foi the maximum bending moment we have (*Stiength of Mateiials*, Part 1),

$$M = \frac{W \times 20}{4} = (5\ W)\text{ ft-lb} = (60\ W)\text{ in-lb}$$

Here $s_b = 20{,}000$, $I = 4.3$, and $c = 2.8$ Substituting these values in formula **3**,

$$60\,W = \frac{20{,}000 \times 4.3}{2.8},$$

whence $$W = \frac{20{,}000 \times 4.3}{60 \times 2.8} = 512\text{ lb, nearly} \quad \text{Ans}$$

EXAMPLE 2 —What must be the section modulus of the cross-section of a simple beam 10 feet long, that the beam may carry a uniform load of 250 pounds per foot, distributed over the whole length of the beam, in addition to a central load of 2,000 pounds, the working flexure strength being 15,000 pounds per square inch?

SOLUTION —The maximum bending moment evidently occurs at the middle of the beam, and is equal to the sum of the bending moments due to the uniform load and the central load, that is (*Strength of Materials*, Part 1), expressing moments in inch-pounds,

$$M = \frac{2{,}000 \times 10 \times 12}{4} + \frac{250 \times 10 \times (10 \times 12)}{8} = 97{,}500\text{ in-lb}$$

From formula **4**, $$Q = \frac{M}{s_b},$$

or, since here $M = 97{,}500$, and $s_b = 15{,}000$,

$$Q = 97{,}500 \div 15{,}000 = 6.5\text{ in.}^3 \quad \text{Ans}$$

EXAMPLE 3 —A timber cantilever 5 feet long and of square cross-section is to carry a weight of 1 ton at its end What must be the side of the cross-section, assuming the working flexure stress to be 800 pounds per square inch?

SOLUTION —The maximum bending moment is

$$M = 2{,}000 \times (5 \times 12) = 120{,}000\text{ in-lb}$$

Let x be the required side of the square cross-section From Table I,

$$I = \frac{x^4}{12},\ c = \frac{x}{2},$$

whence $$Q = \frac{I}{c} = \frac{x^3}{6}$$

Substituting in formula **4**,

$$120{,}000 = 800 \times \frac{x^3}{6} = \frac{400\,x^3}{3},$$

whence $$x = \sqrt[3]{900} = 9.65\text{ in} \quad \text{Ans}$$

EXAMPLES FOR PRACTICE

1 If, in example 3, the cantilever is made 12 inches wide (horizontal dimension), what should be its depth (vertical dimension)?

Ans 8.66 in

2 A simple beam of steel 30 feet long has the form and dimensions shown in Fig 10 What load, uniformly distributed, can the beam carry, taking the working fiber stress as 15,000 pounds per square inch?

Ans 628 lb per ft

3 A white-pine cantilever is 8 feet long and its cross-section is as shown in Fig 11 Find the factor of safety of the cantilever if it is

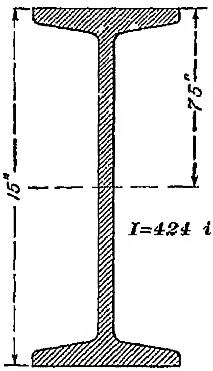


Fig 10

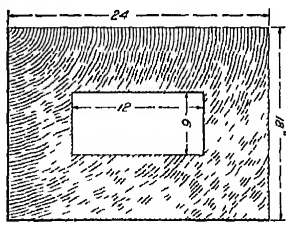


Fig 11

loaded with (*a*) a load of 10,000 pounds at the end, (*b*) a uniform load of 5,000 pounds per foot

Ans $\begin{cases} (a)\ 5 \\ (b)\ 2\frac{1}{2} \end{cases}$

---

27. Shearing Strength —In what precedes, the strength of a beam is considered with regard to bending alone As explained in Art **16**, there is at every section a shearing stress numerically equal to the external shear If the external shear is denoted by V, and the area of the cross-section by A, the average intensity of shearing stress in the section is $\frac{V}{A}$ This shearing stress is not uniformly distributed, and it can be shown that, in beams of rectangular cross-section, the maximum intensity of shearing stress is $\frac{3V}{2A}$ Hence, a rectangular beam must be so designed that this value will not exceed the working shearing strength of the material In metallic beams with thin webs (plate girders), the shearing stress may be considered as uniformly distributed over the cross-section of the web There is,

also, at every horizontal or longitudinal section of the beam, a horizontal shearing stress the intensity of which at any point is equal to the intensity of the vertical shearing stress at that point

Although the maximum intensity of shearing stress, both horizontal and vertical, in wooden beams is usually small, the shearing strength of wood along the grain is also small. As the horizontal external shear usually acts along the grain, the safe load for a wooden beam may depend on its shearing strength and not on its bending strength. For instance, the safe load for a beam 4 in $\times$ 12 in and 4 feet long is 11,200 pounds, uniformly distributed, when based on a fiber strength of 700 pounds per square inch. Such a load will produce a shearing stress per unit area equal to $\frac{3 \times 5{,}600}{2 \times 48} = 175$ pounds per square inch, which exceeds the working shearing stress for the wood along the grain by about 100 pounds per square inch

STIFFNESS OF BEAMS

28. Definition.—The **stiffness** of a beam is the resistance of the beam to deflection in the direction of the external forces acting on the beam. This property is of importance in certain kinds of construction. Thus, most machine parts must be stiff, as their excessive "giving" or yielding may destroy their adjustment. Floor joists and ceilings sustaining a floor with a plastered ceiling below must not deflect too much, or they will crack the plastering

29. Deflection Formulas.—The theory of the deflection of beams is rather complicated, and the formulas for deflection cannot be readily derived without the use of the calculus. Formulas for the most usual cases are given here. In all these formulas, the length of the beam, in inches, is denoted by l, the moment of inertia of the cross-section, in biquadratic inches, by I, Young's modulus of elasticity of the material, in pounds per square inch, by E, and the maximum deflection, in inches, by v. A single load at the middle of the beam is denoted by W (pounds), and a load uniformly distributed over the whole beam, by $w\,l$ (pounds), w denoting

the load per inch The form assumed by the beam when loaded is very much exaggerated in the figures

1 *Simple beam*, AB, Fig 12, supported at A and B Single load W in the middle

$$y = \frac{W l^3}{48 E I} \qquad (1)$$

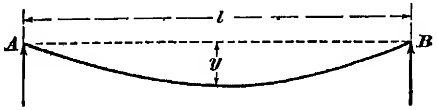


FIG 12

Uniformly distributed load

$$y = \frac{5 w l^4}{384 E I} \qquad (2)$$

2 *Cantilever* AB, Fig 13, fixed at A Single load W at end B

$$y = \frac{W l^3}{3 E I} \qquad (3)$$

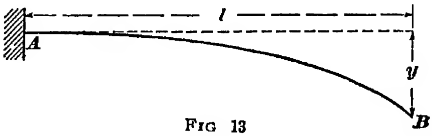


FIG 13

Uniformly distributed load

$$y = \frac{w l^4}{8 E I} \qquad (4)$$

3 *Beam fixed at one end* A, Fig 14, *and simply supported at the other* For single load W in the middle $\left(BC = \frac{l}{5}\sqrt{5}\right)$

$$y = \frac{W l^3}{48 E I} \times \frac{1}{\sqrt{5}} = \frac{W l^3}{E I} \times \frac{\sqrt{5}}{240} \qquad (5)$$

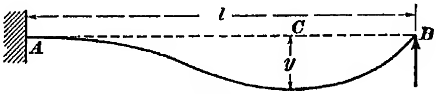


FIG 14

Uniformly distributed load

$$y = \frac{w l^4}{192 E I} \qquad (6)$$

4. *Beam fixed at both ends A* and *B*, Fig. 15. Single load W in the middle.

$$y = \frac{W l^3}{192 E I} \qquad (7)$$

Uniformly distributed load.

$$y = \frac{w l^4}{384 E I} \qquad (8)$$

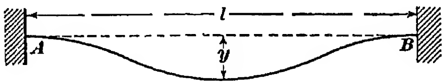


Fig. 15

EXAMPLE 1.—A timber simple beam 10 feet long, and having a width of 4 inches and a depth of 12 inches, carries a uniform load of 400 pounds per foot. What is the deflection?

SOLUTION.—To apply formula 2, we have $w = \frac{400}{12}$, $l = 10 \times 12 = 120$, $E = 1{,}500{,}000$, and $I = \frac{1}{12} \times 4 \times 12^3 = 576$. Substituting in the formula,

$$y = \frac{5 \times 400 \times 120^4}{384 \times 12 \times 1{,}500{,}000 \times 576} = 1 \text{ in. Ans.}$$

EXAMPLE 2.—To determine the deflection of the beam considered in No. 2 of the Examples for Practice following Art. 26, when the beam is loaded to its utmost (working) capacity.

SOLUTION.—Here $w = \frac{628}{12}$, $l = 30 \times 12 = 360$, $E = 30{,}000{,}000$, and $I = 424$. Substituting in formula 2,

$$y = \frac{5 \times 628 \times 360^4}{384 \times 12 \times 30{,}000{,}000 \times 424} = .9 \text{ in. Ans.}$$

BEAMS UNDER INCLINED FORCES

30. Extended Meaning of the Term Beam.—Although the term *beam* usually denotes a horizontal piece acted on by vertical forces, it is often extended to mean any elongated piece having a straight axis containing the centers of gravity of all cross-sections (a cross-section being a section cut by a plane perpendicular to the

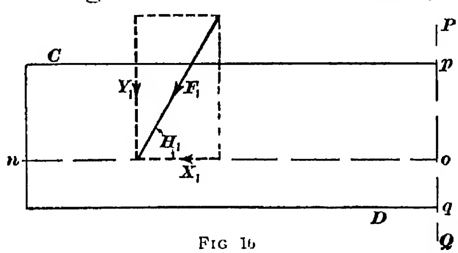


Fig. 16

axis) and acted on by any system of coplanar forces containing the axis. In Fig. 16, CD is a part of a beam of which pq is a cross-section cut by the plane PQ. on, which contains the centers of gravity of all sections perpendicular to it, is the axis of the beam, and F_1 any force intersecting the axis at an angle H_1.

31. General Method of Treatment.—The force F_1 may be resolved into two components X_1 and Y_1, the former along the axis, the latter perpendicular to the axis. The same method of resolution may be applied to all other forces acting on the beam. The beam may then be considered as acted on by two independent systems of forces. The forces perpendicular to the axis, which will be called the system Y, cause shearing and bending stresses in the section pq; these stresses can be computed exactly as for any ordinary beam. The forces acting along the axis, which will be called the system X, cause direct tension or compression, and have no effect on the shearing stress at pq perpendicular to the axis. Both the compressive and the tensile fiber stresses are computed from the forces Y by the methods already explained, and are then combined by algebraic addition with the stress produced by the system X, to obtain the total tensile and compressive stresses.

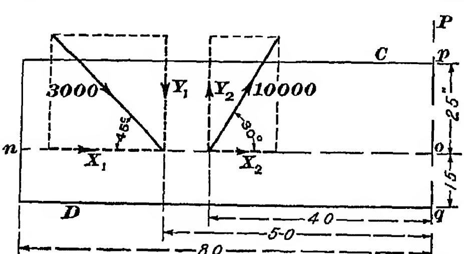


FIG. 17

EXAMPLE.—Forces of 3,000 and 10,000 pounds, respectively, act on a beam CD, Fig. 17, on the left of the section pq, the inclinations and distances being as shown. The moment of inertia of the section is 12 in.^{4}, and the area is 4.5 square inches. To find the stresses in the section pq.

SOLUTION.—Let the components of the forces along the axis be X_1 and X_2, and those perpendicular to the axis Y_1 and Y_2, as shown. Then, considering forces to the right and upward forces as positive,

$X_1 = 3,000 \cos 45° = +2,120$ lb. $\quad X_2 = 10,000 \cos 30° = +8,660$ lb.

$Y_1 = -3,000 \sin 45° = -2,120$ lb. $\quad Y_2 = 10,000 \sin 30° = +5,000$ lb.

$X = X_1 + X_2 = +10,780$ lb. $\quad Y = Y_1 + Y_2 = +2,880$ lb.

Since Y is the external shear on the left of $p\,q$, the shearing stress in the section is 2,880 lb. The force X, being directed toward the right, produces a compressive stress whose intensity is

$$\frac{X}{4.5} = \frac{10{,}780}{4.5} = 2{,}400 \text{ lb. per sq. in.}$$

The bending moment about o is, in inch-pounds (here the sign of Y_1 is disregarded),

$$Y_2 \times (4 \times 12) - Y_1 \times (5 \times 12) = +112{,}800 \text{ in.-lb.}$$

Since this moment is positive, the beam bends downwards; therefore, the upper fibers of the section $p\,q$ are in compression, and the lower in tension. The maximum intensity of stress in the upper fiber is (see formula in Art. **24**, and example 2 of that article)

$$f_c = \frac{112{,}800 \times 2.5}{12} = 23{,}500 \text{ lb. per sq. in.}$$

and that in the lower fiber,

$$f_t = \frac{112{,}800 \times 1.5}{12} = 14{,}100 \text{ lb. per sq. in.}$$

Combining with these stresses the direct compression $\frac{X}{4.5}$ found above, we have, finally

Total compressive stress = 23,500 + 2,400 = 25,900 lb. per sq. in. Ans.

Total tensile stress = 14,100 − 2,400 = 11,700 lb. per sq. in. Ans.

COLUMNS

32. Definition.—As stated in *Strength of Materials*, Part 1, the strength of a compression piece or member depends on its length, and a piece so long that it bends perceptibly before failure is called a **column**.

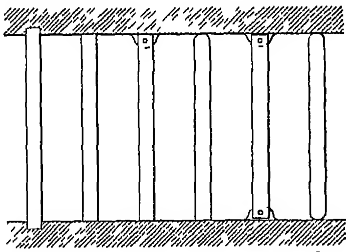


FIG. 18

33. Classification of Columns.—Columns are classified as follows, according to the conditions of the ends:

1. *Columns with fixed ends*, Fig. 18 (*a*). The fixing of the ends is so rigid that bending does not change the direction of the column at its ends.

2. *Columns with pivot ends* (also called *round-ended columns*), Fig. 18 (*b*), in which the ends can slide freely

n which E = Young's modulus of elasticity of the material, in pounds per square inch,

A = area of cross-section of column, in square inches,

P = maximum load, in pounds, that the column can support

n is a number depending on the end condition, and has he following values

$n = 1$ for columns with both ends pivoted,

$n = 2\frac{1}{4}$ for columns with one end pivoted and one fixed,

$n = 4$ for columns with both ends fixed

Fig 19 shows the graph of Euler's formula plotted by taking values of $\frac{l}{r}$ as abscissas and the corresponding values of $\frac{P}{A}$ as ordinates The latter should not exceed the bending working stress of the material

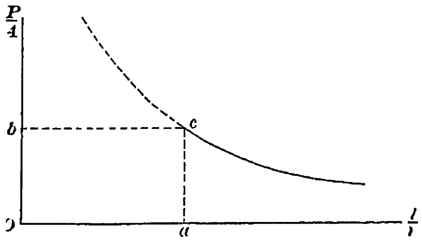


Fig 19

37. Euler's formula is derived from the assumption that ie column will fail by bending due to the moment produced y lateral deflection of the column This deflection will occur nly when the value of $\frac{l}{r}$ exceeds a certain limit below which ie column will fail by crushing For the minimum value f $\frac{l}{r}$, $\frac{P}{A}$ is maximum Assuming this maximum to be the lastic limit of the material L, and substituting it for $\frac{P}{A}$ in uler's formula, $L = \frac{n\pi^2 E}{\left(\frac{l}{r}\right)^2}$, whence, $\frac{l}{r} = \pi\sqrt{\frac{nE}{L}}$

In Fig 19 Ob represents the elastic limit L, and Oa the orresponding value of $\frac{l}{r}$ The part of the curve to the left f ac does not apply, as for values of $\frac{l}{r}$ less than Oa the olumn would fail by direct compression

TABLE III

CONSTANTS FOR STRAIGHT-LINE AND PARABOLA COLUMN FORMULAS

Formula		Mild Steel		Wrought Iron		Cast Iron
		Flat Ends	Pin Ends	Flat Ends	Pin Ends	Flat Ends
Straight-line formula	S_u	52,500	52,500	42,000	42,000	80,000
	k_1	179	220	128	157	438
	limit of $\frac{l}{r}$	195	159	218	178	122
Parabola formula	L	42,000	42,000	34,000	34,000	60,000
	k_2	62	97	43	67	2 25
	limit of $\frac{l}{r}$	190	150	210	170	120
$n\pi^2 E$		712 m	456 m	675 m	432 m	400 m

38. Straight-Line and Parabola Formulas—These are not rational formulas, but rough generalizations from experiments on the failure of columns, and were devised for convenience of practical application They are as follows

Straight-line formula,

$$\frac{P}{A} = s_u - k_1 \times \frac{l}{r} \qquad (1)$$

in which s_u = ultimate compressive strength of material,
k_1 = constant depending on material and on conditions of ends of column

Parabola formula,

$$\frac{P}{A} = L - k_2\left(\frac{l}{r}\right)^2 \qquad (2)$$

in which L = elastic limit of material,
k_2 = constant depending on material and on condition of ends

The values of s_u, L, k_1, and k_2 are given in Table III. Each of these formulas is used only within the limits of the value of $l \div r$ there given Beyond these limits Euler's formula or Rankine's formula, to be treated later, should be applied The values of $n\pi^2 E$ given in Table III are for use in connection with Euler's formula They were computed for $n\pi^2 = 16$ and 25, respectively, the values of n given in Art **36** being only theoretical and holding good for ideal columns, where the ends are either perfectly fixed or perfectly free to turn The letter m in the table means millions

39. In formulas for wooden columns it is convenient to introduce the least width instead of the radius of gyration of the cross-section The following are parabola formulas, for columns with flat ends when $l \div d$ is less than 60

White pine $$\frac{P}{A} = 2{,}500 - 6\left(\frac{l}{d}\right)^2 \qquad (1)$$

Short-leaved yellow pine $$\frac{P}{A} = 3{,}300 - 7\left(\frac{l}{d}\right)^2 \qquad (2)$$

Long-leaved yellow pine $$\frac{P}{A} = 4{,}000 - 8\left(\frac{l}{d}\right)^2 \qquad (3)$$

White oak $$\frac{P}{A} = 3{,}500 - 8\left(\frac{l}{d}\right)^2 \qquad (4)$$

EXAMPLE 1.—What is the safe load for a white-pine column 12 in × 12 in. in cross-section and 16 feet long?

SOLUTION.—Since $\frac{l}{d} = \frac{16 \times 12}{12} = 16$, formula **1** may be used, therefore, $P = 144(2{,}500 - .6 \times 16^2) = 338{,}000$ lb.

This is the probable breaking load; the safe load depends on the factor of safety, which, for a steady load, may be taken as 6, making the safe load 56,300 lb. Ans.

EXAMPLE 2.—What is the safe load for a hollow circular cast-iron column 14 feet long, with flat ends, 8 inches outside diameter, and $\frac{3}{4}$ inch thick? Use a factor of safety of 8.

SOLUTION.—First, $\frac{l}{r}$ should be computed to ascertain whether to use Euler's formula or one of the others. From No. 10 in Table I,

$$r = \tfrac{1}{4}\sqrt{8^2 + 6.5^2} = 2.58 \text{ in.},$$

hence,

$$\frac{l}{r} = \frac{14 \times 12}{2.58} = 65.1$$

This being less than the limiting values of $\frac{l}{r}$ for cast iron given in Table III, either the straight-line or the parabola formula may be used. The former gives, A being 17.08 sq. in.,

$$P = 17.08(80{,}000 - 438 \times 65.1) = 879{,}400 \text{ lb.},$$

which is the probable breaking load. The safe load, with a factor of safety of 8, is

$$879{,}400 \div 8 = 109{,}900 \text{ lb.} \quad \text{Ans.}$$

The parabola formula gives the probable breaking load as

$$17.08(60{,}000 - 2.25 \times 65.1^2) = 861{,}900 \text{ lb.}$$

40. Rankine's Formula.—The following formula, known as **Rankine's formula**, or the **Gordon-Rankine formula**, was derived partly from theoretical considerations and partly from the results of actual tests. It covers a wide range of conditions, is applicable to all values of $\frac{l}{r}$, and seems to agree with the results of experiments better than any other formula so far proposed. It is as follows:

$$\frac{P}{A} = \frac{s_u}{1 + k_3\left(\frac{l}{r}\right)^2}$$

in which s_u = ultimate strength,

k_3 = constant depending on material and class of column.

Many values of s_u and k_3 are in use, in a large steel company's handbook, these are given

For mild steel, flat ends, $s_u = 50,000$, $k_3 = \frac{1}{36000}$

For cast iron, flat ends, $s_u = 80,000$, $k_3 = \frac{1}{6400}$

In many specifications, especially in those for bridges, s is taken as the working stress of the material, in which case the value of P obtained by Rankine's formula (and the same applies to other formulas in which a similar notation is used) is the safe load that the column can support, no factor of safety being necessary

EXAMPLE 1 —To compute the breaking load, by Rankine's formula, for a hollow circular cast-iron column with flat ends, the column being 14 feet long, 8 inches outside diameter, and $\frac{3}{4}$ inch thick

NOTE — This is the same as the second example of Art 39

SOLUTION —From the solution in Art 39, $\frac{l}{r} = 65\ 1$ and $A = 17\ 08$

Making $s_u = 80,000$, $k_3 = \frac{1}{6400}$, and substituting in Rankine's formula, we have

$$P = \frac{17\ 08 \times 80,000}{1 + \frac{65\ 1^2}{6,400}} = 822,140 \text{ lb} \quad \text{Ans}$$

EXAMPLE 2 —To compute by Rankine's formula the factor of safety of a mild-steel column with flat ends, 40 feet long, area of cross-section 11 3 square inches, and least radius of gyration of the cross-section 2 47 inches, when the column sustains a load of 70,000 pounds

SOLUTION —Here $\frac{l}{r} = \frac{40 \times 12}{2\ 47} = 194$, $A = 11\ 3$ Making $s_u = 50,000$, $k_3 = \frac{1}{36000}$, and substituting in the formula,

$$P = \frac{11\ 3 \times 50,000}{1 + \frac{194^2}{36,000}} = 276,200 \text{ lb}$$

This is the breaking load When the column sustains a load of 70,000 pounds, the factor of safety is

$$276,200 \div 70,000 = 4 \text{ nearly} \quad \text{Ans}$$

EXAMPLES FOR PRACTICE

1 What is the breaking load for a steel column with flat ends, the length of the column being 30 feet, area of cross section 41 square inches, and the least radius of gyration 2 5 inches? Use the straight line formula Ans 1,095,700 lb

2 What is the breaking load for a pin-end steel column 40 feet long, area of cross-section 41 square inches, and radius of gyration 3 4 inches? Use the parabola formula Ans 929,350 lb

3 Determine by Rankine's formula the safe load for a flat-end steel column 18 feet long, area of cross-section 8 square inches, and radius of gyration 3 inches Use a factor of safety of 4

Ans 87,400 lb

4 A hollow cast-iron column 10 inches outside diameter, 1 inch thick, and 20 feet long, sustains a load of 164,000 pounds Considering that the column has flat ends, determine the factor of safety by Rankine's formula Ans 7 3

---

41. Design of Columns —By the **design of columns** is not meant the selection of form of cross-section, spacing of rivets in built-up metal columns, etc , but simply the determination of the dimensions of the cross-section The choice of form depends on conditions a discussion of which does not fall within the scope of this Section

The dimensions of the cross-section can be determined by means of the preceding formulas, but the determination, except in special cases, cannot be made directly, because there are two unknown quantities in each formula, namely, A, and r or d Usually, it is easiest to solve by the "trial method," as will be illustrated in connection with bridge design The special cases in which a direct solution is possible are those where a relation between A and r or d is known, as in square, circular, and a few other sections Such cases are illustrated in the following two examples

EXAMPLE 1 —A square white-oak column 10 feet long is to sustain a load of 70,000 pounds with a factor of safety of 6 What must be the side of the cross-section?

SOLUTION —With a factor of safety of 6, the breaking load would be $70,000 \times 6 = 420,000$ lb Also, $A = d^2$, and $l = 120$ in , hence, from formula 4 of Art 39,

$$\frac{420\,000}{d^2} = 3{,}500 - 8 \times \left(\frac{120}{d}\right)^2,$$

whence $$3{,}500\, d^2 = 420{,}000 + 8 \times 14{,}400$$

and, solving for d, $$d = 11\ 1 \text{ in} \quad \text{Ans}$$

Since $\frac{l}{d} = \frac{120}{11\ 1} = 11$, which is below the limit given in Art 39, the use of the parabola formula is justified

EXAMPLE 2.—What size of round wrought-iron column with flat ends is needed to sustain a load of 30,000 pounds, the length of the column being 8 feet? Use a factor of safety of 4.

SOLUTION.—With a factor of safety of 4, the breaking load is 120,000 lb., $A = \frac{1}{4}\pi d^2$, $r = \frac{d}{4}$, and $l = 8 \times 12 = 96$ in. Hence, substituting in the parabola formula,

$$\frac{120,000}{\frac{\pi d^2}{4}} = 34,000 - 43\left(\frac{96}{\frac{d}{4}}\right)^2$$

Clearing of fractions, and solving for d^2,

$$d^2 = \frac{480,000 + 199,197}{3.1416 \times 34,000}, \quad d = 2.52 \text{ in.} \quad \text{Ans.}$$

Now, $\frac{l}{r} = \frac{96 \times 4}{2.52} = 152$. This value is less than the limit given in Table III, hence, it was proper to use the parabola formula.

TORSION

42. Twisting Moment.—A shaft, when in use, is subjected to forces that twist and bend it, and it is also sometimes compressed. For the present, only the twisting effect is considered.

By **twisting moment** at a section of a shaft is meant the algebraic sum of the moments, about the axis of the shaft, of all the forces applied to the shaft on either side of the section. No particular rule for signs is used, except that moments in opposite directions are given opposite signs. It can be proved that the twisting moment at any section of a shaft at rest or rotating at constant speed, whether computed from the forces on the right or from those on the left, is the same numerically.

It is customary to express twisting moments in inch-pounds. It may be convenient to compute the moments of the forces in foot-pounds, and then reduce their sum to inch-pounds. The letter T will be used to denote twisting moment.

EXAMPLE.—Suppose that the moments of the belt tensions represented in Fig. 20 about the shaft axis are, respectively, 4,000, −500, −600, −6,800, −1,000, −700, and −400 foot pounds, beginning from

the left Each moment is obtained by multiplying the corresponding force by the distance of its line of action from the axis It is assumed that all forces, although acting at the surface of the shaft or pulleys,

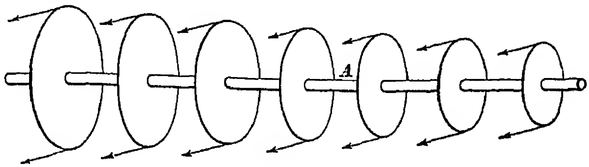


FIG 20

are perpendicular to the axis What is the value of the twisting moment at any section of the shaft?

SOLUTION —The forces on either side of a section consist of the belt pulls, the weight of the shafting and pulleys on that side, and the reactions of the bearings The weights have no moment about the axis of the shaft If the shaft is turning, the frictional parts of the reactions have moments, but these are small and negligible Hence, the pulls only are considered in computing twisting moment At any section between the first and second pulleys, $T = 4{,}000$ ft -lb , between the second and third pulleys, $T = 4{,}000 - 500 = 3{,}500$ ft -lb , between the third and fourth pulleys, $T = 4{,}000 - 500 - 600 = 2{,}900$ ft -lb , etc

43. Torsional Stress —When two cylinders placed end to end are pressed together, and then subjected to twisting forces in opposite directions, there is a tendency to slip, which, if the pressure is large enough, is prevented by the friction between the cylinders Just so in a solid cylinder when it is twisted, there is a tendency for the parts on each side of a cross-section to slip or slide on each other, and the slipping is prevented by the stresses at the section

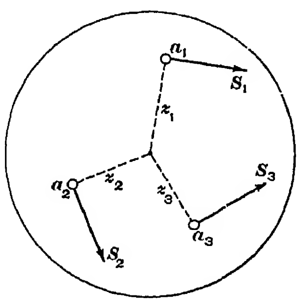


FIG 21

If the cylinder is circular in cross-section, solid or hollow, and the applied forces tend only to twist it, the stress in each cross-section is a shear This shear, which is called **torsional or twisting stress**, is not uniformly distributed, but *its intensity varies as the distance from the axis of the shaft.*

44. Twisting Resisting Moment.—By the **twisting resisting moment** at a section is meant the sum of the moments of the shearing stresses on all the small parts of the section about the axis of the shaft.

Let a_1, a_2, a_3, etc., Fig. 21, be small areas in the section of a shaft, z_1, z_2, z_3, etc., the distances of these areas from the axis, s_1, s_2, s_3, etc., the intensities of stress at these distances, S_1, S_2, S_3, etc., the total stresses at the small areas, and f, the intensity of stress in the outermost fiber, whose distance from the center is $\frac{d}{2}$, denoting the diameter of the shaft by d. Then, according to the principle stated in the last article, the intensity of stress at unit's distance from the axis is $f \div \frac{d}{2} = \frac{2f}{d}$, and

$$s_1 = \frac{2f}{d} \times z_1,\ s_2 = \frac{2f}{d} \times z_2,\ s_3 = \frac{2f}{d} \times z_3, \text{ etc.}$$

Also,

$$S_1 = a_1 s_1 = \frac{2f}{d} \times a_1 z_1,\ S_2 = \frac{2f}{d} \times a_2 z_2, \text{ etc.}$$

The moments of these stresses about the axis are, respectively,

$$S_1 z_1 = \frac{2f}{d} \times a_1 z_1^2,\ S_2 z_2 = \frac{2f}{d} \times a_2 z_2^2, \text{ etc.}$$

and their sum is

$$\frac{2f}{d} \times (a_1 z_1^2 + a_2 z_2^2 + a_3 z_3^2 + \quad) = \frac{2f}{d} \Sigma a z^2$$

As the areas a are made smaller and smaller, or their number increased, $\Sigma a z^2$ finally becomes the polar moment of inertia of the section about the axis (see Art. 11), and $\frac{2f}{d} \Sigma a z^2$ becomes the twisting resisting moment of the section. Denoting the polar moment by J, the resisting moment is $\frac{2f}{d} \times J$, and, since this is equal to the twisting moment T of the external forces, the following fundamental formula is obtained:

$$T = \frac{2f}{d} \times J \qquad (1)$$

If for f is substituted the ultimate shearing stress s_s of the material, formula 1 gives the **torsional strength** of the shaft, namely,

$$T = \frac{2s_s}{d} \times J \qquad (2)$$

45. For a *solid circular shaft*,

$$J = \frac{\pi d^4}{32}, \text{ and } T = \frac{\pi s_s d^3}{16} \qquad (1)$$

For a *hollow circular shaft*, in which d_1 and d_2 denote, respectively, the outside and the inside diameter,

$$J = \frac{\pi(d_1^4 - d_2^4)}{32}, \text{ and } T = \frac{\pi s_s(d_1^4 - d_2^4)}{16\,d_1} \qquad (2)$$

46. For a *square shaft*, the law of variation of the shearing stress is not so simple as for circular shafts The greatest value of the intensity of shearing stress occurs at the middle of the sides of the square The strength of the shaft is given by the following formula, in which d denotes one side of the cross-section

$$T = \frac{s_s d^3}{5}$$

EXAMPLE—A hollow shaft whose inner and outer diameters are 4 and 10 inches, respectively, is subjected to a twisting moment of 250,000 foot-pounds What is the value of the maximum shearing stress?

SOLUTION From formula 2 of Art 45,

$$s_s = \frac{16\,T d_1}{\pi(d_1^4 - d_2^4)}$$

Here, $T = 250{,}000 \times 12 = 3{,}000{,}000$ in -lb, $d_1 = 10$, $d_2 = 4$ Substituting in the foregoing equation,

$$s_s = \frac{16 \times 3{,}000{,}000 \times 10}{3.1416(10^4 - 4^4)} = 15{,}680 \text{ lb per sq in} \quad \text{Ans}$$

47. Torsional Stiffness. Angle of Torsion.—When a shaft is transmitting power, it is twisted, and the pulleys on it turn slightly with respect to each other The amount of this twisting depends on the stiffness or rigidity, and not on the strength of the material Large amounts of twist in a shaft are objectionable, hence, stiffness rather than strength may determine the size of the shaft Rules for designing

are based on experience rather than on theory, and do not fall within the scope of this Course

48. In any shaft in which the twisting moment is constant, a line on its surface, as $m\,n$, Fig 22, which is parallel to the axis before twisting, takes a form such as $m'\,n$ after twisting The line $m'\,n$ is a helix if the stress does not exceed the elastic limit If a piece of tracing paper were wrapped about the cylinder, and the lines $m\,n$ and $m'\,n$ traced on it, they would be found to be straight when the paper is laid out flat The angle between the two lines is called the **helix angle**, and that between the two lines $o\,m$ and $o\,m'$ is called the **angle of torsion** For a solid circular shaft, the angle of torsion a (degrees), Fig 22, for any section

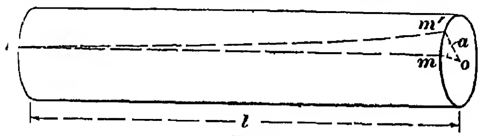


FIG 22

distant l from the end n of the shaft is given by the formula

$$a = \frac{5{,}760\,T\,l}{\pi^2\,E\,d^4}$$

in which E is the shearing modulus of elasticity

NOTE —The last formula is the basis of the method for determining the shearing modulus of elasticity of any material A specimen of the material is twisted, and the angle of torsion and the twisting moment are measured These values and those of l and d are next substituted in the formula, which is then solved for E

EXAMPLE —What is the angle of torsion for a 20-foot length of a wrought-iron shaft whose diameter is 3 inches, when it is subjected to a twisting moment of 17,500 inch-pounds?

SOLUTION —Here $T = 17{,}500$, $l = 20 \times 12 = 240$, $E = 10{,}000{,}000$ (*Strength of Materials*, Part 1), and $d = 3$ Substituting in the formula,

$$a = \frac{5{,}760 \times 17{,}500 \times 240}{3\,1416^2 \times 10{,}000{,}000 \times 3^4} = 3° \quad \text{Ans}$$

49. Bending of Shafts —The foregoing formulas do not take into account the bending of the shaft, hence, for long shafts carrying loads, such as pulleys between supports, they should not be used Where shafts are subjected to bending only, they are treated as beams, although they

may rotate The formulas for beams apply directly to such cases, but not to cases in which there is combined bending and torsion In the latter case, an equivalent twisting moment may be found, by the use of advanced mathematics, that will take the place of the twisting and bending moments combined

Let M = bending moment for any section,
T = twisting moment for same section,
T_1 = equivalent twisting moment

Then, $$T_1 = M + \sqrt{M^2 + T^2}$$

EXAMPLES FOR PRACTICE

1 If the twisting moment in a solid circular wrought-iron shaft is 2,500 foot-pounds, what should be its diameter? Use a factor of safety of 10 Ans $3\frac{1}{8}$ in

2 What is the angle of torsion for a 12-foot length of wrought-iron shaft whose diameter is 6 inches, when it is subjected to a twisting moment of 18,000 foot-pounds? Ans 1 4°

3 A hollow cast-iron shaft has an outside diameter of 8 inches and a thickness of 1 inch If the twisting moment that the shaft can safely sustain is 80,000 inch-pounds, what is the factor of safety? Ans 17

STRENGTH OF ROPES AND CHAINS

ROPES

50. Hemp and Manila Ropes —The strength of **hemp** and **manila ropes** varies greatly, depending not so much on the material and area of cross-section as on the method of manufacture and the amount of twisting

Hemp ropes are about 25 to 30 per cent stronger than manila ropes or tarred hemp ropes Ropes laid with tar wear better than those laid without tar, but their strength and flexibility are greatly reduced For most purposes, the following formula may be used for the safe working load of any of the three ropes mentioned above

$$P = 100\,C^2 \qquad (1)$$

in which P = working load, in pounds,
C = circumference of the rope, in inches

This formula gives a factor of safety of from $7\frac{1}{2}$ for manila or tarred hemp rope to about 11 for the best three-strand hemp rope

When the load P is given, the circumference is obtained by the formula

$$C = \frac{\sqrt{P}}{10} \qquad (2)$$

When excessive wear is likely to occur, it is better to make the circumference of the rope considerably larger than that given by formula **2.**

51. Wire Ropes —Wire rope is made by twisting a number of wires (usually nineteen) together into a strand, and then twisting several strands (usually seven) together to form the rope Wire rope is very much stronger than hemp rope, and may be much smaller in size to carry the same load

For iron-wire rope of seven strands, nineteen wires to the strand, the following formula may be used, the letters having the same meaning as in the formula in the preceding article

$$P = 600\,C^2 \qquad (1)$$

Steel-wire ropes should be made of the best quality of steel wire, when so made, they are superior to the best iron-wire ropes If made from an inferior quality of steel wire, the ropes are not so good as the better class of iron-wire ropes When substituting steel for iron ropes, the object in view should be to gain an increase of wear rather than to reduce the size The following formula may be used in computing the size or working strength of the best steel-wire rope, seven strands, nineteen wires to the strand

$$P = 1{,}000\,C^2 \qquad (2)$$

Formulas **1** and **2** are based on a factor of safety of 6

52. Long Ropes —When using ropes for the purpose of raising loads to a considerable height, the weight of the rope itself must also be considered and added to the load The weight of the rope per running foot, for different sizes, may be obtained from the manufacturer's catalog

EXAMPLE 1.—What should be the allowable working load of an iron-wire rope whose circumference is $6\frac{3}{4}$ inches? The weight of the rope is not to be considered.

SOLUTION.—Using formula **1** of Art. **51**,

$$P = 600 \times (6\tfrac{3}{4})^2 = 27{,}337.5 \text{ lb.} \quad \text{Ans.}$$

EXAMPLE 2.—The working load, including the weight, of a hemp rope is to be 900 pounds. What should be its circumference?

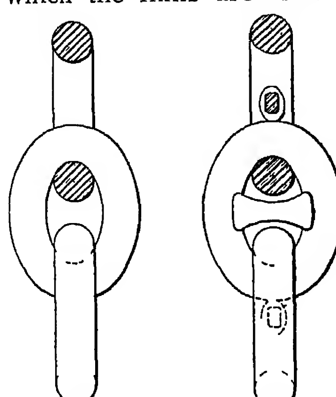
SOLUTION.—Using formula **2** of Art. **50**,

$$C = \frac{\sqrt{900}}{10} = 3 \text{ in.} \quad \text{Ans.}$$

53. Size of Ropes.—In measuring ropes, the circumference is used instead of the diameter, because the ropes are not round and the circumference is not equal to 3.1416 times the diameter. For three strands the circumference is about 2.86 d; for seven strands, it is about 3 d, d being the diameter.

CHAINS

54. The size of a **chain** is always specified by giving the diameter of the iron from which the links are made. The two kinds of chain most generally used are the **open-link** chain and the **stud-link** chain. The former is shown in Fig. 23 (*a*), and the latter in Fig. 23 (*b*). The stud prevents the two sides of a link from coming together when under a heavy pull, and thus strengthens the chain.



(*a*) FIG. 23 (*b*)

It is good practice to anneal old chains that have become brittle by overstraining. This renders them less liable to snap from sudden jerks. The annealing process reduces their tensile strength, but increases their toughness and ductility—two qualities that are sometimes more important than mere strength.

Let P = safe load, in pounds,

d = diameter of link, in inches

Then, for open-link chains, made from a good quality of wrought iron,

$$P = 12{,}000\,d^2 \qquad (1)$$

and, for stud-link chains,

$$P = 18{,}000\,d^2 \qquad (2)$$

EXAMPLE 1 —What load will be safely sustained by a $\frac{3}{4}$-inch open-link chain?

SOLUTION —Using formula 1,

$$P = 12{,}000\,d^2 = 12{,}000 \times \left(\frac{3}{4}\right)^2 = 6{,}750 \text{ lb} \quad \text{Ans}$$

EXAMPLE 2 —What must be the diameter of a stud-link chain to carry a load of 28,125 pounds?

SOLUTION —Solving formula 2 for d,

$$d = \sqrt{\frac{P}{18{,}000}} = \sqrt{\frac{28{,}125}{18{,}000}} = 1\tfrac{1}{4} \text{ in} \quad \text{Ans}$$

HYDRAULICS

(PART 1)

FLOW OF WATER THROUGH ORIFICES AND TUBES

FUNDAMENTAL FACTS AND PRINCIPLES

1. Introduction.—In hydrostatics, the principles deduced for perfect liquids apply with very little error to liquids that are more or less imperfect. Thus, the laws relating to pressure on surfaces are scarcely affected by the viscosity of the liquid, and, so long as the liquid is at rest or moving very slowly, internal friction, or viscosity, may be neglected. But in dealing with the flow of liquids, which is the province of hydraulics, the viscosity becomes of great importance. Formulas derived for the flow of ideally perfect liquids must be modified to take account of the internal friction due to eddies, cross-currents, friction between the liquid and the walls of the enclosing pipe or conduit, etc.

In the study of hydraulics, therefore, the following method is adopted: (1) Formulas for flow are deduced on the assumption that the liquid is perfect. These formulas are called **rational formulas**. (2) In order to obtain results that will apply to imperfect liquids, and take into account all the conditions of actual flow, rational formulas are modified by introducing into them certain numbers determined by experiment. Such numbers are called **empirical constants**, and the resulting modified formulas are called **empirical formulas**.

As hydraulics is thus largely a matter of empirical constants, it is not to be expected that problems on the flow of water can be solved with the same accuracy as problems in interest or mensuration The calculated result of the flow through an orifice or over a weir is likely to be in error by from 1 to 3 per cent, that for the flow in a long pipe, by 5 per cent, and that for the flow in a channel or conduit, perhaps by 5 to 10 per cent

2. Discharge, Velocity, and Cross-Section Assume water to flow through a pipe, and the pipe to be full Let F denote the area of a cross-section of the pipe, and Q the volume of water flowing past this cross-section in a second The volume or quantity Q is called the **discharge** of the pipe If all the particles move past the section F with the same velocity v, it is evident that the quantity passing in 1 second is equal to the volume of a prism or cylinder whose base is F and whose length is v, hence,

$$Q = Fv$$

In this and subsequent formulas given in this Section, F will express the area, in square feet, v, the velocity, in feet per second, Q, the discharge, in cubic feet per second, and h, the height, or head, in feet

Actually, the particles of water passing through any section will have different velocities, those near the walls

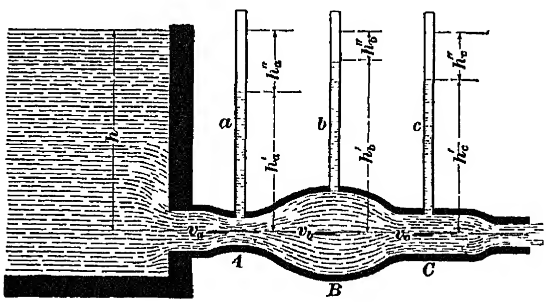


Fig 1

of the pipe moving more slowly than those near the center The formula just given may, however, be used, with the understanding that v signifies the *mean velocity* of the flow

3. If the area of the pipe varies, as shown in Fig. 1, the mean velocities at different sections will vary. Since water is practically incompressible, the same quantity must flow through each section per unit of time; hence, denoting the areas of the cross-sections at A, B, and C by F_a, F_b, and F_c, respectively, we must have

$$Q = F_a v_a,\quad Q = F_b v_b,\quad Q = F_c v_c,$$

whence

$$F_a v_a = F_b v_b = F_c v_c$$

Also,

$$\frac{v_a}{v_b} = \frac{F_b}{F_a},\quad \frac{v_a}{v_c} = \frac{F_c}{F_a},\quad \frac{v_c}{v_b} = \frac{F_b}{F_c}$$

These last equations show that *the velocities at any two cross-sections are inversely as the areas of those cross-sections*. This is a fundamental principle, and should be memorized.

4. Energy of Water.—As explained in *Kinematics and Kinetics*, a body has energy when it is capable of doing work, and energy may be either kinetic or potential.

A mass of liquid may possess a store of energy due either to its motion, its position, or the pressure to which it is subjected. If it is in motion, as in a stream or river, it has kinetic energy; if it is in motion and at the same time under pressure, as in a waterworks pipe line, it has, in addition to the kinetic energy due to its velocity, a certain potential energy due to the pressure and called **pressure energy**. Finally, if the liquid is at a certain elevation above a level that it eventually reaches, it has another form of potential energy, called **energy of position**; water in a stand pipe is an example.

5. The relation between the different kinds of energy may be explained with the aid of Fig. 2. The tank m is filled with water to the level a, which is h feet above the reference level b. Let any intermediate level, as c, be chosen at random. With the leg n two cylinders are connected, one at the level b and the other at c. Assume the water to be entirely at rest. A weight consisting of W pounds of water lying at the upper level a has energy of position, the amount of which is easily found. In descending to the reference level b, the weight W acts through a distance h; hence, its capacity

for work, referred to the level b, is Wh. This weight W at the upper level has no pressure energy, since the gauge pressure at the surface is zero (the atmospheric pressure is neglected), and it has no kinetic energy, since it is at rest.

Consider, now, a mass of water of weight W at the reference level b in the leg n. This water has no energy of position, for it is already at the lower level, and can fall no farther. It is, however, subjected to a pressure due to the head h, and, because of that pressure, may be made to do work. Suppose a valve to be opened into the cylinder e; then, the water will enter the cylinder, and, because of its

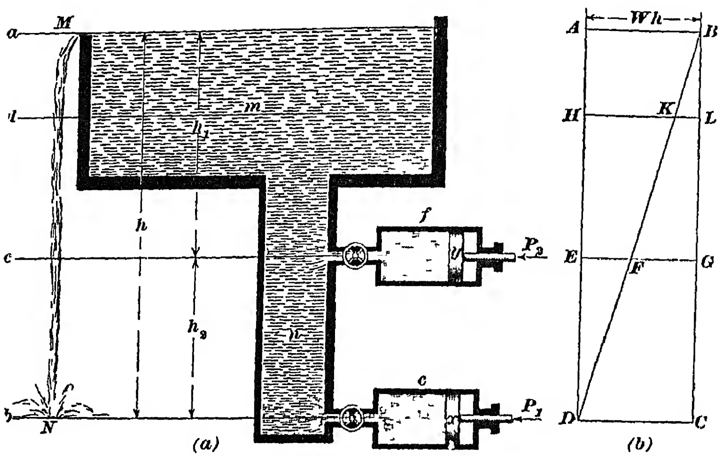


Fig. 2

pressure, will overcome a resistance P_1 and push the piston i to the right. When the weight W has entered the cylinder, let the valve be closed and let a second valve opening to the atmosphere be opened. The gauge pressure in the cylinder is now zero, the velocity is zero, and the distance of the mass of water above the level b is zero; hence, its total energy is zero, and the energy it had before entering the cylinder is precisely equal to the work done on the piston. This work is calculated as follows. The volume of the weight W that enters the cylinder is $\frac{W}{62.5}$ cubic feet, taking

the weight of 1 cubic foot of water as 62.5 pounds. Let A denote the piston area, in square inches, and L the distance, in feet traveled by the piston, then, the volume swept through by the piston is $\left(\frac{A}{144} \times L\right)$ cubic feet, and, since this is also the volume of the entering water,

$$\frac{AL}{144} = \frac{W}{62.5},$$

whence $AL = W \times \frac{144}{62.5} = \frac{W}{.434}$, since $\frac{62.5}{144} = .434$.

The pressure p at the level b is $.434\,h$ pounds per square inch (see *Hydrostatics*). The total pressure on the piston is pA, and the work done by this force pA pounds acting through the distance L feet is

$$pAL = .434\,h \times \frac{W}{.434} = Wh \text{ foot-pounds}$$

It appears, therefore, that the energy of W pounds at the level b due to the pressure is the same as that of W pounds at the level a due to position.

It is more convenient in subsequent formulas to express the pressure energy in terms of the intensity of pressure. As shown in *Hydrostatics*, the intensity of pressure p due to a head of h feet is equal to wh pounds per square inch, where w ($= .434$ pound) is the weight of 12 cubic inches of water. Therefore,

$$h = \frac{p}{w}$$

and

$$Wh = W \times \frac{p}{w}$$

Hence, to find the pressure energy, in foot-pounds, of a given weight of liquid, multiply the weight, in pounds, by the pressure, in pounds per square inch, and divide the result by the weight of 12 cubic inches of the liquid.

Take, now, a mass of water at the intermediate level c. If the liquid is allowed to enter the cylinder f, it will do work on the piston because of its pressure, and, as just shown, the amount of this pressure energy is Wh_1 foot-pounds. But, on leaving the cylinder f, the water can still do the work Wh_2 in falling to the level b, that is, it has energy of

position equal to Wh_2 foot-pounds The sum of the two is $Wh_1 + Wh_2$, or $W(h_1 + h_2)$, that is, Wh foot-pounds

It appears, then, that if the water is at rest, the energy of a given mass with reference to the level b is the same at all levels At the upper level a, the energy is all due to position, passing downwards from a to b, the energy of the position decreases and the pressure energy increases, until at the level b the entire energy is pressure energy

6. The change from one form of energy to another may be represented graphically as in Fig 2 (b) The rectangle $ABCD$ is drawn with AB in the level a and CD in the level b The constant width represents the constant energy Wh The diagonal BD is drawn, then, the horizontal intercepts between AD and BD represent the energy of position at different levels, while those between BD and BC represent the corresponding pressure energy Thus, for the level c,

$$EF = \text{energy of position} = Wh$$
$$FG = \text{pressure energy} = Wh_1$$

Similarly, for the level d, HK and KL represent, respectively, the energy of position and the pressure energy

7. In case of a stream of freely falling water, there is a similar change from energy of position to kinetic energy In Fig 2, MN shows such a stream At M, the weight W of a mass of liquid about to fall has a potential energy of Wh foot-pounds, with reference to the level b The same mass at N has no longer any energy of position, but has acquired kinetic energy, due to the velocity of its fall Now, it was shown in *Kinematics and Kinetics* that a body weighing W pounds and moving with a velocity of v feet per second has a kinetic energy $\frac{Wv^2}{2g}$, also, that the velocity v of a body falling freely through a distance h is equal to $\sqrt{2gh}$, and, therefore, $v^2 = 2gh$ Let K represent the kinetic energy of the body after it has fallen through the distance h Then,

$$K = \frac{Wv^2}{2g} = \frac{W \times 2gh}{2g} = Wh$$

At the intermediate level c, the velocity of the falling vater is $\sqrt{2 g h_1}$, and the kinetic energy is $W h_1$. The energy of position is $W h_2$. The total energy is $W h_1 + W h_2 = W h$. Fig. 2 (b) represents also this change from energy of position to kinetic energy. The decreasing intercepts between $A D$ and $B D$ give the decreasing energies due to position, while the increasing intercepts between $B D$ and $B C$ represent the increasing kinetic energy.

BERNOULLI'S LAW FOR FRICTIONLESS FLOW

8. Statement of Bernoulli's Law.—The energy ossessed by a mass of liquid may be expended in two ways: 1) useful work may be done by the liquid, as exemplified in aterwheels and hydraulic engines; (2) work may be done overcoming various frictional resistances. In either case, ie loss of energy is juivalent to the ork done.

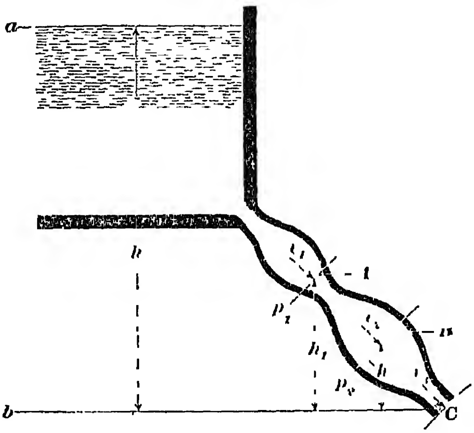


FIG.

In the case of a juid merely flowing a pipe or channel, ere is no work done other bodies—no heels turned, no stons moved—and, it is assumed that e liquid is friction-ss, no energy is pended in friction. follows that during e flow the amount of energy remains constant, and at every oss-section the energy of a given mass of liquid is the me. This statement is *Bernoulli's law, without friction*.

In Fig. 3, water flows from a tank through a pipe of rying cross-section. The level b through the end of the ie is taken as the reference level or datum plane. Consider y section 1 of the pipe, and assume the velocity at that

section to be v_1, denote the mean height of this section above the reference level by h_1, and let p_1 indicate the gauge pressure. At this section, a mass of water of weight W has energies as follows:

$$\text{Energy of position} = W h_1$$

$$\text{Kinetic energy} = \frac{W v_1}{2g}$$

$$\text{Pressure energy} = W \times \frac{p_1}{w}$$

The total energy E at this section is, therefore,

$$E = W h_1 + W \times \frac{v_1}{2g} + W \times \frac{p_1}{w} = W\left(h_1 + \frac{v_1^2}{2g} + \frac{p_1}{w}\right)$$

Similarly, at section B,

$$E = W\left(h_2 + \frac{v_2^2}{2g} + \frac{p_2}{w}\right),$$

According to Bernoulli's law, the energy at B is the same as at A, hence,

$$W\left(h_1 + \frac{v_1^2}{2g} + \frac{p_1}{w}\right) = W\left(h_2 + \frac{v_2^2}{2g} + \frac{p_2}{w}\right),$$

whence
$$h_1 + \frac{v_1^2}{2g} + \frac{p_1}{w} = h_2 + \frac{v_2^2}{2g} + \frac{p}{w}$$

9. Velocity Head and Pressure Head.—In the foregoing equation, h_1 and h_2 are heads or distances, h_1 is the distance of section A above the level b, and h_2 that of section B above the same level. It is convenient to give to a height above the reference level, as h_1 or h_2, the name **potential head**. The term $\frac{v_1^2}{2g}$ represents the height that a body must fall from rest in order to attain the velocity v_1, this height is called the **velocity head**. The term $\frac{p_1}{w}$ represents the head necessary to produce the pressure of p_1 pounds per square inch (see Art. 5), hence, this term is known as the **pressure head**.

Bernoulli's law may now be stated as follows:

In the flow of a constant quantity of water through a pipe or channel, with friction disregarded, the sum of the potential head, velocity head, and pressure head is the same at all sections.

It will be observed that each of the heads here defined, when multiplied by the weight W, gives energy in one form or another. Thus,

$W \times$ potential head = energy of position
$W \times$ velocity head = kinetic energy
$W \times$ pressure head = pressure energy

The constant sum of the three heads is readily determined for a case like that shown in Fig. 3, in which the liquid surface has a free surface. At the level a, $v = 0$, and $p = 0$; hence, the velocity and pressure heads are both equal to zero, and the sum of the three heads is merely the head h, the height of the level a above the reference level. This head h is called the hydrostatic head, or, more commonly, the **static head**. Hence, for any section the sum of the three heads is equal to the static head, with reference to the datum level b.

EXAMPLE.—In Fig. 3, $h = 40$ feet, $h_1 = 12$ feet, and $h_2 = 8$ feet. The area of the section c through which the water is discharging is 36 square inches, and the areas of the sections I and II are, respectively, 60 square inches and 180 square inches. The flow is uniform and is assumed to be frictionless. Required the quantity discharged and the velocity and pressure heads at sections I and II.

SOLUTION.—The tank is so large, compared with the pipe, that it may be assumed without any appreciable error that the water at the level a has no velocity; hence, at a the velocity head is zero, the pressure head is also zero, and the potential head, with reference to the level c, is h. At c, the pressure head is zero, since the water discharges freely into the atmosphere, the potential head, with reference to the level c, is also zero, and the velocity head is $\frac{v^2}{2g}$. According to Bernoulli's law, the sum of the heads at a must be equal to the sum of the heads at c; that is,

$$h = \frac{v^2}{2g},$$

whence

$$v = \sqrt{2gh} = \sqrt{2 \times 32.16 \times 40} = 50.7 \text{ ft. per sec.}$$

From the formula in Art. 2,

$$Q = Av = \tfrac{36}{144} \times 50.7 = 12.7 \text{ cu. ft. per sec., nearly. Ans.}$$

The velocities along the pipe vary inversely as the areas of the sections (Art. 3); hence

$$v_1 = v \times \frac{A}{A_1} = 50.7 \times \tfrac{36}{60} = 30.42 \text{ ft. per sec.}$$

and

$$v_2 = v \times \frac{A}{A_2} = 50.7 \times \tfrac{36}{180} = 10.14 \text{ ft. per sec.}$$

The corresponding velocity heads are

$$\frac{v_1^2}{2g} = \frac{30.42^2}{2 \times 32.16} = 14.4 \text{ ft} \quad \text{Ans}$$

and

$$\frac{v_2^2}{2g} = \frac{10.14^2}{2 \times 32.16} = 1.6 \text{ ft} \quad \text{Ans}$$

The sum of the three heads for all sections is 40 ft Hence, at section A, the pressure head is

$$40 - h_1 - \frac{v_1^2}{2g} = 40 - 12 - 14.4 = 13.6 \text{ ft} \quad \text{Ans}$$

and at section B it is

$$40 - 8 - 1.6 = 30.4 \text{ ft} \quad \text{Ans}$$

10. Piezometric Measurements —In Fig 4, which is a reproduction of Fig 1, is shown a horizontal pipe

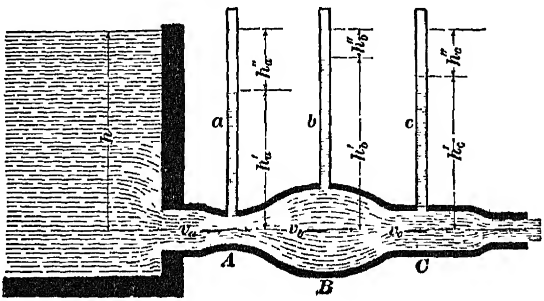


FIG 4

discharging into the atmosphere Taking the axis of the pipe as the reference level, the potential heads at that level are all zero, and, therefore, the last equation of Art 8 becomes

$$\frac{v_1^2}{2g} + \frac{p_1}{w} = \frac{v_2^2}{2g} + \frac{p_2}{w} = h$$

That is, the sum of the velocity head and pressure head is equal to the static head at the level of the axis of the pipe At the sections A, B, and C, tubes a, b, and c are inserted in the pipe If the end of the pipe is stopped so that there is no flow, the water will rise in the tubes until it reaches the level of the water in the tank As soon, however, as the pipe is opened and flow begins, the water falls in the tubes

Let h_a' be the height of the water in the tube a above the reference level This is evidently the pressure head at

section A If the velocity at this section is denoted by v_a, the last equation of Art 8 gives

$$h = \frac{v_a^2}{2g} + h_a',$$

whence $$\frac{v_a^2}{2g} = h - h_a' = h_a''$$

denoting by h_a'' the velocity head at A

The same reasoning applies to the other tubes In each case, the height of the water in the tube measures the *pressure head* for the section, and the difference between the level in the tube and the level in the reservoir or tank measures the *velocity head*

Where the cross-section has the greatest area the velocity of flow is least, and in consequence the velocity head is least and the pressure head is greatest, and, vice versa, at the smallest section the velocity and velocity head are greatest and the pressure head is therefore least

11. A gauge or tube inserted in a pipe to show the pressure of the water is called a **piezometer** When a piezometer is to be placed in a pipe through which water is flowing, the tube should always be so inserted as to be at right angles to the current in the pipe, as shown at a, Fig 5

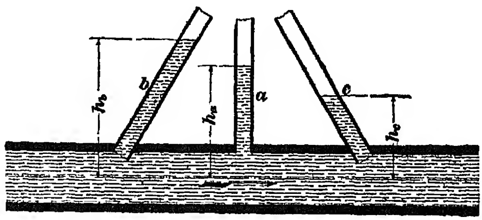


FIG 5

For, if the tube is so inclined that the current flows against the end, as shown at b, the action of the current will force the water into the tube, and cause it to rise higher than the head due to the pressure, and, if inclined in the opposite direction, as at c, the action of the current will reduce the indicated pressure The end of the tube should be made smooth and flush with the inner surface of the pipe While

it is usual to tap the tube into the top of the pipe, it is advisable for accurate measurements to connect tubes with the sides and bottom also. Results obtained from piezometer measurements are liable to some uncertainty.

EXAMPLE.—The velocities v_a, v_b, and v_c, Fig. 4, being, respectively, 30, 5, and 16 feet per second, and the static head h, 20 feet, required the velocity head and height of water in the tube at each section.

SOLUTION.—The three velocity heads are

$$h_a'' = \frac{v_a^2}{2g} = \frac{30^2}{2 \times 32.16} = 14 \text{ ft.} \quad \text{Ans.}$$

$$h_b'' = \frac{5^2}{2 \times 32.16} = .39 \text{ ft.} \quad \text{Ans.}$$

$$h_c'' = \frac{16^2}{2 \times 32.16} = 3.98 \text{ ft.} \quad \text{Ans.}$$

In tube a, $h_a' = 20 - 14 = 6$ ft. Ans.

In tube b, $h_b' = 20 - .39 = 19.61$ ft. Ans.

In tube c, $h_c' = 20 - 3.98 = 16.02$ ft. Ans.

12. Remarks on Bernoulli's Law.—Bernoulli's law without friction is the basis of all the formulas for the flow of water through orifices, weirs, and short tubes. In these cases, the friction between the liquid and the enclosing surfaces is very small in comparison with the other quantities that enter the calculation.

In the case of flow in long pipes, channels, streams, etc., the friction becomes a very important factor, and Bernoulli's law must be modified accordingly. This modified form will be taken up with the flow of water in long pipes, channels, and streams.

FLOW OF WATER THROUGH ORIFICES

THEORETICAL VELOCITY AND DISCHARGE

13. Flow Through a Small Orifice Into the Atmosphere.—In Fig. 6, it is assumed that the vessel is kept full to the level a. Openings, or **orifices**, are made at the levels m, b, and c. These orifices are supposed to be so small, compared with their distances below the surface, that their dimensions may be neglected. The head on any of

them may, therefore, be taken as the distance of any part of it from the surface

If the level of any orifice whose head is h is taken as a plane of reference, and the water is assumed to discharge freely into the atmosphere with a velocity v, both the potential and the pressure head are equal to zero, and, therefore, the velocity head $\frac{v^2}{2g}$ must be equal to the static head h (Art 9), that is,

$$\frac{v^2}{2g} = h,$$

whence

$$v = \sqrt{2gh} \qquad (1)$$

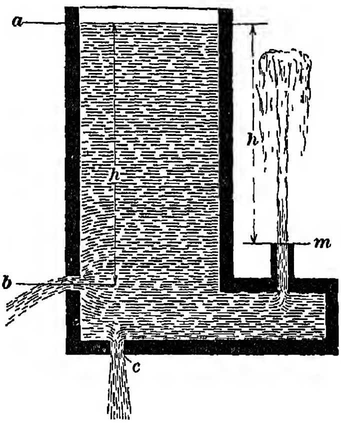


FIG 6

This value of v is called the **theoretical velocity** of efflux through the orifice Owing to friction and other resistances, the actual velocity is a little less than the theoretical, as will be explained in another place It will be observed that the theoretical velocity of efflux through a small orifice is the same as the velocity of a body falling freely in a vacuum through a distance equal to the head on the orifice

Using the value 32 16 for g, or 8 02 for $\sqrt{2g}$, formula **1** may be written

$$v = 8\ 02\sqrt{h} \qquad (2)$$

Also, the head h necessary to produce a velocity v is given by the formula

$$h = \frac{v^2}{2g} = \frac{v^2}{2 \times 32\ 16} = 01555\,v^2 \qquad (3)$$

14. Flow Under Pressure—In Fig 7, the water at the upper level a is loaded with a weight W, which produces a pressure on the surface of the liquid The water flows through the orifice o into a second vessel, in which the liquid level c is h_2 feet above the orifice To determine the

theoretical velocity of discharge through the orifice, the level b is taken as the reference level, and, applying Bernoulli's law, it is found that, at the level a, the potential head is h_1, the velocity head is zero, and the pressure head is $\frac{p'}{w}$, where p' denotes the pressure per square inch on the water surface due to the weight W. If F is the area of the liquid surface, p' is equal to $\frac{W}{F}$. For the jet emerging from the orifice, the potential head is zero, the velocity head is $\frac{v^2}{2g}$, and the pressure head is h_2. From the last equation of Art. 8,

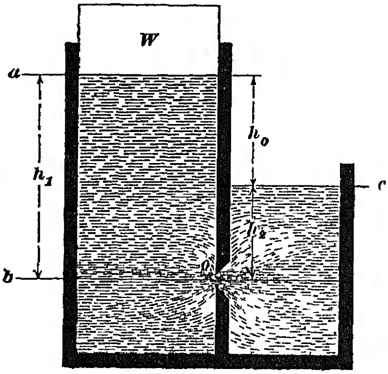


FIG. 7

$$h_1 + 0 + \frac{p'}{w} = 0 + \frac{v^2}{2g} + h_2,$$

whence

$$\frac{v^2}{2g} = \frac{p'}{w} + h_1 - h_2$$

Let h' denote the head that gives the pressure p', then,

$$\frac{v^2}{2g} = h' + h_1 - h_2 = h' + h_o$$

and $\qquad v = \sqrt{2g(h' + h_o)} = 8.02\sqrt{h' + h_o} \qquad (1)$

If the jet discharges into the atmosphere, as in Fig. 6,

$$h_2 = 0,\ h_o = h_1,$$

and, therefore, $\qquad v = 8.02\sqrt{h' + h_1} \qquad (2)$

On the other hand, if there is no extra pressure on the surface at a, and the jet still discharges into the atmosphere,

$$h' = 0,\ h_2 = 0,\ h_o = h_1$$

and $\qquad v = 8.02\sqrt{h_1},$

which is the same as formula 2, Art. 13.

EXAMPLE.—Let the area of the liquid surface at a, Fig. 7, be 5 square feet, and let W equal 2 tons. The orifice o is 20 feet below the level a and 8 feet below the level c. (a) Find the velocity of flow through the orifice. (b) Find the velocity of flow at the level b, the discharge being into the atmosphere.

SOLUTION.—(*a*) The external pressure is

$$\frac{2{,}000 \times 2}{5 \times 144} = 5.56 \text{ lb. per sq. in.}$$

and from the equation in *Hydrostatics*, $h = 2.304\,p$, the corresponding head h' is $2.304 \times 5.56 = 12.8$ ft., $h_1 = 20$, $h_2 = 8$, and $h_0 = 20 - 8 = 12$. From formula **1**,

$$v = 8.02\sqrt{h' + h_0} = 8.02\sqrt{12.8 + 12} = 39.94 \text{ ft. per sec.} \quad \text{Ans.}$$

(*b*) From formula **2**,

$$v = 8.02\sqrt{h' + h_1} = 8.02\sqrt{12.8 + 20} = 45.93 \text{ ft. per sec.} \quad \text{Ans.}$$

15. Large Orifice in Bottom of Vessel.—If the dimensions of the orifice are large compared with those of the enclosing vessel, the formula for the theoretical velocity of efflux is obtained as follows. Let a denote the area of the orifice, and A the area of the liquid surface at the upper level (see Fig. 8). Further, let v denote the velocity through the orifice, and v_0 the velocity with which the water at the upper surface descends. According to Art. **3**, the velocities are inversely as the areas, that is,

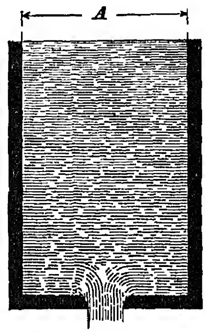


FIG. 8

$$\frac{v_0}{v} = \frac{a}{A},$$

whence $$v_0 = v\frac{a}{A}$$

The velocity head at the upper level is, therefore,

$$\frac{v_0^2}{2g} = \frac{\left(v \times \frac{a}{A}\right)^2}{2g} = \frac{v^2}{2g} \times \frac{a^2}{A^2}$$

As in the example in Art. **9**, the pressure heads are both zero, and the potential head at the orifice is zero, while at the surface it is h. The last equation of Art. **8** becomes therefore,

$$h + \frac{v_0^2}{2g} + 0 = 0 + \frac{v^2}{2g} + 0,$$

or $$h + \frac{v^2 a^2}{2gA^2} = \frac{v^2}{2g},$$

whence $$v = \sqrt{\frac{2gh}{1 - \frac{a^2}{A^2}}} = 8.02\sqrt{\frac{h}{1 - \left(\frac{a}{A}\right)^2}}$$

If A is more than twenty times a, the factor $\frac{a^2}{A^2}$ in the formula just given may be neglected, and the formula $v = \sqrt{2\,g\,h}$ may be used

EXAMPLE.—A vessel has a rectangular cross-section, 11 in × 14 in., and the upper surface of the water is 14 feet above the bottom. If an orifice 4 inches square is made in the bottom of the vessel, what is the velocity of efflux?

SOLUTION.—The area of the cross-section is $14 \times 11 = 154$ sq. in. The area of the orifice is $4 \times 4 = 16$ sq. in. Since $154 \div 16 = 9\frac{5}{8}$, the area of the base is less than twenty times the area of the orifice, hence, using the formula

$$v = 8.02\sqrt{\frac{14}{1-\left(\frac{16}{154}\right)^2}} = 30.17 \text{ ft. per sec.} \quad \text{Ans.}$$

16. Large Orifice in Side of Vessel.—When the dimensions of the orifice are large, and the orifice is in the side of the vessel, as shown in Fig. 9, the head is different for different parts of the orifice, and consequently the theoretical velocity varies at different parts of the orifice. Thus, at the top, $v_1 = \sqrt{2\,g\,h_1}$, while at the bottom, $v_2 = \sqrt{2\,g\,h_2}$. If the mean head h is more than about four times the vertical dimension c, formula **1**, Art. **13**, gives the mean velocity with sufficient exactness, and the discharge is given by the combination of the formula in Art. 2 and formula **1**, Art. **13**; thus,

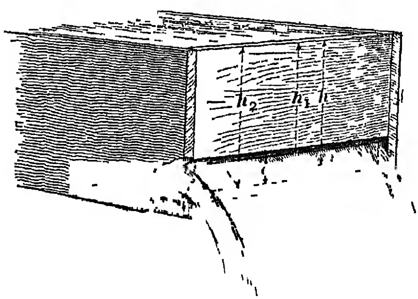


FIG. 9

$$Q = F\,v = F\sqrt{2\,g\,h} = 8.02\,F\sqrt{h} \qquad (1)$$

For a circular orifice, $F = \frac{1}{4}\pi c^2$, and for a rectangular orifice, $F = b\,e$, denoting the width of the orifice by b.

When h is less than $4\,c$, a more exact formula for a rectangular orifice is the following:

$$Q = \tfrac{2}{3}\,b\sqrt{2\,g}\left(h_2^{\frac{3}{2}} - h_1^{\frac{3}{2}}\right) \qquad (2)$$

17. Let the top of the orifice be at or above the liquid level, and let the head on the lower edge be denoted by H, as shown in Fig 10 Then, $h_1 = 0$, $h_2 = H$, and formula **2**, Art **16**, becomes

$$Q = \tfrac{2}{3} b \sqrt{2g} H^{\frac{3}{2}}$$
$$= \tfrac{2}{3} b H \sqrt{2gH},$$

or, since bH = area F of orifice,

$$Q = \tfrac{2}{3} F \sqrt{2gH}$$

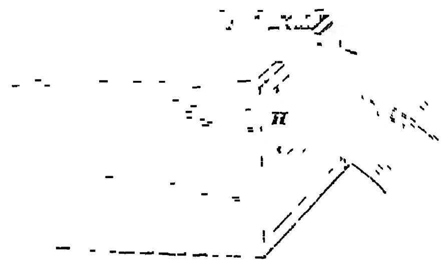


FIG 10

EXAMPLE 1 —Compute the theoretical discharge from a vertical circular orifice 6 inches in diameter whose center is 9 feet below the water level

SOLUTION — $F = \frac{1}{4}\pi d^2 = .7854 \times (\frac{6}{12})^2 = .19635$ sq ft, $h = 9$ Substituting in formula **1**, Art **16**,

$$Q = 8\ 02 \times .19635 \times \sqrt{9} = 4\ 72 \text{ cu ft per sec} \quad \text{Ans}$$

EXAMPLE 2 —What is the theoretical discharge for a rectangular orifice whose length is 4 5 feet and whose height is 9 inches, the top edge being at the level of the water?

SOLUTION —To apply the formula, we have $F = 4\ 5 \times \frac{9}{12} = 3\ 375$ sq ft, $H = \frac{9}{12} = \frac{3}{4}$ ft Substituting in the formula,

$$Q = \tfrac{2}{3} \times 3\ 375 \times \sqrt{2 \times 32\ 16 \times \tfrac{3}{4}} = 15\ 63 \text{ cu ft per sec} \quad \text{Ans}$$

EXAMPLES FOR PRACTICE

1 The velocities in three parts of a horizontal pipe with varying cross-section are 3, 8, and 11 feet per second, respectively If piezometric tubes are placed at these three points, determine the height of the water in the tubes, assuming that the static head is 25 feet

$$\text{Ans} \begin{cases} 24\ 86 \text{ ft} \\ 24\ 00 \text{ ft} \\ 23\ 12 \text{ ft} \end{cases}$$

2 A weight of 12 2 tons is placed on the surface of the water contained in a rectangular box whose cross-section is 10 2 square feet Calculate the velocity of flow through an orifice 9 feet below the surface and 1 square inch in cross-section Ans 55 15 ft per sec

3 The area of an orifice in the bottom of a rectangular tank is 2 5 inches square The surface of the water is 12 5 feet above the orifice and the area of the cross-section of the tank is 125 square inches Calculate the velocity of efflux Ans 28 39 ft per sec

4 A 2-inch circular hole is tapped in the side of a stand pipe 20 feet in diameter and 100 feet high If the water level is 81 5 feet above the orifice, determine the velocity of efflux

Ans 72 4 ft per sec

ACTUAL DISCHARGE THROUGH STANDARD ORIFICES

18. Standard Orifices —An orifice is called a **standard orifice** when the flow through it takes place in such a manner that the jet touches the opening on the inside edge only A hole in a thin plate, as shown in Fig 11, is such an orifice, as is also a square-edged hole in the side of the vessel, as in Fig 12, when the side is so thin that the jet does not touch it beyond the inner edge If the sides of the vessel are very thick, a standard orifice can be made by beveling the outer edges, as shown in Fig 13

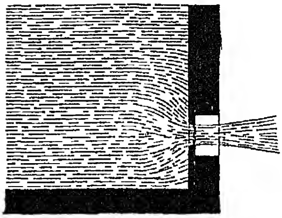


Fig 11

Small quantities of water can be measured comparatively accurately by means of standard orifices These are usually placed in the vertical sides of the tank or reservoir, though an orifice may be placed in the bottom The head on the orifice should preferably be greater than four times the vertical dimension of the orifice If measurements are made carefully, the calculated discharge should not vary from the actual discharge by more than 1 or 2 per cent

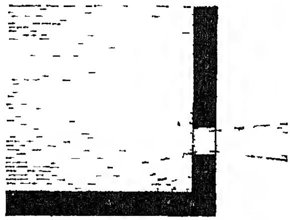


Fig 12

19. Contraction of the Jet. When a jet issues from a standard orifice, it *contracts*, so that the diameter is least at a distance from the edge equal to about one-half the diameter of the orifice Beyond this point the jet gradually

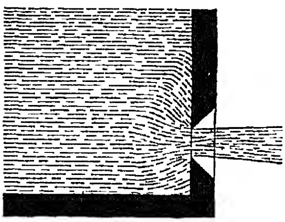


Fig 13

enlarges and becomes broken by the effect of the resistance of the air

The ratio of the area of the smallest section of the jet to the area of the orifice is called the **coefficient of contraction.** Let F_0 and F denote, respectively, the area of the contracted section and the area of the orifice, and let c_1 denote the coefficient of contraction, then,

$$c_1 = \frac{F_0}{F},$$

and, therefore, $F_0 = c_1 F$

The value of c_1 is almost the same for all forms of orifice and for all heads The most reliable experiments indicate that this value lies between 60 and 64 A probable mean value is 62

20. Coefficient of Velocity—Because of the slight friction at the edge of the orifice, and also because of the internal friction of the water due to contraction, the actual velocity of the jet at the smallest cross-section is slightly less than the theoretical velocity as calculated by formulas **1** and **2** of Art **13.** The ratio of the actual velocity to the theoretical velocity is called the **coefficient of velocity.** Let this coefficient be denoted by c_2, then, if v_0 denotes the actual and v the theoretical velocity,

$$c_2 = \frac{v_0}{v},$$

and, therefore, $v_0 = c_2 v$

It is found that c_2 is greater for high heads than for low, and values ranging from 975 to nearly 1 have been obtained by different experimenters An average value usually taken is 98, which means that the actual velocity is but 98 per cent of the theoretical velocity

21. Coefficient of Discharge—The actual discharge through an orifice is much less than the theoretical discharge, because of the contraction of the jet and also because of the diminution of the theoretical velocity The ratio of the actual to the theoretical discharge is called the **coefficient of discharge**

Let Q = theoretical discharge, in cubic feet per second,
Q_a = actual discharge,
c_3 = coefficient of discharge

Then,
$$c_3 = \frac{Q_a}{Q}, \text{ and } Q_a = c_3 Q$$

The coefficient of discharge is equal to the product of the coefficient of velocity and the coefficient of contraction. For we have (Art. **2**)

$$Q_a = F_a v_a$$

or, substituting the values of F_a and v_a (Arts. **19** and **20**),

$$Q_a = c_1 F \times c_2 v = c_1 c_2 \times F v$$

or, since Fv is equal to the theoretical discharge Q,

$$Q_a = c_1 c_2 \times Q,$$

whence
$$\frac{Q_a}{Q} = c_1 c_2, \text{ that is, } c_3 = c_1 c_2$$

Using for c_1 and c_2 the mean values .62 and .98, respectively, the last equation becomes

$$c_3 = .62 \times .98 = .61, \text{ nearly}$$

22. Formulas for Actual Discharge.—The formulas of Arts. **16** and **17** give the theoretical discharge Q for vertical orifices. For the actual discharge Q_a, it is necessary to introduce the coefficient c_3 in the second member. Provided the head is at least four times the vertical dimension of the orifice, we have

$$Q_a = 8.02\, c_3 F \sqrt{h} \qquad (1)$$

For a circular orifice of diameter d feet, $F = .7854\, d^2$, and, therefore,

$$Q_a = 8.02 \times .7854\, c_3 d^2 \sqrt{h} = 6.299\, c_3 d^2 \sqrt{h} \qquad (2)$$

If the orifice is a square whose side is d feet, $F = d^2$, and

$$Q_a = 8.02\, c_3 d^2 \sqrt{h} \qquad (3)$$

For a rectangular orifice of width b and depth d, $F = bd$, and

$$Q_a = 8.02\, c_3 b d \sqrt{h} \qquad (4)$$

If the head on the rectangular orifice is less than four times the dimension d, the discharge is found by introducing c_3 in the more exact formula **2**, Art. **16**; that is,

$$Q_a = \tfrac{2}{3} b c_3 \sqrt{2g}\left(h_2^{\frac{3}{2}} - h_1^{\frac{3}{2}}\right)$$

or

$$Q_a = 5.35\, b c_3 \left(h_2^{\frac{3}{2}} - h_1^{\frac{3}{2}}\right) \qquad (5)$$

The head h in formulas **1**, **2**, **3**, and **4** is measured to the center of the orifice, and h and all orifice dimensions are to be taken in feet The quantity Q_0 will then be given in cubic feet per second The values of c_3 are to be taken from Tables I, II, and III Experiments show that c_3 varies with the head h, with the kind of orifice, and with the vertical

TABLE I

COEFFICIENTS OF DISCHARGE FOR CIRCULAR VERTICAL ORIFICES

Head h Feet	Diameter of Orifice, in Feet						
	02	04	07	1	2	6	1
4		637	624	618			
6	655	630	618	613	601	593	
8	648	626	615	610	601	594	590
1 0	644	623	612	608	600	595	591
1 5	637	618	608	605	600	596	593
2 0	632	614	607	604	599	597	595
2 5	629	612	605	603	599	598	596
3 0	627	611	604	603	599	598	597
4 0	623	609	603	602	599	597	596
6 0	618	607	602	600	598	597	596
8 0	614	605	601	600	598	596	596
10 0	611	603	599	598	597	596	595
20 0	601	599	597	596	596	596	594
50 0	596	595	594	594	594	594	593
100 0	593	592	592	592	592	592	592

dimension of the orifice The tables, which are taken from high authorities on hydraulics, apply only to standard orifices

In the table of coefficients for rectangular orifices, it will be observed that values apply only to an orifice 1 foot wide Experiments show that c_3 increases somewhat with the breadth, so that for a breadth much in excess of 1 foot it is advisable to increase the tabular value

For approximate calculations, the value of c_s in formulas **2**, **3**, and **4** may be taken as 615 The calculated discharge will not vary by more than 3 or 4 per cent from the actual discharge, provided the orifice has a height of not more than 18 inches nor less than 1 inch, and that the head lies between 1 foot and 30 feet

In the solution of problems, it is entirely permissible to use the value of c_s that most nearly fits the given conditions

TABLE II

COEFFICIENTS OF DISCHARGE FOR SQUARE VERTICAL ORIFICES

Head h Feet	Side of Square, in Feet						
	02	04	07	1	2	6	1
4		643	628	621			
6	660	636	623	617	605	598	
8	652	631	620	615	605	600	597
1 0	648	628	618	613	605	601	599
1 5	641	622	614	610	605	602	601
2 0	637	619	612	608	605	604	602
2 5	634	617	610	607	605	604	602
3 0	632	616	609	607	605	604	603
4 0	628	614	608	606	605	603	602
6 0	623	612	607	605	604	603	602
8 0	619	610	606	605	604	603	602
10 0	616	608	605	604	603	602	601
20 0	606	604	602	602	602	601	600
50 0	602	601	601	600	600	599	599
100 0	599	598	598	598	598	598	598

It is useless refinement to keep more than three significant figures in expressing its value, and no more than four figures of the final result should be used

EXAMPLE 1 —What is the discharge from a circular orifice $1\frac{1}{2}$ inches in diameter if the head is 7 feet?

TABLE III

COEFFICIENTS OF DISCHARGE FOR RECTANGULAR ORIFICES 1 FOOT WIDE

Head h on Center of Orifice Feet	Depth of Orifice, in Feet						
	125	25	5	75	1	1 5	2
4	634	633	622				
6	633	633	619	614			
8	633	633	618	612	608		
1 0	632	632	618	612	606	626	
1 5	630	631	618	611	605	626	628
2 0	629	630	617	611	605	624	630
2 5	628	628	616	611	605	616	627
3 0	627	627	615	610	605	614	619
4 0	624	624	614	609	605	612	616
6 0	615	615	609	604	602	606	610
8 0	609	607	603	602	601	602	604
10 0	606	603	601	601	601	601	602
20 0				601	601	601	602

SOLUTION — The diameter of the orifice is 125 ft, from Table I, the coefficient is found to be 600 for an orifice 1 ft in diameter under a head of 6 ft, and the same for a head of 8 ft In the same way, the coefficient for a diameter of 2 ft is 598 from 6 ft to 8 ft head Hence, take c_1 = 600, as the diameter is nearer 1 ft than 2 ft From formula 2,

$Q_a = 6.299 \times 6 \times 125^2 \times \sqrt{7}$
= 1562 cu ft per sec Ans

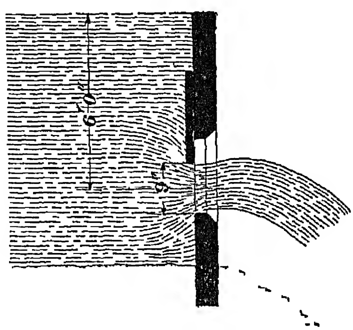


FIG 14

EXAMPLE 2 — A dam across a stream has an opening closed by a sluice gate (see Fig 14) in such a way that the gate when opened forms a standard orifice of rectangular cross-section The width of the opening is 1 foot, and it is found that the water keeps flush with the top of the dam when the gate is opened 9 inches What is the rate

of flow of the stream, if the center of the opening is 6 feet below the surface of the water in the dam?

SOLUTION —From Table III, the value of c_3 for an orifice 1 ft wide and 75 ft deep is found to be 604 when the head is 6 ft Since the head is more than four times the depth, formula **4** should be used The substitution of known values in that formula gives

$$Q_0 = 8\ 02 \times 604 \times 1 \times 75\sqrt{6} = 8\ 9 \text{ cu ft per sec} \quad \text{Ans}$$

EXAMPLE 3 —Calculate the discharge through a rectangular orifice 2 feet deep and 3 feet wide with its upper edge 5 feet below the liquid level Assume c_3 = 615

SOLUTION —Since the mean head, 6 ft, is less than $4d$, formula **5** should be used Here, $h_1 = 5$ ft, $h_2 = h_1 + 2$ ft $= 7$ ft Substituting in the formula,

$$Q_0 = 5\ 35 \times 3 \times 615\,(7^{\frac{3}{2}} - 5^{\frac{3}{2}}) = 72\ 45 \text{ cu ft per sec} \quad \text{Ans}$$

23. Submerged Orifice —An example of a submerged rectangular orifice is shown in Fig 15 From formula **2**, Art **13**, the theoretical velocity v is $8\ 02\sqrt{h_0}$, hence, for the theoretical discharge we have

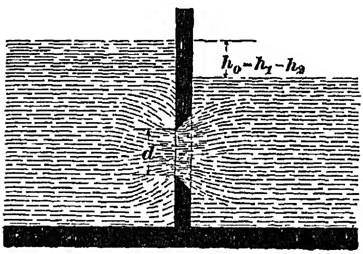


FIG 15

$$Q = 8\ 02\,F\sqrt{h_0}$$
$$= 8\ 02\,b\,d\sqrt{h_0} \qquad (1)$$

Using the mean value 615 for c_3, the actual discharge is given by the formula

$$Q_0 = 615 \times 8\ 02\,b\,d\sqrt{h_0} = 4\ 932\,b\,d\sqrt{h_0} \qquad (2)$$

24. Reduced Contraction Rounded Orifices —If the orifice is made at the side of the vessel, as shown at a, Fig 16, or at the bottom, as shown at b, the contraction of the stream is reduced and the discharge is increased Experiments show the increase to be about 3 5 per cent for a, and from 6 to 12 per cent for b These values have not been accurately determined, and where accurate measurements are to be

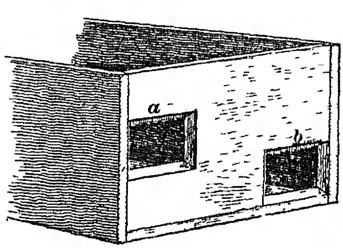


FIG 16

made the orifice should always be arranged as shown in Figs. 11, 12, and 13.

If the inner edge of the orifice is rounded, as shown in Figs. 17 and 18, the coefficient of discharge is increased, and may be made nearly 1, if the edge is rounded as shown in Fig. 18.

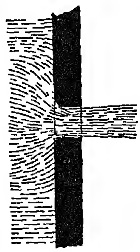


FIG. 17

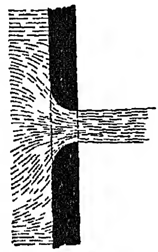


FIG. 18

25. The Miners' Inch. The **miners' inch** is an arbitrary unit for measuring water by its flow through an orifice. This unit is mainly used in measuring water for irrigation and mining purposes. It is the quantity of water flowing in a certain time through an orifice of specified dimensions under a specified head. Both the dimensions of the orifice and the head vary in different localities, so that the miners' inch has not a fixed value. The orifice is sometimes taken as 1 inch square, the head as 6.5 inches, and the discharge is expressed in cubic feet per minute or in gallons per day. For the dimensions just stated Table II gives .624 as the approximate value of the coefficient of discharge. Therefore, by formula **1**, Art. **22**,

$$Q_a = \frac{.624 \times 8.02}{144} \times \sqrt{\frac{6.5}{12}} = .0256 \text{ cu. ft. per sec.}$$

$$= 1.536 \text{ cu. ft. per min.} = 16{,}540 \text{ gal. per day,}$$

which is the value of the miners' inch for the specified conditions.

EXAMPLES FOR PRACTICE

1. What is the discharge, in cubic feet per minute, from a standard circular orifice whose diameter is $2\frac{1}{2}$ inches, if the head is 20 feet?
Ans. 43.72 cu. ft.

2. A square orifice in the side of a reservoir measures .2 foot on each side, and the head on the center is 22 feet. What is the discharge in cubic feet per second?
Ans. .9058 cu. ft.

3. What is the discharge from a rectangular orifice 1 foot wide, if the head on the upper edge is $2\frac{1}{2}$ feet and the depth of the orifice is $10\frac{1}{2}$ inches?
Ans. 7.309 cu. ft. per sec.

4 What is the approximate discharge from a rectangular gate in the side of a dam when the breadth is 15 inches, the depth 6 inches, and the head on the upper edge $4\frac{1}{2}$ feet? Use the approximate coefficient of discharge .615 Ans 6.719 cu ft per sec

5 What is the discharge from a submerged rectangular orifice $1\frac{1}{2}$ feet wide and 1 foot deep, if the difference in the level of the water on the two sides of the orifice is $3\frac{1}{2}$ feet? Ans 13.84 cu ft per sec

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FLOW THROUGH SHORT TUBES

26. Efflux From a Standard Tube—The manner in which water flows through a short tube in the side of a reservoir is shown in Fig 19 The jet at first contracts to a section smaller than that of the tube, but afterwards expands again and fills the tube as it emerges into the atmosphere That this result may be obtained, several conditions are required (1) The tube must be *standard*, that is, the inner corners must be sharp and the length must not be less than about $2\frac{1}{2}$ times the diameter (2) The head must not be more than about 40 or 50 feet (3) The interior of the tube must be wetted by the water, that is, it must not be greasy

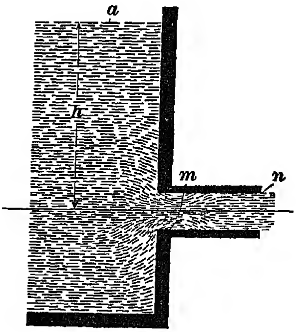

FIG 19

The pressure at the section n at the end of the tube is that of the atmosphere Since the velocity at the contracted section m must be greater than at the section n, it follows that the pressure at m must be less than that at n Hence, the pressure at m is less than that of the atmosphere This fact may be shown experimentally as in Fig 20 If a

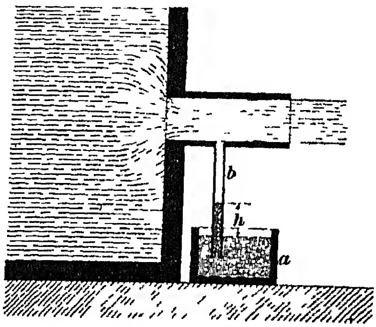

FIG 20

small branch tube b is carried down into a cup of mercury a, it will be observed that the mercury rises in the tube to a height h. This shows that the pressure of the air on the mercury in the cup a is greater than the pressure in the tube b, and the excess of atmospheric pressure over the pressure in the tube at the contracted jet is measured by the height h.

If Bernoulli's law is applied, the theoretical velocity, as in the case of an orifice, is given by the formula

$$v = \text{velocity at } n = \sqrt{2gh}$$

The actual velocity v_0 is given by the formula

$$v_0 = c_1' v = c_1' \sqrt{2gh}$$

in which c_1' is the coefficient of velocity.

The values of c_1' vary slightly with the head, but a mean is .815 or .82. For low heads it rises to .83, and for high heads it drops to .80.

Since the issuing jet has the same area as the tube, the coefficient of contraction is 1, and, therefore, the coefficient of discharge is the same as the coefficient of velocity.

27. Conical Tubes and Nozzles.—For a conical tube, as shown in Fig. 21, the coefficient of discharge

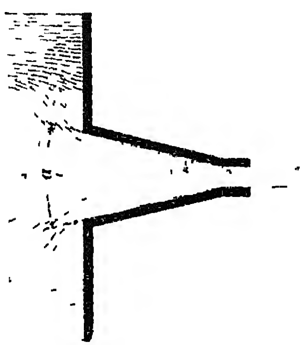

FIG. 21

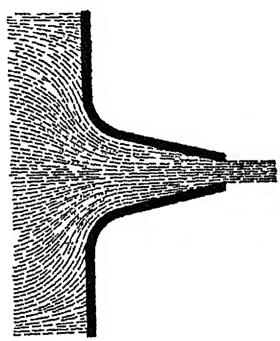

FIG. 22

reaches a maximum value of .946 when the angle a of the cone is about $13\frac{1}{2}°$. The coefficient of velocity increases with the angle of the cone until it becomes about the same as the coefficient for a standard orifice. If the inner edge of

the tube is well rounded, as shown in Fig 22, the coefficient of discharge is still further increased and may be made nearly 1

28. A **nozzle** is a kind of conical tube with a cylindrical tip Nozzles are used when it is desired to deliver water with a high velocity for any purpose Their most common application is in connection with hose for fire purposes, etc By means of nozzles, a very high coefficient of velocity is obtained, and a large percentage of the energy of the jet is thereby utilized The theoretical height to which a stream from a nozzle can be thrown is equal to the head that would produce the velocity with which the jet flows from the nozzle If v is this velocity, the theoretical height is $\frac{v^2}{2g}$ The resistance of the air always reduces this height Under low heads, the coefficient of velocity for a nozzle as ordinarily constructed is about 98

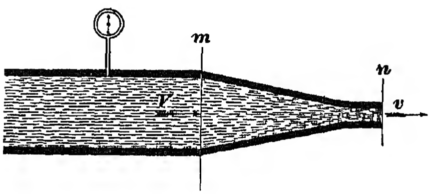

Fig 23

29. With the aid of Bernoulli's law, the velocity of the jet issuing from a nozzle can be found when the pressure of the water entering the nozzle is known In Fig 23, let p be the pressure at the section m as determined by a gauge, and let V be the velocity at this section At the end n the pressure is zero (on the gauge), and the velocity is denoted by v

$$\text{Total head at } m = \frac{p}{w} + \frac{V^2}{2g}$$

$$\text{Total head at } n = \frac{v^2}{2g}$$

If friction is neglected, Bernoulli's law gives

$$\frac{v^2}{2g} = \frac{p}{w} + \frac{V^2}{2g}$$

whence

$$v = \sqrt{2g\left(\frac{p}{w} + \frac{V^2}{2g}\right)}$$

This is the theoretical velocity. For the actual velocity, we have (Art. 26)

$$v_0 = c_1' \sqrt{2g\left(2.304\,p + \frac{V^2}{2g}\right)} \qquad (1)$$

Let D and d denote, respectively, the diameters at sections m and n; then, since the velocities V and v are inversely as the areas at m and n (Art. 3), we have

$$V : v_0 = \tfrac{1}{4}\pi d^2 : \tfrac{1}{4}\pi D^2,$$

whence $\quad V = v_0 \times \dfrac{d^2}{D^2}$, and $V^2 = v_0^2 \times \dfrac{d^4}{D^4}$

From equation (1),

$$\frac{v_0^2}{2g c_1'^2} = 2.304\,p + \frac{V^2}{2g} = 2.304\,p + \frac{v_0^2}{2g} \times \frac{d^4}{D^4}$$

Transposing,

$$\frac{v_0^2}{2g} \times \left(\frac{1}{c_1'^2} - \frac{d^4}{D^4}\right) = 2.304\,p$$

from which

$$v_0 = \sqrt{\frac{2g \times 2.304\,p}{\dfrac{1}{c_1'^2} - \dfrac{d^4}{D^4}}} = \frac{12.17\,c_1'\sqrt{p}}{\sqrt{1 - c_1'^4\left(\dfrac{d}{D}\right)^4}}$$

EXAMPLE.—The pressure on a nozzle is 70 pounds per square inch, the large diameter is $1\frac{1}{2}$ inches, and the diameter of the tip is $\frac{1}{2}$ inch. (*a*) Find the velocity of the jet with $c_1' = .98$. (*b*) To what height will the jet rise, neglecting the resistance of the air?

SOLUTION.—(*a*) From the above formula,

$$v_0 = \frac{12.17 \times .98 \times \sqrt{70}}{\sqrt{1 - .98^2 \times \left(\dfrac{.5}{1.5}\right)^4}} = 100.4 \text{ ft. per sec.} \quad \text{Ans.}$$

(*b*) $$h = \frac{v_0^2}{2g} = 156.7 \text{ ft.} \quad \text{Ans.}$$

30. Diverging and Compound Tubes.—In Fig. 24 is shown a conical diverging tube with sharp inner corners. The tubes shown in Figs. 25 and 26 are **compound**, the tube beyond the smallest section *a* is divergent, while the part leading to the minimum section is either a rounded converging entrance, as shown in Fig. 25, or a conical converging tube, as shown in Fig. 26. The tube shown in Fig. 26 is called a **Venturi tube.**

It is found by experiment that the discharge through a tube of the form shown in Fig 25 is much in excess of the discharge through an orifice or a straight tube of the same minimum diameter In fact, the discharge is greater than the theoretical discharge due to the head, that is, the coefficient of discharge c_3 exceeds 1 Experiments by Eytelwein on a Venturi tube 8 inches long and with maximum and minimum diameters of 1 8 inches and 1 inch respectively, gave c_3 = 1 55 With a compound tube of different form, Francis obtained the high value c_3 = 2 43

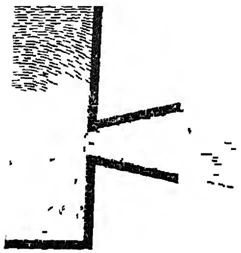

FIG 24

Let v_a = velocity of jet at section a,
v_b = velocity at section b,
p_a = pressure at section a,
p_b = pressure at section b

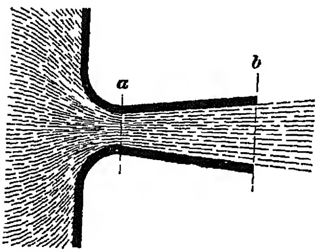

FIG 25

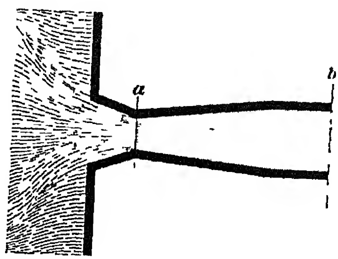

FIG 26

Evidently, p_b is the pressure of the atmosphere Also, let h = head on axis of tube Applying Bernoulli's law to sections a and b,

$$\frac{v_a^2}{2g} + \frac{p_a}{w} = \frac{v_b^2}{2g} + \frac{p_b}{w}$$

Using gauge pressures, $p_b = 0$, whence

$$\frac{p_a}{w} = \frac{v_b^2 - v_a^2}{2g}$$

If F_a and F_b denote the areas of the sections a and b respectively, then $\frac{v_b}{v_a} = \frac{F_a}{F_b}$ Since F_a is smaller than F_b,

evidently v_b is less than v_a, and the fraction $\frac{v_b^2 - v_a^2}{2g}$ is negative This indicates that $\frac{p_a}{w}$ is negative, or that the pressure at section a is less than atmospheric pressure

If, now, Bernoulli's law is applied to the surface and to section a, we have

$$h = \frac{p_a}{w} + \frac{v_a^2}{2g},$$

whence $$v_a = \sqrt{2g}\sqrt{h - \frac{p_a}{w}} = \sqrt{2gh'}$$

where $h' = h - \frac{p_a}{w}$ As $\frac{p_a}{w}$ is negative, it is clear that h' is greater than h, and therefore v_a is greater than the velocity due to the head h It is this fact—that is, that the pressure at the minimum section is less than atmospheric pressure—that accounts for the coefficients of velocity and discharge being greater than 1

A numerical example will make this point clear Let the areas F_a and F_b be 1 and 3 square inches, respectively, and let $h = 4$ feet The theoretical velocity at section b is

$$v_b = \sqrt{2gh} = 16\ 04 \text{ feet per second}$$

Assume the coefficient of velocity to be 5 at this section, then, the actual velocity is $16\ 04 \times\ 5 = 8\ 02$ feet per second Since $v_a : v_b = F_b : F_a$, we have $v_a : 8\ 02 = 3 : 1$, hence, the velocity at section a is $8\ 02 \times 3 = 24\ 06$ feet per second Therefore,

$$\frac{p_a}{w} = \frac{8\ 02^2 - 24\ 06^2}{2 \times 32\ 16} = -8 \text{ feet}$$

The corresponding pressure is $-8 \times\ 434 = -3\ 47$ pounds per square inch The negative sign indicates that the pressure at section a is 3 47 pounds less than the atmospheric pressure, which is 14 7 pounds per square inch Hence, the absolute pressure at a is $14\ 7 - 3\ 47 = 11\ 23$ pounds per square inch The effective head producing the flow is $h' = h - \frac{p_a}{w} = 4 - (-8) = 12$ feet, hence, the theoretical velocity at section a is $v_a = 8\ 02\sqrt{12} = 28$ feet per second,

nearly, whereas if the tube were cut off at section a, the theoretical velocity would be only $8.02\sqrt{4} = 16.04$ feet per second

Using the same data, let us assume the flow to be entirely frictionless We shall then have

$$v_b = 16.04,\ v'_a = 16.04 \times 3 = 48.12$$

$$\frac{p_a}{w} = \frac{16.04^2 - 48.12^2}{2 \times 32.16} = -32 \text{ feet}$$

$$h' = 4 - (-32) = 36 \text{ feet, and } v'_a = 8.02\sqrt{36} = 48.12$$

Without the diverging part of the tube, $v'_a = 8.02\sqrt{4} = 16.04$, hence, the coefficient of velocity for the section a is $48.12 \div 16.04 = 3$, which is the ratio $F_b : F_a$ of the sectional areas

31. Inward Projecting Tubes —When a tube projects into a vessel, as shown in Fig 27 (a) and (b), the contrac-

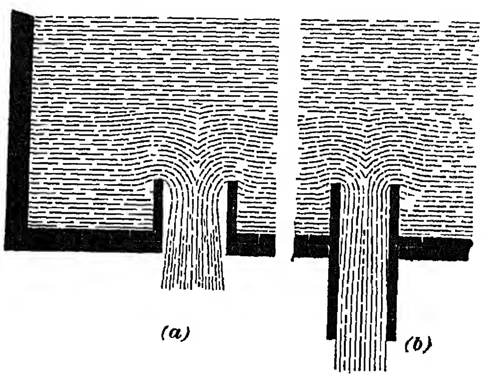

FIG 27

tion is increased and the discharge greatly reduced The coefficient of discharge for the arrangement shown at (a) is about .5, and for the tube shown at (b), about .72

HYDRAULICS

(PART 2)

FLOW OF WATER IN PIPES

RESISTANCES TO FLOW IN PIPES

BERNOULLI'S LAW FOR ANY FLOW

1. Statement of the Law.—In the case of frictionless flow, Bernoulli's law asserts that the total energy contained in a given mass of liquid at any section is the same as the energy of the same mass at any other section It has been shown, however, that perfectly frictionless flow is never attained, and that, as the flow proceeds, the energy of the mass of liquid gradually decreases The energy that is apparently lost is merely transformed into heat, but it loses its availability for doing mechanical work

Let m and n be two sections anywhere in a system containing flowing water, and suppose that the flow is from m toward n The weight of 1 cubic foot of water will, as usual, be denoted by w If the height of the section m above the reference level is h_m, and the velocity of flow and the pressure at m are v_m and p_m, respectively, the total energy of W pounds of water at that section is

$$W h_m \text{ (energy of position)}$$

$$+ W \times \frac{v_m^2}{2g} \text{ (kinetic energy)}$$

$$+ W \times \frac{p_m}{w} \text{ (pressure energy)}$$

The same expressions, with the subscript n, give the three parts of the total energy at n. If there is no energy lost between the two sections, then,

$$Wh_m + W \times \frac{v_m^2}{2g} + W \times \frac{p_m}{w} = Wh_n + W \times \frac{v_n^2}{2g} + W \times \frac{p_n}{w}$$

Suppose, however, that work is done in overcoming various frictional resistances between m and n, and let the energy expended in doing this work be denoted by E_{mn}. This energy is taken from the stock of energy the water has at section m; hence, when the water reaches section n, its energy is less by the amount E_{mn}. Or, to state the matter in another way: *The energy at m is equal to the energy at n plus the energy that has been expended between m and n in overcoming frictional resistances.* This statement is Bernoulli's law, with friction, and may be expressed algebraically by the equation

$$Wh_m + W \times \frac{v_m^2}{2g} + W \times \frac{p_m}{w}$$
$$= Wh_n + W \times \frac{v_n^2}{2g} + W \times \frac{p_n}{w} + E_{mn}$$

2. Loss of Head.—Each term in the preceding equation, except the last, is the product of the weight W and a head. If both members are divided by W, the resulting equation is

$$h_m + \frac{v_m^2}{2g} + \frac{p_m}{w} = h_n + \frac{v_n^2}{2g} + \frac{p_n}{w} + \frac{E_{mn}}{W}$$

and the last term $\frac{E_{mn}}{W}$ is also a head that may be expressed in feet, like h_m or $\frac{v_m^2}{2g}$. Evidently, this head $\frac{E_{mn}}{W}$ is the difference between the total available head at section m and that at section n; in other words, it is the loss of head between m and n due to friction and other resistances.

The symbol Z will be used to denote generally a lost head, and $Z_{mn} = \frac{E_{mn}}{W}$ will denote the loss between the two sections m and n. The lost head will in general be made up of several parts, due, respectively, to skin friction between

water and pipe, sudden enlargements or contractions in the pipe, bends and elbows, and valves or other obstructions in the pipe The magnitudes of the losses due to these various causes will now be discussed

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COEFFICIENTS OF HYDRAULIC RESISTANCE

3. Friction in Pipes—When water flows through a pipe, it meets with resistances due to the friction of the particles on the sides of the pipe and on each other These resistances absorb energy, and cause a loss in head, which will be denoted by Z_f This loss is called the **friction head**

Experiments have shown that the friction of water flowing through a pipe follows, approximately, the following laws

1 *The loss in friction is proportional to the length of the pipe*
2 *It varies nearly as the square of the velocity*
3 *It varies inversely as the diameter of the pipe*
4 *It increases with the roughness of the pipe*
5 *It is independent of the pressure in the pipe*

In accordance with these laws, the friction head Z_f is expressed by the equation

$$Z_f = f \times \frac{l}{d} \times \frac{v^2}{2g}$$

in which l = length of pipe,
d = diameter of pipe,
v = mean velocity of flow,
f = a coefficient depending on the roughness of pipe

It is customary to express every loss of head as the product of a fractional factor, called a **coefficient of hydraulic resistance**, and the velocity head $\frac{v^2}{2g}$

It has been found that f varies with the diameter of the pipe and the velocity of flow Table I at the end of this Section gives values of f for clean cast-iron pipes well laid

EXAMPLE —What is the loss of head due to friction in a pipe 10 inches (= 8333 ft) in diameter and 1,000 feet long, if the mean velocity of flow is 8 feet per second?

SOLUTION.—From Table I, the coefficient f for a pipe 10 in. in diameter is found to be .0213, when the velocity of flow is 8 ft. per sec.; therefore, substituting in the formula,

$$Z_f = .0213 \times \frac{1,000}{.8333} \times \frac{8^2}{2 \times 32.16} = 25.434 \text{ ft.} \quad \text{Ans.}$$

4. Sudden Enlargement of Pipe.—When the cross-section of a pipe suddenly changes, as shown in Fig. 1, and the flow is from the smaller to the larger part, there is a loss of energy due to the formation of eddies. Consequently, there is a loss of head, the magnitude of which may be calculated by the following formula:

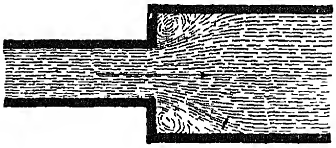


FIG. 1

Let Z_a = loss of head due to change of cross-section;
F_1 = area of cross-section of smaller part;
F_2 = area of cross-section of larger part;
$r = \frac{F_2}{F_1}$ = ratio of two areas;
v = velocity in larger part.

Then,
$$Z_a = (r - 1)^2 \times \frac{v^2}{2g}$$

EXAMPLE.—A pipe 1 foot in diameter discharges into one 2 feet in diameter, and the velocity in the larger pipe is 3 feet per second. Calculate the loss of head due to the enlargement.

SOLUTION.— $r = \frac{F_2}{F_1} = \frac{\frac{1}{4}\pi \times 2^2}{\frac{1}{4}\pi \times 1^2} = \frac{2^2}{1^2} = 4$

Hence, by the formula,

$$Z_a = (4 - 1)^2 \times \frac{3^2}{2 \times 32.16} = \frac{81}{64.32} = 1.26 \text{ ft.} \quad \text{Ans.}$$

5. Sudden Contraction of Pipe.—When the section of the pipe is suddenly made smaller, as shown in Fig. 2,

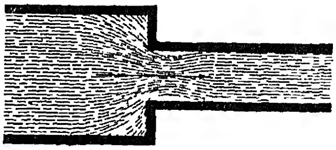


FIG. 2

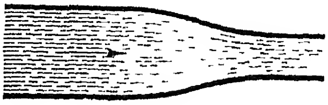


FIG. 3

there is likewise a loss of head, though this loss is small

compared with the loss due to an enlargement As in the case of the standard tube, the jet is contracted as it enters the smaller pipe, but at once expands and fills it

Let F_0 = minimum area of contracted jet,

F = area of cross-section of smaller pipe,

$c_1 = \frac{F_0}{F}$ = coefficient of contraction,

v = velocity in smaller pipe,

Z_b = loss of head due to contraction

Then,
$$Z_b = \left(\frac{1}{c_1} - 1\right)^2 \times \frac{v^2}{2g}$$

The loss of head due either to an enlargement or to a contraction may be made so small that it may be neglected if the change in section is made gradual, as shown in Figs 3

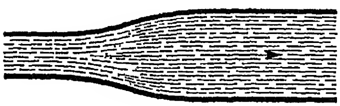


FIG 4

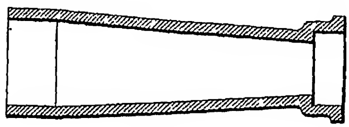
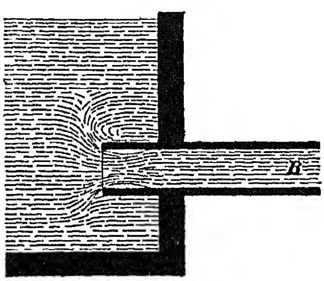


FIG 5

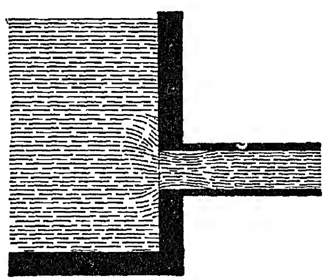
and 4 In practice, a change in section is made by a **reducer**, as shown in Fig 5

6. Loss of Head at Entrance—When water flows from a reservoir into a pipe, it meets with resistances, due to friction, contraction, etc , that absorb part of its energy,



$m = 93$

FIG 6



$m = 49$

FIG 7

and this causes a loss of head similar to the loss when water flows through an orifice or a short tube Such resistances

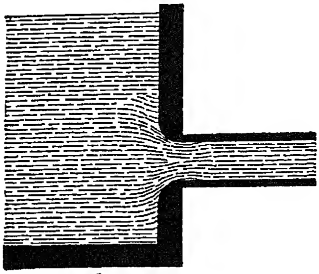
are called **resistance at entrance**, and the correspond-ing loss of head is called **loss of head at entrance**.

The lost head depends on the form of the end of the pipe where it enters the reservoir, and can be expressed by

$$Z_e = m \times \frac{v^2}{2g}$$

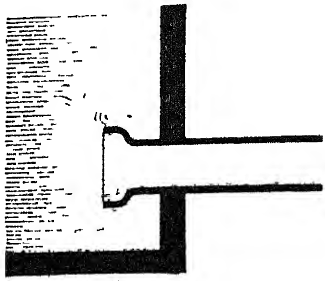
in which Z_e = head lost at entrance.

The value of m for different cases are shown in Figs. 8 and 9.



$m = 0$

FIG. 8



$m = .5$

FIG. 9

7. Elbows and Bends.—Where the pipe has sudden bends, as shown in Figs. 10 and 11, there will occur a loss

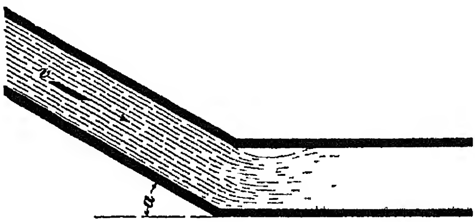


FIG. 10

of head due to shocks and eddies, contraction, and partial increase in velocity. For a sharp bend, the loss of head

$$Z_d = k_s \times \frac{v^2}{2g} \qquad (1)$$

in which k_s is a coefficient depending on the angle a, Fig. 10, and whose values are given in Table II. For a curved bend, the loss of head is, according to Weisbach,

$$Z'_d = k_c \times \frac{a}{180} \times \frac{v^2}{2g} \qquad (2)$$

where a is the angle of bend in degrees and k_c is found from the empirical formula

$$k_c = .131 + 1.847\left(\frac{r}{R}\right)^{\frac{7}{2}} \qquad (3)$$

where r is the radius of the pipe and R is the radius of the bend, Fig. 11. According to formula **3**, k_c decreases rapidly as the ratio of r to R decreases, until this ratio becomes 1 : 10. More recent experiments, however, tend to show that no advantage can be gained from making R greater than $5r$. Table III gives the values of k_c for different values of $r : R$ as calculated by formula **3**.

8. Valves and Obstructions.—If a pipe is partly closed by a valve, or if it contains any other obstruction, a

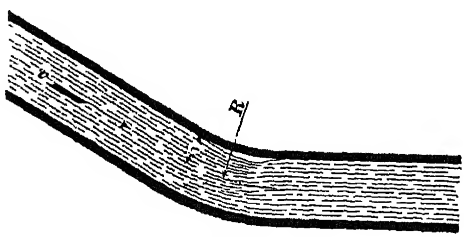


Fig. 11

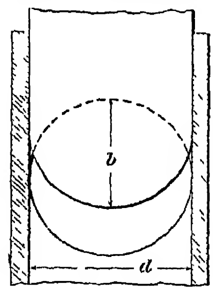


Fig. 12

loss of head occurs at every obstruction. The loss due to this cause may be denoted by Z_c, and, as usual,

$$Z_c = f \times \frac{v^2}{2g}$$

The value of f depends on the amount of opening and on the nature of the obstruction. For a gate valve (see Fig. 12), the experiments of Weisbach show values of f as follows for varying values of the ratio $b : d$.

$b : d =$	0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$
$f =$	.00	.07	.26	.81	2.1	5.5	17	98

In the case of an obstruction, it may be assumed that the cross-section of the pipe is reduced from its original area F to some smaller area F'. Then, as in Art. **5**, the loss of head is

$$Z'_c = \left(\frac{F}{F'} - 1\right)^2 \times \frac{v^2}{2g}, \text{ that is, } f = \left(\frac{F}{F'} - 1\right)^2$$

From this formula it is seen that, when F' is small compared with F, the coefficient j becomes large

9. Total Loss of Head—The entire loss of head Z between two given sections is made up of the various losses enumerated in the preceding articles, that is,

$$Z = Z_f + Z_s + Z_b + Z_c + Z_d + Z'_d + Z_e + Z'_e$$

With a pipe carefully designed and laid, some of these partial losses become negligible in comparison with Z_f, the friction head. If the pipe is of uniform size, Z_s and Z_b are both zero, if it has few bends and these are of long radius, Z_d and Z'_d are small, if the valves are open wide and there are no accidental obstructions, Z_c and Z'_e are zero. Frequently, the losses reduce to Z_f, the friction head, and Z_e, the loss at entrance, if the pipe is very long, the latter is very small compared with the former, and may be neglected

EXAMPLE—A water main of clean cast-iron pipe is 6,000 feet long and 6 inches in diameter, and is laid in practically a horizontal plane. It has four 90° bends of $2\frac{1}{2}$ feet radius, and two sharp bends with angle $a = 40°$. The entrance arrangement is similar to that shown in Fig. 4. Calculate the total loss of head with a mean velocity of flow of $2\frac{1}{4}$ feet per second

SOLUTION—From Table I, the value of f for a pipe 6 inches in diameter is 0259 for $v = 2$ ft per sec, and 0249 for $v = 3$ ft per sec. The difference in the value of f for a difference in velocity of 2 25 − 2 = 25 ft per sec is

$$(0259 - 0249) \times 25 = 0003$$

The value of f for a velocity of 2 25 ft per sec is, therefore

$$0259 - 0003 = 0256$$

Substituting in the formula of Art **3**,

$$Z_f = 0256 \times \frac{6,000}{5} \times \frac{2\ 25^2}{2 \times 32\ 16} = 24\ 18 \text{ ft}$$

From the formula in Art **6**, the loss at entrance is found to be

$$Z_e = 5 \times \frac{2\ 25^2}{2 \times 32\ 16} = 039 \text{ ft}$$

From Table II, the coefficient for each sharp bend is found to be 139. The loss of head due to sharp bends is, then, by formula 1 in Art **7**,

$$Z_d = 2 \times 139 \times \frac{2\ 25^2}{2 \times 32\ 16} = 022 \text{ ft}$$

For curved bends, $\frac{r}{R} = \frac{3}{30} = 1$. From Table III, the coefficient for each curved bend is found to be 131

The loss of head due to curved bends is

$$Z'_d = 4 \times 131 \times \frac{90}{180} \times \frac{2\ 25^2}{2 \times 32\ 16} = 021 \text{ ft}$$

The total lost head is, therefore,

$$Z = 24\ 18 + 039 + 022 + 021 = 24\ 26 \text{ ft} \quad \text{Ans}$$

NOTE—This example shows the insignificance of the losses due to entrance and bends compared with the loss due to friction

GENERAL FORMULAS FOR THE FLOW OF WATER IN PIPES

10. Application of Bernoulli's Law.—In the consideration of flow through pipes, it is assumed that there is full flow, that is, that the pipe is filled from end to end

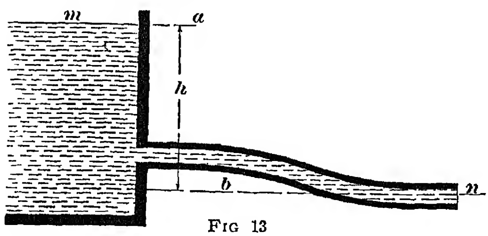


FIG 13

The pipe is fed from a reservoir, as shown in Figs 13 and 14, and the discharge may be into the atmosphere, as in Fig 13, or into a second reservoir, as in Fig 14 The length of the pipe is measured along its axis

The fundamental formula for the velocity in the pipe is obtained with the aid of Bernoulli's law In Fig 13, consider a mass of water at m (the level in the reservoir) and an equal mass at n The reservoir is so large that the velocity at m is inappreciable, hence, the kinetic energy $W \times \frac{v_m^2}{2g} = 0$ The gauge pressure at the surface is zero, and therefore the pressure energy $W \times \frac{p_m}{W} = 0$ If the level b through the end of the tube is taken as a reference level, the energy of position of the mass at m is $W h$, where h is the potential head, or the height of the level a above the level b The mass emerges from the end n with a velocity v, and has therefore the kinetic energy $W \times \frac{v^2}{2g}$ The pressure

energy at b is zero, and the energy of position is likewise zero

Let E denote the energy expended in overcoming frictional resistances, the resistance at entrance, etc Then, by Bernoulli's law,

$$\text{energy at } m = \text{energy at } n + E,$$

that is,
$$Wh = W \times \frac{v^2}{2g} + E,$$

whence

$$h = \frac{v^2}{2g} + \frac{E}{W}$$

The quotient $\frac{E}{W}$, as in Art **2**, is the lost head between the sections m and n, and is denoted by Z, hence,

$$h = \frac{v^2}{2g} + Z$$

that is, *the hydrostatic head on the end n is equal to the velocity head plus the loss of head*

11. When the discharge is into a reservoir, as shown in Fig 14, the reference level is taken, as before, at the level

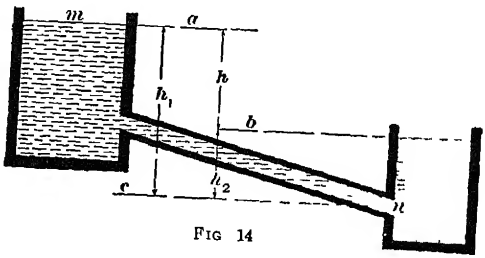


FIG 14

of the discharging end n In this case, the mass at n has a pressure energy Wh_2 due to the head h_2 on n, in addition to the kinetic energy $W \times \frac{v^2}{2g}$ Hence, the equation for this case is

$$Wh_1 = W \times \frac{v^2}{2g} + Wh_2 + E,$$

whence
$$h_1 = \frac{v^2}{2g} + h_2 + Z$$

and, by transposition,

$$h_1 - h_2 = \frac{v'^2}{2g} + Z$$

The difference $h_1 - h_2$ is clearly the distance between the levels a and b, and this is the *effective head* that produces the flow Denoting this head by h, we have, as in the first case,

$$h = \frac{v^2}{2g} + Z$$

12. Formula for Velocity —The head Z is made up of several terms, each of which is the product of $\frac{v^2}{2g}$ and a coefficient of resistance The leading term being the friction head $Z_f = f \times \frac{l}{d} \times \frac{v^2}{2g}$, we may write

$$Z = f \times \frac{l}{d} \times \frac{v^2}{2g} + c \times \frac{v^2}{2g}$$

in which c is the sum of all the coefficients for losses due to entrance, bends, valves, sudden enlargements, etc Substituting this value of Z, the value of h given in the last article becomes

$$h = \frac{v'^2}{2g} + f \times \frac{l}{d} \times \frac{v^2}{2g} + c \times \frac{v^2}{2g}$$

Solving for v,

$$v = \sqrt{\frac{2gh}{1 + f \times \frac{l}{d} + c}} = 8\,02 \sqrt{\frac{h}{1 + f \times \frac{l}{d} + c}} \qquad (1)$$

Table I gives the mean values of f that may be used for clean iron pipes, either smooth or coated with coal tar Since f depends on v, which is unknown, it is first necessary to take from the table a mean value of f depending on the diameter of the pipe, and then solve for v This gives an approximate value for v from which to find a new value of f, and solve again for v If the last value of f is nearly the same as would be given in the table for the value of v last found, the result is satisfactory If not, the last value of v must be taken as an approximation from which a new value of f is to be found, and the process repeated

If m has the value 5 given in Fig 9 for the common case of a pipe with a bell end, and there are no sharp bends or similar resistances, then $c =$ 5, and the formula for v becomes

$$v = \sqrt{\frac{2 g h}{1\ 5 + f \times \frac{l}{d}}} = 8\ 02 \sqrt{\frac{h}{1\ 5 + f \times \frac{l}{d}}} \qquad (2)$$

EXAMPLE —A pipe 12 inches in diameter, 900 feet long, with a bell end, enters the reservoir in such a way that the coefficient m may be taken as 5 The pipe has two 45° bends, each with a radius of 2 feet If the head on the discharge end of the pipe is 35 feet, what will be the velocity of flow?

SOLUTION —The loss of head from the bends depends on the ratio between the radius of the pipe and the radius of the bend This ratio is $\frac{6}{24} =$ 25 From Table III, the coefficient k_c for a ratio of 2 is 138, and for the ratio 3, $k_c =$ 158, therefore, for 25 the coefficient is $\frac{138 + 158}{2} =$ 148, hence, $c =$ 5 + 2 × 148 × $\frac{45}{180} =$ 574 Assuming 0223 as an approximate value for f for use in this case, and substituting the values of the coefficients in formula **1**, we have

$$v = 8\ 02\sqrt{\frac{35}{1 + 0223 \times \frac{900}{1} + 574}} = 10\ 20 \text{ ft per sec}$$

From Table I, the value of f for a velocity of 10 20 ft per sec is 0203 Using this value of f, the velocity becomes

$$v = 8\ 02\sqrt{\frac{35}{1 + 0203 \times \frac{900}{1} + 574}} = 10\ 65 \text{ ft per sec}$$

From Table I, the difference in the value of f for a difference in velocity of 10 65 − 10 = 65 ft per sec is

$$\frac{0203 - 0200}{2} \times 65 = 0001$$

The value of f corresponding to $v =$ 10 65 ft per sec is, therefore, 0203 − 0001 = 0202 This is very nearly equal to 0203, therefore, 10 65 ft per sec is practically the required velocity Ans

13. Long Pipes.—Pipes in which the length l is greater than about 1,000 d are called **long pipes** In them, the velocity head and loss of head at entrance become so small in comparison with the loss due to friction that they may be neglected, and the formula for velocity may be written

$$v = \sqrt{\frac{2 g h d}{f l}} = 8\ 02\sqrt{\frac{h d}{f l}} \qquad (1)$$

For d in inches, formula **2**, Art **12**, becomes

$$v = 2\ 315\sqrt{\frac{h\,d}{f\,l + 125\,d}} \qquad (2)$$

and instead of formula **1** we have

$$v = 2\ 315\sqrt{\frac{h\,d}{f\,l}} \qquad (3)$$

EXAMPLE 1 —A pipe 10 inches in diameter and 8,000 feet long is so laid that there is practically no loss of head from bends or valves If the head is 150 feet, what is the mean velocity of flow?

SOLUTION —Since the length is more than 1,000 times the diameter, formula **1** may be used Taking 0230 as a mean value of f, the approximate velocity of flow is

$$v = 8\ 02\sqrt{\frac{150 \times \frac{10}{12}}{0230 \times 8{,}000}} = 6\ 61 \text{ ft per sec}$$

From Table I, the difference in the value of f corresponding to a difference in velocity of 6 61 − 6 00 = 61 ft per sec is

$$\frac{0219 - 0213}{2} \times 61 = 0002$$

The value of f corresponding to a velocity of 6 61 ft per sec is, therefore, 0219 − 0002 = 0217 Substituting this value in the formula for v,

$$v = 8\ 02\sqrt{\frac{150 \times \frac{10}{12}}{0217 \times 8{,}000}} = 6\ 81 \text{ ft per sec}$$

Interpolating from Table I as before, the value of f corresponding to a velocity of 6 81 ft per sec is found to be 0217 Since this is the same as the assumed value of f, 6 81 ft per sec is the required velocity Ans

EXAMPLE 2 —What would have been the value of v in example 1, if formula **2**, Art **12**, had been used?

SOLUTION —Substituting in the formula,

$$v = 8\ 02\sqrt{\frac{150}{1\ 5 + 0217 \times \frac{8{,}000}{\frac{10}{12}}}} = 6\ 78 \text{ ft per sec} \quad \text{Ans}$$

By comparing these two examples, it is seen that for long pipes the effect of resistances at entrance may be neglected without affecting the practical accuracy of the result

14. Head Required to Produce a Given Velocity. A formula for the head required to produce a given velocity of flow v can be found from the formulas given in Art **12**

by solving for h. Thus, from formula **1**, Art. **12**, the value of the head is

$$h = \frac{v^2\left(1 + f \times \frac{l}{d} + c\right)}{64.32} \qquad (1)$$

For a straight cylindrical pipe, in which the effect of bends disappears and c is taken equal to .5, the preceding formula becomes

$$h = \frac{f\,l\,v^2}{64.32\,d} + .0233\,v^2 = .07\,v^2\left(\frac{.222\,f\,l}{d} + \frac{1}{3}\right) \qquad (2)$$

Formulas **1** and **2** apply when d is in feet; for d in inches, formula **2** becomes

$$h = \frac{f\,l\,v^2}{5.36\,d} + .0233\,v^2 = .07\,v^2\left(\frac{2.664\,f\,l}{d} + \frac{1}{3}\right) \qquad (3)$$

EXAMPLE.—A pipe 8 inches in diameter and 2,500 feet long has three 75° bends, the radius of each being the same as the diameter of the pipe. If the coefficient for loss at entrance is $m = .5$, and $f = .0220$, what must be the head to produce a velocity of flow of 7 feet per second?

SOLUTION.—The ratio between the radius of the pipe and the radius of the bend is $\frac{4}{8} = .5$; therefore, from Table III, the coefficient k, is .294, and $c = .5 + 3 \times .294 \times \frac{75}{180} = .868$.

Substituting in formula **1**,

$$h = \frac{7^2\left(1 + .022 \times \frac{2{,}500}{\frac{2}{3}} + .868\right)}{64.32} = 64.27 \text{ ft.} \quad \text{Ans.}$$

15. Formulas for Discharge.—The formulas just given are made use of in ascertaining the quantity of water that will be discharged from a pipe in a given time, with a given head. This is readily found by the formula $Q = Fv$, where F is the area of the cross-section of the pipe and v is the mean velocity, as determined by the preceding formulas.

When the diameter is given in feet, the discharge, in cubic feet per second, is

$$Q = .7854\,d^2\,v \qquad (1)$$

When d is in inches,

$$Q = \frac{.7854\,d^2\,v}{144} = .00545\,d^2\,v \qquad (2)$$

Since 1 cubic foot contains 7 48 gallons, the discharge, in gallons per second, when d is in feet, is

$$Q = 7854\, d^2 v \times 7\ 48 = 5\ 875\, d^2 v \qquad (3)$$

and when d is in inches,

$$Q = 0408\, d^2 v \qquad (4)$$

EXAMPLE 1 —What is the discharge, in gallons per minute, from a pipe 6 inches in diameter, if the mean velocity of efflux is 5 6 feet per second?

SOLUTION —Substituting in formula **4**,

$$Q = 0408 \times 36 \times 5\ 6 = 8\ 225 \text{ gal per sec}$$
$$8\ 225 \times 60 = 493\ 5 \text{ gal per min} \quad \text{Ans}$$

EXAMPLE 2 —The length of a pipe is 6,270 feet, its diameter is 8 inches, and the total head at the point of discharge is 215 feet How many gallons are discharged per minute?

SOLUTION —First find the approximate value of v from formula **3**, Art **13**, taking the value of $f = 0235$ Substituting in the formula,

$$v = 2\ 315 \sqrt{\frac{h\, d}{f\, l}} = 2\ 315 \sqrt{\frac{215 \times 8}{0235 \times 6{,}270}} = 7\ 91 \text{ ft per sec, nearly}$$

From Table I, the value of f for a pipe 8 in in diameter and a velocity of 7 91 ft per sec is

$$0225 - \frac{(\ 0007 \times 1\ 91)}{2} = 0218$$

With this value for f used in formula **3**, Art **13**, the velocity is

$$v = 2\ 315 \sqrt{\frac{215 \times 8}{0218 \times 6{,}270}} = 8\ 21 \text{ ft per sec}$$

From Table I, the value of f for a velocity of 8 21 ft per sec is found to be 0217 This is so near the assumed value for f that $v = 8\ 21$ ft per sec may be considered correct Substituting in formula **4**,

$$Q = 0408 \times 8^2 \times 8\ 21 = 21\ 438 \text{ gal per sec}$$
$$21\ 438 \times 60 = 1{,}286\ 28 \text{ gal per min} \quad \text{Ans}$$

16. Formulas for Diameter —With h, l, and d in feet and the quantity Q in cubic feet per second, the formula for the diameter of a pipe without sharp bends is

$$d = 479\left[(1\ 5\, d + f\, l)\frac{Q^2}{h}\right]^{\frac{1}{5}} \qquad (1)$$

The derivation of this formula is as follows From formula **2**, Art **12**,

$$v^2 = \frac{2\, g\, h}{1\ 5 + f \times \frac{l}{d}} = \frac{2\, g\, h\, d}{1\ 5\, d + f\, l}$$

and from formula **1**, Art. **15**,

$$v^2 = \frac{Q^2}{.7854^2 d^4}$$

Hence, equating the two values of v^2,

$$\frac{2\,g\,h\,d}{1.5\,d + f\,l} = \frac{Q^2}{.7854^2\,d^4},$$

and, therefore, $$d^5 = \frac{Q^2(1.5\,d + f\,l)}{.7854^2 \times 2\,g\,h}$$

Extracting the fifth root,

$$d = \left(\frac{1}{.7854^2 \times 64.32}\right)^{\frac{1}{5}} \times \left[(1.5\,d + f\,l)\frac{Q^2}{h}\right]^{\frac{1}{5}}$$

$$= .479\left[(1.5\,d + f\,l)\frac{Q^2}{h}\right]^{\frac{1}{5}}$$

In using this formula, take the approximate value of f as .0200, and compute an approximate value for d, neglecting the term 1.5 d in the second member of the formula. With this value of d, find the value of v from the formula $v = \frac{Q}{.7854\,d^2}$, and find the corresponding value of f from Table I.

Repeat the computation for d by placing the approximate values of d and f just found in the second member of the formula. One or two repetitions of this process will give a close approximation to d from which to select the pipe from the standard market sizes.

For pipes in which the length is more than 1,000 times the diameter, the following formula may be used:

$$d = .479\left(\frac{f\,l\,Q^2}{h}\right)^{\frac{1}{5}} \qquad (2)$$

EXAMPLE 1.—What must be the diameter of a pipe to discharge 1,000,000 gallons of water per 24 hours, if the length is 1,250 feet, and the head 75 feet?

SOLUTION.—The discharge, in cubic feet per second, is

$$\frac{1,000,000}{86,400 \times 7.48} = 1.5474$$

The approximate diameter of the pipe, by formula **2**, is

$$d = .479\left(\frac{.02 \times 1,250 \times 1.5474^2}{75}\right)^{\frac{1}{5}} = .458 \text{ ft.}$$

The velocity corresponding to this value of d is (formula **1**, Art. **15**)

$$v = \frac{1.5474}{.7854 \times .458^2} = 9.40$$

From Table I, the value of f for a pipe 6 in. (= .5 ft.) in diameter, and a velocity of flow of 10 ft. per sec., is .0220. Substituting this value of f and the approximate value of d in formula **1**,

$$d = .479\left[(1.5 \times .458 + .022 \times 1{,}250)\,\frac{1.5474^2}{75}\right]^{\frac{1}{5}} = .469 \text{ ft.}$$

The next higher commercial size is a 6-in. pipe; hence, that size may be taken. Ans.

Example 2.—A water main 17,320 feet long must supply a city with 10,000,000 gallons of water per 24 hours under a steady flow. If the head is 120 feet, what must be the diameter of the pipe?

Solution.—The discharge, in cubic feet per second, is

$$Q = \frac{10{,}000{,}000}{86{,}400 \times 7.48} = 15.474$$

The approximate value of d is, therefore,

$$d = .479\left(\frac{.0200 \times 17{,}320 \times 15.474^2}{120}\right)^{\frac{1}{5}} = 1.771 \text{ ft.}$$

The velocity of flow corresponding to this diameter is

$$v = \frac{15.474}{.7854 \times 1.771^2} = 6.28 \text{ ft. per sec.}$$

From Table I, the coefficient f for a pipe 20 in. in diameter, and a velocity of 6 ft. per sec., is .0193. Since the length of the pipe is more than 1,000 times the approximate diameter, formula **2** may be used; hence,

$$d = .479\left(\frac{.0193 \times 17{,}320 \times 15.474^2}{120}\right)^{\frac{1}{5}} = 1.759 \text{ ft.}$$

The next higher available market size may be used. Ans.

17. Commercial Sizes of Cast-Iron Pipes.—The diameters, in inches, of cast-iron pipe commonly found in the market are as follows:

3	8	16	30	54
4	10	18	36	60
5	12	20	42	72
6	14	24	48	84

It should be stated, however, that pipes larger than the 48-inch are not made by all manufacturers. Pipes smaller than 4 inches in diameter are also seldom made.

In determining the size of a pipe, it is wiser to select a commercial size larger than the computed size rather than smaller. For example, suppose the computation to give

$d = 30.3$ inches. The market size lower is 30 inches, and would probably do when the pipe is new and clean, but if the next larger size, 36 inches, is taken, the pipe is sure to give the required discharge when it becomes tuberculated or somewhat foul.

EXAMPLES FOR PRACTICE

1. What is the loss of head due to friction in a 16-inch pipe 2,150 feet long with a velocity of flow of 3 feet per second?
Ans. 1.85 ft.

2. Determine the velocity, in feet per second, in a 12-inch water main 1,720 feet long with a head of 90 feet. Ans. 13 ft. per sec.

3. A city requires a supply of water amounting to 2,000,000 gallons per 24 hours, the reservoir is located 21,400 feet from the city, and has an elevation of 312 feet. Determine the commercial diameter of the pipe necessary to give this discharge. Ans. 10 in.

18. Flow Under Pressure.—In Fig. 15 is represented a long pipe L discharging under pressure. If h_1 represents the total head and h_2 the depth of the reservoir B, the effect-

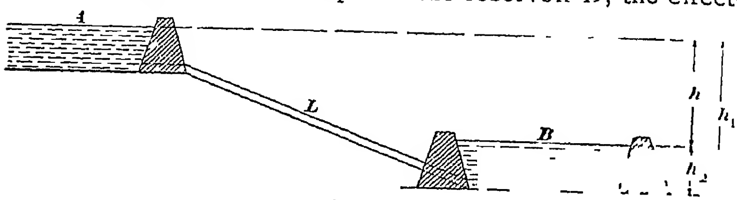


FIG. 15

ive head is the difference between these two, or h. In problems where the pipe is discharging against some resistance or pressure, as in the case of a water motor or hydraulic engine, this pressure or resistance must be converted into the equivalent head, by the formula $h_2 = 2.304\,p$, in which p represents the pressure and h_2 is the equivalent head.

EXAMPLE 1.—Calculate the diameter of a pipe 21,000 feet long and laid on a grade of 1 foot in 300 feet that will deliver 1,000,000 gallons per 24 hours to a turbine at a working pressure of 20 pounds per square inch.

SOLUTION.—The grade of 1 in 300 for the distance 21,000 ft. makes the total head $21{,}000 \div 300 = 70$ ft. $= h_1$. The pressure $p = 20$ lb., the equivalent head h_2 of which is $2.304 \times 20 = 46.08$ ft.; $h = h_1 - h_2$

$= 70 - 46\ 08 = 23\ 92$ ft , effective head This value of h substituted in formula **2**, Art **16**, gives, for the approximate diameter,

$$d = 479\left(\frac{02 \times 21{,}000 \times 1\ 5474^2}{23\ 92}\right)^{\frac{1}{5}} = 1\ 012 \text{ ft}$$

The velocity corresponding to this value of d is

$$v = \frac{1\ 5474}{7854 \times 1\ 012^2} = 1\ 92 \text{ ft per sec}$$

It is found from the table that the value of f is 0237 for a 1-ft pipe with a velocity of 2 ft per sec Using the approximate value of d in formula **1**, Art **16**, we have

$$d = 479\left[(1\ 5 \times 1\ 012 + 0237 \times 21{,}000) \times \frac{1\ 5474^2}{23\ 92}\right]^{\frac{1}{5}} = 1\ 05 \text{ ft Ans}$$

The nearest commercial size to this diameter is a 12-in pipe Hence, this may be taken

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FLOW THROUGH VERY SHORT PIPES

19. A **standard tube** is one whose length is not over $2\frac{1}{2}$ times its diameter A **very short pipe** is one whose length does not exceed 60 times its diameter A **short pipe** is one whose length is less than 1,000 times its diameter, and a **long pipe** is one whose length is greater than 1,000 times its diameter

From the experiments of Eytelwein and others, a few coefficients of velocity have been obtained for *very short pipes* with small diameters and low heads For larger pipes the values are too high The formulas for velocity corresponding to these coefficients are as follows

For $l = 3\,d$, $\quad v = 82\sqrt{2gh}$

For $l = 12\,d$, $\quad v = 77\sqrt{2gh}$

For $l = 24\,d$, $\quad v = 73\sqrt{2gh}$

For $l = 36\,d$, $\quad v = 68\sqrt{2gh}$

For $l = 48\,d$, $\quad v = 63\sqrt{2gh}$

For $l = 60\,d$, $\quad v = 60\sqrt{2gh}$

EXAMPLE 1 —Calculate the discharge from a pipe 4 inches in diameter and 15 feet long with a head of 7 feet

SOLUTION —Here, the length is $45\,d$ Taking the nearest coefficient,

$$v = 63\sqrt{2gh} = 63\sqrt{2 \times 32\ 16 \times 7} = 13\ 37 \text{ ft per sec}$$

The area of a 4-in pipe is 0873 sq ft , hence, from the formula $Q = Fv$,

$$Q = 0873 \times 13\ 37 = 1\ 17 \text{ cu ft per sec} \quad \text{Ans}$$

EXAMPLE 2 —A reservoir is tapped through its masonry dam by a horizontal pipe 24 inches in diameter, 20 feet long, and whose center lies 20 feet below the surface of the water in the reservoir Required the discharge

SOLUTION —The length in this instance is 10 times the diameter Using the nearest coefficient,

$$v = .77\sqrt{2gh} = .77\sqrt{2 \times 32\ 16 \times 20} = 27\ 6 \text{ ft per sec}$$

The area of the pipe is 3 1416 sq ft , hence,

$$Q = 27\ 6 \times 3\ 1416 = 86\ 7 \text{ cu ft per sec} \quad \text{Ans}$$

This result is probably 10 per cent too high, owing to lack of data for pipes of this size Problems such as this are, fortunately, seldom met with in practice

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THE HYDRAULIC GRADE LINE

20. The **hydraulic grade line**, or **hydraulic gradient**, is a line drawn through a series of points to which water would rise in piezometer tubes attached to a pipe through which water flows With a smooth pipe of uniform

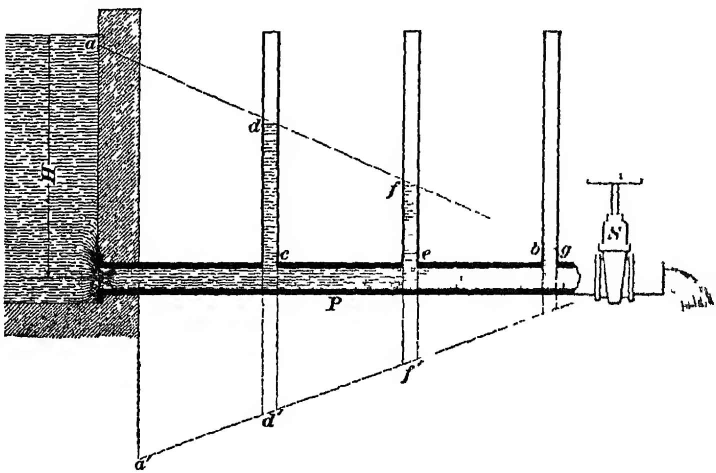

FIG 16

cross-section and without bends or other obstructions to flow, the hydraulic grade line is a straight line extending from the reservoir to the end of the pipe

In Fig 16 is shown a horizontal pipe that may be assumed to have an indefinite length, leading from a reservoir to a

stop-valve S When the valve is open so that water from the pipe discharges freely into the atmosphere, the hydraulic grade line is the line $a\,d\,f\,g$ The distance of the point a below the surface of the water in the reservoir represents the head absorbed in overcoming the resistances of entrance to the pipe, and in producing the velocity with which the water flows In the same way, the difference in the height to which the water rises in any two piezometer tubes represents the head absorbed in overcoming the resistance to flow in the pipe between the points at which the tubes are inserted

21. The flow of water through the pipe P would be the same whether the pipe were horizontal, as shown in the figure, or whether it were laid along the grade line $a\,d\,f\,g$ The flow would also be the same if the reservoir were deepened and the pipe laid along the line $a'\,d'\,f'$ The pressures in the pipe, however, would vary greatly with the different positions If the pipe were laid along the line $a\,d\,f\,g$, there would be little or no pressure in any part of it, and if it were perforated at the top, little or no water would flow from the perforations In the horizontal position, however, and still more in the position $a'\,d'\,f'$, there would be pressure at all points, the pressure for any point in the pipe being equivalent to the head represented by the vertical distance from that point to the hydraulic grade line, and, if the pipe were perforated anywhere, water would issue from the perforations

22. Position of Hydraulic Grade Line—In laying a line of pipe to connect two points lying at different levels, it is of the utmost importance to ascertain the position of the hydraulic grade line Let A and B, Fig 17, represent two reservoirs, connected by a pipe line of uniform diameter, through which the water flows by gravity from the upper to the lower level The hydraulic grade line is the straight line connecting the two reservoirs, in order to cover the most unfavorable conditions, it is usually drawn between the two ends of the pipe line, and not from surface to surface of the water in the two reservoirs, as the level of these

surfaces may vary. The slope of the grade line will be represented by $\frac{h}{l}$.

In order that the pipe may flow full, no part of it should rise above the hydraulic grade line EF. The following considerations will make this point clear.

Assume the pipe to be laid above the hydraulic gradient, along the broken line EDF. From the fundamental formula $Q = .7854\, d^2 v$, it follows that the discharge Q varies with

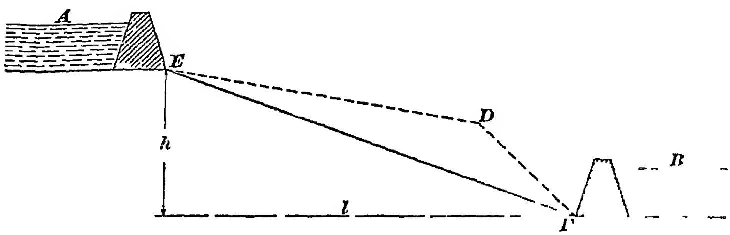

Fig. 17

the velocity. It has also been demonstrated that, other things being equal, the velocity increases with the ratio $\frac{h}{l}$, or the sine of the angle of inclination of the pipe to the horizon. It follows that the line ED, whose inclination is less than that of DF, cannot deliver as much water as DF can carry. Consequently, DF cannot run full, but acts merely as a trough or open channel. In order that the line DF may flow full, it must have a smaller diameter than ED, and, where conditions exist that make this form of construction unavoidable, a smaller diameter is selected for DF, which diameter is determined by methods that will be explained later.

23. The Siphon.—The part of a pipe that rises above the hydraulic gradient is called a siphon. The principles on which the action of a siphon depends are explained in *Pneumatics*. If the siphon is kept filled, the flow through it will take place in accordance with the laws given for pipes laid below the hydraulic gradient, and the same formulas apply.

The total head producing the flow in a siphon is the vertical distance from the discharge end of the pipe to the level

of the water in the reservoir If the siphon is of uniform section, without sharp bends or obstructions, the hydraulic gradient will be a straight line EF, Fig 17, from the reservoir to the discharge end F, and the pressure in all parts of the pipe that rise above the line will be less than the atmospheric pressure Air always tends to collect in the highest point of a siphon, and means must be provided for its removal, in order to keep up the flow This is effected by means of an air pump or air valve, as will be explained elsewhere Such means of removing the air should be provided for whenever circumstances make it unavoidable to place part of a pipe above the hydraulic gradient.

HYDRAULIC TABLE FOR LONG PIPES

24. In the preceding articles have been given all the rules and formulas necessary to solve any practical problems that may arise. A little ingenuity will sometimes be needed in their application, and both care and good judgment must be exercised in the selection of the proper coefficient in cases where great accuracy is required

In the design of water pipes, fractional diameters are usually found The commercial sizes of pipe are cast in even inches, and only pipes of a certain size are commonly manufactured When the calculation calls for some fractional diameter, the next larger size of even inches should be taken One is then perfectly sure that all losses of head and frictional resistances are provided for The interior and varying conditions of pipes and the lack of sufficiently extended experimental data permit errors of from 2 to 10 per cent to affect the most careful calculations Every effort should be made to avoid errors, but, for the reasons just given, small errors are practically unimportant in solving problems relating to the flow of water in pipes

Table IV at the end of this Section greatly facilitates the solution of all practical problems likely to occur It is very conveniently arranged, and will save much time and labor It comprises every commercial size of cast-iron water pipe,

and is equally applicable to wrought-iron and steel pipe, not riveted

It must be borne in mind that the projecting rivet heads in a steel riveted pipe reduce its carrying capacity very much more than by the decrease of the diameter caused by the annular ring of rivets For instance, a 12-inch riveted steel pipe with a row of rivet heads projecting into the interior, 1 inch in depth, will not have the same discharge as a 40-inch smooth pipe Costly and embarrassing mistakes have been committed by neglecting this fact

25. The quantities given in Table IV are

d = diameter, both in inches and in feet (the value used in the formulas is always in feet, unless otherwise stated)

v = velocity of flow, in feet per second

$s = \frac{h}{l}$ = slope, or head per unit of length of pipe, or sine of the average inclination of the pipe to the horizontal (here, h is the total head, and l is the length of the pipe)

$s_m = 5{,}280 \frac{h}{l}$ = head, in feet, per mile of pipe

$G = \frac{l}{h}$ = grade = length of pipe for which the head, or rise, is 1 If l and h are in feet, G is in feet, and indicates the number of feet of pipe in which the rise is 1 foot Thus, if G = 750 feet, the grade is 1 foot in 750 feet

Q = discharge, for clean or tar-coated pipes, in either cubic feet per second, gallons per minute, or gallons per day of 24 hours

$Q' = \frac{Q}{\sqrt{2}}$, corresponding quantities for extremely foul pipes

The head h and length l are easily found when either the slope or the grade is given, since $h = s\,l$, and $l = G\,h$

The table has been constructed from the formulas (Arts 13 and 15)

$$v = \sqrt{\frac{2\,g\,h\,d}{f\,l}} = \sqrt{\frac{2\,g\,d}{f}} \times \frac{h}{l} \qquad (1)$$

$$Q(\text{cu ft}) = \frac{\pi\,d^2}{4}\,v \qquad (2)$$

Assuming values for v and d, values of f have been found from a table similar to Table I, but more complete. Having v, d, and f, the slope $\frac{h}{l}$ has been computed from equation (1), which gives

$$\frac{h}{l} = \frac{f v^2}{2 g d}$$

The values of f here used are for clean pipes, either smooth or coated with coal-tar varnish. For extremely rough or foul pipes, the value of f has been taken as twice that for clean pipes. If the velocity in a rough pipe is denoted by v', and $2f$ is used instead of f, equation (1) becomes

$$v' = \sqrt{\frac{2 g d}{2 f} \times \frac{h}{l}}$$

Therefore, $$\frac{v}{v'} = \frac{\sqrt{\frac{2 g d}{f} \times \frac{h}{l}}}{\sqrt{\frac{2 g d}{2 f} \times \frac{h}{l}}} = \sqrt{2}$$

and, as the discharges are proportional to the velocities,

$$\frac{Q}{Q'} = \frac{v}{v'} = \sqrt{2}, \text{ whence, } Q' = \frac{Q}{\sqrt{2}}$$

In determining the diameter of a pipe, it is always advisable to determine it for both of the extreme conditions, that is, both assuming it perfectly clean and assuming it extremely foul or rough. Also, when the diameter of a pipe is known, the values of Q and Q' show the extreme limits between which the discharge may vary.

EXAMPLE 1.—What is the discharge, in cubic feet per second, of a 14-inch pipe in which the velocity is 3.2 feet per second?

SOLUTION.—Find, in Table IV, under diameter 14 inches, 3.2 in the column headed v. Opposite this value and in column headed Cubic Feet per Second, the discharge is found to be 3.4208 cu. ft. per sec. Ans.

EXAMPLE 2.—Determine the velocity, in feet per second, in a 16-inch water main 1,500 feet long, with a head of 54 feet.

SOLUTION.—The ratio $\frac{l}{h}$ is $\frac{1{,}500}{54}$, or 27.778. Looking in the table, in the column headed G, under the diameter 16 in., it is seen that the

value 27 778 falls between that corresponding to a velocity of 12 5 ft per sec and that corresponding to a velocity of 13 ft per sec The difference in the value of v for a difference in G of 28 782 − 26 681 = 2 101 is 13 0 − 12 5 = 5 ft per sec For a difference in G of 28 782 − 27 778 = 1 004, the difference in v is

$$\frac{5\times 1\ 004}{2\ 101}=24$$

Therefore, the velocity is

$$12\ 5+24=12\ 74 \text{ ft per sec} \quad \text{Ans}$$

EXAMPLE 3 —What is the loss of head due to friction in a 10-inch pipe 2,000 feet long, with a velocity of flow of 2 8 feet per second?

SOLUTION —Since $\frac{h}{l}=\frac{f\,v^2}{2\,g\,d}$, and $\frac{f\,l}{d}\times\frac{v^2}{2\,g}$ is the friction head (formula of Art **3**), the quantity given in the second column multiplied by the length of the pipe will give the loss of head due to friction From the table, $\frac{h}{l}$ corresponding to a diameter of 10 in and a velocity of 2 8 ft per sec, is 0034446 Then,

$$Z_f=\ 0034446\times 2{,}000=6\ 89 \text{ ft} \quad \text{Ans}$$

By the formula of Art **3**,

$$Z_f=\ 0236\times\frac{2{,}000}{\frac{10}{12}}\times\frac{2\ 8^2}{64\ 32}=6\ 90 \text{ ft}$$

By a comparison of these two solutions, it is seen that a great deal of calculation is saved by the use of the table

EXAMPLE 4 —Required, the diameter of pipe necessary to deliver 700,000 gallons per day of 24 hours, if the reservoir is situated 90 feet above the city and at a distance of 15,300 feet

SOLUTION —Here, $\frac{l}{h}=\frac{15{,}300}{90}=170$ Looking for the number 700,000 in the column headed Gallons per Day, under diameter 6 inches, it is seen that the next higher, 761,360, requires a grade (column G) of 1 ft in 38 374 ft Since the available grade is only 1 ft in 170 ft, look in the same column under the diameter 8 in The next higher value is 721,890, and the required grade (column G) is 1 ft in 175 36 ft Therefore, an 8-in pipe could be used, but the discharge would be somewhat greater than 721,890 gal per day Ans

EXAMPLE 5 —It is required to deliver 2,500,000 gallons per day of 24 hours with a 20-inch pipe If the reservoir is 9 miles from the city (*a*) what must be the head? (*b*) what is the velocity in the pipe?

SOLUTION —(*a*) Looking in Table IV, under diameter 20 in, in the column headed Gallons per Day, the value 2,537,000 is found Opposite this value in the column headed s_m, the quantity 3 1086 is

found This is the head per mile of length, therefore, the required head is

$$3.4086 \times 9 = 30.6774 \text{ ft} \quad \text{Ans}$$

(*b*) Opposite 2,537,900, in the column headed v, the value 1.8 is found Therefore, $v = 1.8$ ft per sec Ans

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EXAMPLES FOR PRACTICE

1 Check, by Table IV, the values found in examples **1** and **2** of Art **15**

2 Check, by Table IV, example 1 of Art **16**.

3 What is the loss of head due to friction in a pipe 20 inches in diameter, 5 miles long, if the velocity of flow is 3.5 feet per second?

Ans 60.6 ft

4 Required, (*a*) the diameter of pipe necessary to deliver 3,500,000 gallons per day, if the length of pipe is 8,000 feet, and the head is 40 feet, (*b*) the velocity in the pipe

Ans $\begin{cases} (a)\ 16 \text{ in} \\ (b)\ 3.9 \text{ ft per sec} \end{cases}$

5 The velocity of flow in a 30-inch pipe, 8.3 miles long, is 3.2 feet per second If the head is 50 feet, calculate the discharge in million gallons per day

Ans 10 million gal

6 Required the diameter of pipe necessary to deliver 5,000,000 gallons per day, with a velocity not higher than 4 feet per second, and a grade not less than 1 in 500

Ans 20 in

7 The total head for a 24-inch pipe line was 40 feet, the quantity discharged was 5.07 million gallons per day Determine the length of the line

Ans 41,840 ft

TABLE I

VALUES OF THE COEFFICIENT OF FRICTION f FOR SMOOTH CAST- OR WROUGHT-IRON PIPES

Diameter		Velocity in Feet per Second									
Inches	Feet	1	2	3	4	5	6	8	10	12	15
½ =	04166	0440	0345	0301	0289	0282	0276	0265	0258	0252	0248
¾ =	06250	0408	0332	0298	0284	0277	0271	0261	0254	0248	0244
1 =	08333	0380	0324	0294	0281	0274	0268	0258	0251	0246	0241
1½ =	12500	0357	0316	0289	0277	0270	0264	0255	0248	0243	0239
1¾ =	14583	0336	0303	0283	0273	0266	0260	0251	0244	0240	0236
2 =	16667	0316	0292	0277	0268	0262	0256	0247	0240	0236	0233
3 =	25000	0297	0280	0268	0260	0254	0249	0240	0234	0233	0229
4 =	33333	0285	0271	0260	0252	0247	0242	0235	0229	0224	0221
6 =	50000	0274	0259	0249	0243	0238	0233	0225	0220	0216	0213
8 =	66667	0264	0250	0240	0234	.0229	0225	0218	0212	0209	0206
10 =	8333	0257	0244	0234	0227	0223	0219	0213	0208	0205	0202
12 =	1 0000	0250	0237	0228	0221	0217	0214	0208	0203	0200	0197
14 =	1 1667	0244	0230	0222	0216	0212	0208	0201	0197	0195	0192
16 =	1 3333	0235	0224	0215	0210	0206	0203	0196	0193	0191	0189
18 =	1 5000	0228	0217	0209	0204	0200	0198	0193	0189	0187	0185
20 =	1 6667	0222	0212	0204	0199	0196	0193	0188	0185	0183	0180
24 =	2 0000	0210	0200	0193	0189	0186	0184	0180	0177	0176	0173
30 =	2 5000	0197	0188	0181	0176	0174	0172	0169	0166	0165	0163
36 =	3 0000	0185	0177	0170	0166	0164	0162	0159	0157	0156	0154
42 =	3 5000	0168	0163	0159	0157	0155	0154	0151	0149	0148	0147
48 =	4 0000	0158	0154	0150	0148	0146	0145	0143	0141	0140	0139
54 =	4 5000	0149	0146	0143	0140	0138	0136	0134	0132	0132	0131
60 =	5 0000	0141	0138	0130	0133	0132	0130	0128	0126	0125	0125
72 =	6 0000	0134	0131	0128	0127	0125	0124	0122	0120	0120	0119

TABLE II

COEFFICIENTS FOR ANGULAR BENDS

a = angle of bend in degrees

a	10°	20°	40°	60°	80°	90°	100°	110°	120°	130°	140°	150°
k_s	017	046	139	364	74	984	1 26	1 56	1 86	2 16	2 43	2 81

TABLE III

COEFFICIENTS FOR CIRCULAR BENDS

r = radius of pipe R = radius of bend

$\frac{r}{R}$	1	2	3	4	5	6	7	8	9	1 0
k_c	131	138	158	206	294	440	661	977	1 408	1 978

TABLE IV

HYDRAULIC TABLE FOR CAST-IRON PIPES

$d = 4$ *inches* $= \frac{1}{3}$ *foot*

v	$s = \frac{h}{l}$	$s_m = 5\,280\,\frac{h}{l}$	$G = \frac{l}{h}$	Q			Q'		
				Cubic Feet per Second	Gallons per Minute	Gallons per Day	Cubic Feet per Second	Gallons per Minute	Gallons per Day
1	000014235	075159	70,250	0087265	3 9165	5,639 6	0061704	2 7693	3,987 8
2	000056343	29749	17,748	017453	7 8329	11,279	012341	5 5386	7,975 5
3	00012593	6649	7,940 9	026179	11 749	16,919	018511	8 3079	11,963
4	00022149	1 1694	4,514 9	034906	15 666	22,558	024682	11 077	15,951
5	00034375	1 8150	2,909 1	043632	19 582	28,108	030852	13 847	19,939
6	00049103	2 5958	2,034 0	052359	23 498	33 838	037023	16 616	23,927
7	00060460	3 5091	1,504 7	061086	27 415	39 477	043194	19 385	27 915
8	[illegible]	4 5518	1,160 0	069812	31 331	45 117	049364	22 154	31 902
9	[illegible]	5 7288	921 [illegible]	078538	[illegible]	[illegible]	055534	24 924	35 890
10	[illegible]	[illegible]	752 [illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	39 878
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	43 865
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	47 853
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	51 841
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	55 829
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

16	0032955	17 400	303 44	13962	62 663	90 234	098728	44 309	63 804
17	0037043	19 558	269 97	14835	66 580	95,874	10490	47 078	67 792
18	0041346	21 830	241 86	15708	70 495	101,510	11107	49 847	71 779
19	0045797	24 181	218 36	16580	74 412	107,150	11724	52 616	75,767
20	0050596	26 715	197 64	17453	78 328	112,790	12341	55 386	79 755
21	0055535	29 323	180 07	18326	82 245	118,430	12958	58 155	83 743
22	0060679	32 038	164 80	19198	86 160	124 070	13575	60 924	87,730
23	0065928	34 810	151 68	20071	90 078	129,710	14192	63 694	91,719
24	0071461	37 731	139 94	20944	93 994	135,350	14809	66 463	95,706
25	0077192	40 757	129 55	21816	97 910	140,990	15426	69 232	99,694
26	0083111	43 882	120 32	22689	101 83	146,630	16043	72 001	103,680
27	0089217	47 106	112 09	23561	105 74	152,270	16660	74 770	107 670
28	0095660	50 508	104 54	24434	109 66	157,910	17278	77 541	111,660
29	0102300	54 015	97 750	25307	113 58	163,550	17895	80 310	115 650
30	010914	57 625	91 626	26179	117 49	169,190	18511	83 079	119,630
31	011618	61 341	86 076	27052	121 41	174,830	19129	85 848	123,620
32	012341	65 161	81 029	27925	125 33	180,470	19745	88 618	127,610
33	013084	69 083	76 430	28797	129 24	186,100	20362	91 386	131 600
34	013846	73 107	72 222	29670	133 16	191,750	20980	94 156	135,580
35	014627	77 229	68 368	30543	137 08	197,390	21597	96 926	139,570

TABLE IV—(*Continued*)

$d = 4$ *inches* $= \frac{1}{3}$ *foot*

v	$s = \frac{h}{l}$	$s_m = 5{,}280\,\frac{h}{l}$	$G = \frac{l}{h}$	Q Cubic Feet per Second	Q Gallons per Minute	Q Gallons per Day	Q' Cubic Feet per Second	Q' Gallons per Minute	Q' Gallons per Day
3 6	015426	81 448	64 826	31415	140 99	203,030	22214	99 694	143,560
3 7	016244	85 767	61 562	32288	144 91	208,670	22831	102 46	147,550
3 8	017106	90 321	58 457	33160	148 82	214,310	23448	105 23	151,530
3 9	017962	94 837	55 674	34033	152 74	219,940	24065	108 00	155 520
4 0	018836	99 451	53 091	34906	156 66	225,580	24682	110 77	159,510
4 1	019750	104 28	50 634	35778	160 57	231,220	25299	113 54	163 500
4 2	020684	109 21	48 346	36651	164 49	236,800	25916	116 31	167 490
4 3	021629	114 20	46 234	37524	168 41	242 500	26533	119 08	171 470
4 4	022601	119 33	44 240	38309	172 32	248 1[illegible]	27150	121 85	175,460
4 5	023503	124 57	42 380	39269	17[illegible] 24	253 [illegible]	27767	124 62	179 450
4 [illegible]	[illegible]	129 91	40 043	[illegible]	[illegible]	259 420	[illegible]	127 30	183 440
4 7	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	130 16	187 430
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	132 93	191 410
4 [illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	135 72	195 410
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	199 390

5 5	034510	182 21	28 977	47995	215 40	310,180	33938	152 31	219 330
6 0	040634	214 55	24 610	52359	234 99	338,380	37023	166 16	239,270
6 5	047293	249 71	21 145	56722	254 57	366,570	40108	180 00	259,200
7 0	054394	287 20	18 384	61086	274 15	394,780	43194	193 85	279,150
7 5	062021	327 47	16 124	65449	293 73	422 970	46279	207 70	299,080
8 0	070089	370 07	14 268	69812	313 31	451,170	49364	221 54	319,020
8 5	078518	414 57	12 736	74175	332 90	479,370	52449	235 39	338,960
9 0	087345	461 18	11 449	78538	352 48	507,560	55534	249 24	358,900
9 5	096814	511 18	10 329	82901	372 06	535,760	58619	263 08	378,830
10 0	10672	563 46	9 3707	87265	391 64	563,960	61705	276 93	398,780
10 5	11704	617 95	8 5444	91628	411 23	592,160	64790	290 78	418,720
11 0	12777	674 62	7 8265	95991	430 80	620,360	67875	304 62	438,650
11 5	13891	733 45	7 1988	1 0036	450 39	648,560	70961	318 47	458,600
12 0	15045	794 35	6 6469	1 0472	469 97	676,750	74046	332 32	478,530
12 5	16281	859 61	6 1423	1 0908	489 55	704,950	77131	346 16	498,470
13 0	175620	927 26	5 6942	1 1344	509 13	733,150	80216	360 01	518,410
13 5	18887	997 24	5 2946	1 1781	528 71	761 340	83301	373 85	538,340
14 0	20258	1,069 6	4 9363	1 2217	548 30	789,550	86387	387 70	558,290
14 5	21712	1,146 4	4 6059	1 2654	567 89	817,750	89473	401 55	578,230
15 0	23213	1,225 7	4 3078	1 3090	587 46	845,940	92557	415 39	598,160

TABLE IV—(*Continued*)

$d = 6$ *inches* $= \ 5$ *foot*

v	$s = \frac{h}{l}$	$s_m = 5\,280\,\frac{h}{l}$	$G = \frac{l}{h}$	Q Cubic Feet per Second	Q Gallons per Minute	Q Gallons per Day	Q' Cubic Feet per Second	Q' Gallons per Minute	Q' Gallons per Day
1	0000090795	047939	110,140	019635	8 8121	12,689	013884	6 2310	8 972 6
2	000036020	19018	27,763	039270	17 624	25,379	027768	12 462	17,945
3	000080595	42554	12,408	058905	26 436	38,068	041651	18 693	26,918
4	00014189	74917	7,047 7	078540	35 248	50,758	055535	24 924	35,891
5	00022015	1 1624	4,542 4	098175	44 061	63,447	069419	31 155	44,863
6	00031433	1 6596	3,181 4	11781	52 873	76,136	083303	37 386	53,836
7	00042479	2 2428	2,354 1	13745	61 685	88,826	097187	43 617	62,809
8	00055163	2 9126	1,812 8	15708	70 497	101,520	11107	49 848	71,781
9	00069312	3 6597	1,442 7	17671	79 309	114,200	12495	56 079	80,753
1 0	00085075	4 4919	1,175 4	19635	88 121	126,890	13884	62 310	89,726
1 1	0010219	5 3954	978 61	21508	96 933	139,580	15272	68 541	98,698
1 2	0012071	6 3737	828 40	23562	105 75	152 270	16661	74 772	107,670
1 3	0014083	7 4357	710 08	25525	114 50	104 000	18049	81 003	116,640
1 4	0016230	8 5724	615 93	27489	123 37	177 650	19437	87 235	125,620
1 5	0018520	9 7816	539 7[illegible]	29452	132 18	190 340	20820	93 465	134,590

1 6	0020951	11 062	477 30	31416	140 99	203,030	22214	99 697	143,560
1 7	0023544	12 431	424 74	33380	149 81	215,720	23603	105 93	152,540
1 8	0026274	13 873	380 60	35343	158 62	228,410	24991	112 16	161,510
1 9	0029184	15 409	342 65	37306	167 43	241,100	26379	118 39	170,480
2 0	0032238	17 022	310 19	39270	176 24	253,790	27768	124 62	179,450
2 1	0035379	18 680	282 66	41234	185 05	266,480	29156	130 85	188,430
2 2	0038647	20 406	258 75	43197	193 87	279,160	30544	137 08	197,400
2 3	0042044	22 199	237 85	45161	202 68	291,860	31933	143 31	206,370
2 4	0045564	24 057	219 47	47124	211 49	304,540	33321	149 54	215,340
2 5	0049284	26 022	202 90	49087	220 30	317,230	34710	155 78	224,320
2 6	0053137	28 056	188 19	51051	229 11	329,920	36098	162 01	233,290
2 7	0057122	30 160	175 06	53014	237 93	342,610	37486	168 24	242,260
2 8	0061238	32 333	163 30	54978	246 74	355,300	38875	174 47	251,230
2 9	0065376	34 518	152 96	56942	255 55	367,990	40263	180 70	260,210
3 0	0069738	36 821	143 39	58905	264 36	380,680	41651	186 93	269,180
3 1	0074225	39 190	134 73	60868	273 17	393,370	43040	193 16	278,150
3 2	0078837	41 626	126 84	62832	281 99	406,060	44428	199 39	287,120
3 3	0083705	44 196	119 47	64795	290 80	418 750	45816	205 62	296,090
3 4	0088569	46 764	112 91	66759	299 61	431,440	47205	211 86	305,070
3 5	0093551	49 395	106 89	68723	308 43	444,130	48594	218 09	314,040

TABLE IV—(*Continued*)

$d = 6$ *inches* $= .5$ *foot*

v	$s = \frac{h}{l}$	$s_m = 5\,280\,\frac{h}{l}$	$G = \frac{l}{h}$	Q: Cubic Feet per Second	Q: Gallons per Minute	Q: Gallons per Day	Q': Cubic Feet per Second	Q': Gallons per Minute	Q': Gallons per Day
3 6	009881	52 171	101 20	70685	317 23	456,810	49982	224 32	323,010
3 7	010404	54 930	96 121	72649	326 05	469,500	51370	230 55	331,990
3 8	010956	57 844	91 279	74612	334 86	482,190	52758	236 78	340,960
3 9	011502	60 728	86 944	76576	343 67	494,880	54146	243 01	349,930
4 0	012080	63 779	82 785	78540	352 48	507,570	55535	249 24	358 910
4 1	012665	66 868	78 961	80503	361 29	520,260	56923	255 47	367,870
4 2	013263	70 028	75 398	82467	370 11	532,950	58312	261 70	376,850
4 3	013868	73 220	72 111	84431	378 92	545,640	59701	267 94	385,820
4 4	014490	76 505	69 014	86393	387 73	558,330	61089	274 16	394,790
4 5	015124	79 855	66 120	88357	396 54	571 020	62477	280 30	403,770
4 6	015771	83 272	63 406	90321	405 36	583 710	63866	286 63	412,740
4 7	016430	86 750	60 864	92285	414 17	596 400	65254	292 86	421 720
4 8	017093	90 253	58 522	94248	422 98	[illegible]	66642	299 09	430 690
4 9	017770	93 858	56 255	96212	431 80	[illegible]	68032	305 32	439 660
5 0	018470	97 522	54 14	98175	440 61	634 472	69420	311 55	448 630

5 5	022123	116 81	45 202	1 0799	484 66	697,910	76361	342 70	493,490
6 0	026059	137 59	38 374	1 1781	528 73	761,360	83303	373 86	538,360
6 5	030321	160 09	32 981	1 2763	572 78	824,800	90244	405 01	583,220
7 0	034861	184 07	28 685	1 3745	616 85	888,260	97187	436 17	628,090
7 5	039668	209 45	25 209	1 4726	660 91	951,700	1 0413	467 33	672,950
8 0	044736	236 21	22 353	1 5708	704 97	1,015,100	1 1107	498 48	717,810
8 5	050188	264 99	19 925	1 6690	749 03	1,078,600	1 1801	529 64	762,680
9 0	055913	295 22	17 885	1 7671	793 09	1,142,000	1 2495	560 79	807,530
9 5	061961	327 15	16 139	1 8653	837 14	1,205,500	1 3190	591 94	852,390
10 0	068283	360 53	14 645	1 9635	881 21	1,268,900	1 3884	623 10	897,260
10 5	074974	395 86	13 338	2 0617	925 27	1,332,400	1 4578	654 26	942,130
11 0	081945	432 66	12 203	2 1598	969 33	1,395,800	1 5272	685 41	986,980
11 5	089195	470 94	11 211	2 2580	1,013 4	1,459,300	1 5967	716 57	1,031,900
12 0	096714	510 65	10 340	2 3562	1,057 5	1,522,700	1 6661	747 72	1,076,700
12 5	10460	552 29	9 5600	2 4544	1,101 5	1,586,200	1 7355	778 88	1,121,600
13 0	11277	595 42	8 8677	2 5525	1,145 6	1,649,600	1 8049	810 03	1,166,400
13 5	12121	640 00	8 2499	2 6507	1 189 6	1,713,000	1 8743	841 18	1,211,300
14 0	12994	686 06	7 6961	2 7489	1,233 7	1,776,500	1 9438	872 35	1,256,200
14 5	13919	734 90	7 1846	2 8471	1,277 8	1,840,000	2 0132	903 50	1,301,000
15 0	14874	785 33	6 7233	2 9452	1,321 8	1,903,400	2 0826	934 65	1,345,900

TABLE IV—(*Continued*)

$d = 8$ *inches* $=$ 6667 *foot*

v	$s = \frac{h}{l}$	$s_m = 5{,}280\frac{h}{l}$	$G = \frac{l}{h}$	Q: Cubic Feet per Second	Q: Gallons per Minute	Q: Gallons per Day	Q′: Cubic Feet per Second	Q′: Gallons per Minute	Q′: Gallons per Day
1	0000065670	034674	152,280	034907	15 666	22,559	024683	11 077	15,951
2	000026045	13751	38,395	069814	31 332	45,118	049365	22 155	31,903
·3	000058180	30719	17,188	10472	46 998	67,677	074047	33 232	47,854
·4	00010269	54218	9,738 5	13963	62 664	90,236	098730	44 310	63,806
5	00015904	83975	6,287 5	17453	78 330	112,800	12341	55 387	79,757
6	00022735	1 2004	4,398 6	20944	93 996	135,350	14810	66 465	95,708
7	00030762	1 6242	3,250 7	24435	109 66	157,910	17278	77 543	111,660
8	00039880	2 1057	2,507 5	27925	125 33	180,470	19746	88 620	127,610
9	00050094	2 6450	1,996 2	31416	140 99	203,030	22214	99 697	143,560
1 0	00061474	3 2458	1,626 7	34907	156 66	225,590	24683	110 77	159,510
1 1	00073818	3 8975	1,354 7	38397	172 33	248,150	27151	121 85	175,460
1 2	00087581	4 6242	1,141 8	41888	187 99	270,710	29619	132 93	191,420
1 3	0010215	5 3936	978 93	45378	203 66	293,260	32087	144 01	207 370
1 4	0011775	6 2170	849 28	48870	219 33	315,830	34556	155 09	223 320
1 5	0013433	7 0923	744 46	52360	234 99	338 380	37024	166 16	239,270

1 6	0015212	8 0317	657 39	55851	250 66	360,940	39492	177 24	255,220
1 7	0017065	9 0101	586 00	59342	266 32	383,500	41960	188 32	271,180
1 8	0019041	10 053	525 19	62832	281 99	406,060	44428	199 39	287,120
1 9	0021148	11 166	472 87	66322	297 65	428,620	46896	210 47	303,070
2 0	0023283	12 293	429 50	69814	313 32	451,180	49365	221 55	319,030
2 1	0025546	13 488	391 44	73304	328 99	473,740	51833	232 63	334,980
2 2	0027902	14 732	358 40	76794	344 65	496,300	54301	243 70	350,930
2 3	0030398	16 050	328 97	80286	360 32	518,860	56770	254 78	366,890
2 4	0032937	17 390	303 61	83776	375 98	541,410	59238	265 86	382,830
2 5	0035622	18 808	280 72	87267	391 65	563,980	61706	276 94	398,790
2 6	0038403	20 276	260 40	90757	407 31	586,530	64174	288 01	414,730
2 7	0041209	21 758	242 66	94248	422 98	609,090	66642	299 09	430,690
2 8	0044173	23 323	226 38	97739	438 65	631,660	69111	310 17	446,640
2 9	0047228	24 936	211 74	1 0123	454 32	654 210	71580	321 25	462,590
3 0	0050372	26 596	198 52	1 0472	469 98	676,770	74047	332 32	478,540
3 1	0053697	28 352	186 23	1 0821	485 65	699,330	76516	343 40	494,490
3 2	0057026	30 109	175 36	1 1170	501 31	721,890	78984	354 48	510,450
3 3	0060544	31 967	165 17	1 1519	516 98	744,440	81452	365 55	526,390
3 4	0064055	33 821	156 12	1 1868	532 65	767,010	83921	376 63	542,350
3 5	0067763	35 778	147 57	1 2218	548 31	789,570	86389	387 71	558 300

TABLE IV—(*Continued*)

$d = 8$ *inches* $= 6667$ *foot*

v	$s = \frac{h}{l}$	$s_m = 5\ 280 \frac{h}{l}$	$G = \frac{l}{h}$	Q: Cubic Feet per Second	Q: Gallons per Minute	Q: Gallons per Day	Q′: Cubic Feet per Second	Q′: Gallons per Minute	Q′: Gallons per Day
3 6	0071568	37 788	139 73	1 2566	563 98	812,120	88857	398 79	574,250
3 7	0075344	39 781	132 72	1 2915	570 64	834,680	91325	409 86	590,200
3 8	0079338	41 890	126 04	1 3264	595 31	857,240	93793	420 94	606,150
3 9	0083284	43 973	120 07	1 3614	610 97	879,790	96261	432 02	622,100
4 0	0087462	46 180	114 34	1 3963	626 64	902,360	98730	443 10	638,060
4 1	0091692	48 413	109 06	1 4312	642 30	924,910	1 0120	454 17	654,000
4 2	0096015	50 696	104 15	1 4661	657 98	947,480	1 0367	465 25	669,960
4 3	010038	53 002	99 619	1 5010	673 64	970,040	1 0614	476 33	685,910
4 4	010488	55 376	95 348	1 5359	689 30	992,590	1 0860	487 41	701,860
4 5	010946	57 796	91 355	1 5708	704 97	1,015,100	1 1107	498 48	717,810
4 6	011414	60 266	87 611	1 6057	720 64	1,037,700	1 1354	509 57	733,770
4 7	011890	62 778	84 105	1 6406	736 31	1,060,300	1 1601	520 64	749 720
4 8	012369	65 307	80 848	1 6755	751 97	1,082,800	1 1848	531 72	765,670
4 9	012862	67 909	77 750	1 7104	767 64	1,105,400	1 2095	542 80	781,630
5 0	013363	70 554	74 836	1 7453	783 30	1,128,000	1 2341	553 87	797,570

5 5	016014	84 551	62 447	1 9199	861 63	1,240,700	1 3575	609 26	877,320
6 0	018873	99 648	52 986	2 0944	939 96	1,353,500	1 4809	664 65	957,080
6 5	021952	115 91	45 553	2 2689	1,018 3	1,466,300	1 6044	720 03	1,036,800
7 0	025231	133 22	39 633	2 4435	1,096 6	1,579,100	1 7278	775 43	1,116,600
7 5	028754	151 82	34 778	2 6180	1,175 0	1,691,900	1 8512	830 81	1,196,400
8 0	032477	171 48	30 791	2 7925	1,253 3	1,804,700	1 9746	886 20	1,276,100
8 5	036462	192 52	27 426	2 9671	1,331 6	1,917,500	2 0980	941 59	1,355,900
9 0	040650	214 63	24 600	3 1416	1,409 9	2,030,300	2 2214	996 97	1,435,600
9 5	044955	237 36	22 244	3 3161	1,488 3	2,143,100	2 3448	1,052 3	1,515,400
10 0	049440	261 04	20 226	3 4907	1,566 6	2,255,900	2 4683	1,107 7	1,595,700
10 5	054301	286 71	18 416	3 6652	1,644 9	2,368,700	2 5917	1,163 1	1,674,900
11 0	059370	313 47	16 843	3 8397	1,723 3	2,481,500	2 7151	1,218 5	1,754,600
11 5	064644	341 32	15 469	4 0143	1 801 6	2,594,300	2 8385	1,273 9	1,834,400
12 0	070118	370 22	14 262	4 1888	1,879 9	2,707,100	2 9619	1,329 3	1,914,200
12 5	075865	400 56	13 181	4 3633	1,958 3	2,819,900	3 0853	1,384 7	1,993,900
13 0	081818	432 00	12 222	4 5378	2,036 6	2,932,600	3 2087	1,440 1	2,073,700
13 5	087977	464 52	11 367	4 7124	2,114 9	3,045,400	3 3321	1,495 4	2,153,400
14 0	094343	498 13	10 600	4 8870	2,193 3	3,158,300	3 4556	1,550 9	2,233,200
14 5	101c6	533 57	9 8956	5 0615	2,271 6	3,271,100	3 5790	1,606 2	2,313,000
15 0	10799	570 16	9 2604	5 2360	2,349 9	3,383,800	3 7024	1,661 6	2,392,700

TABLE IV—(*Continued*)

d = 10 *inches* = 8333 *foot*

v	$s = \frac{h}{l}$	$s_m = 5\,280\,\frac{h}{l}$	$G = \frac{l}{h}$	Q: Cubic Feet per Second	Q: Gallons per Minute	Q: Gallons per Day	Q′: Cubic Feet per Second	Q′: Gallons per Minute	Q′: Gallons per Day
1	0000051045	026951	195,910	054542	24 478	35,248	038566	17 309	24,924
2	000020239	10686	49,411	10908	48 956	70,497	077133	34 617	49,848
3	000045201	23866	22,123	16363	73 434	105,750	11570	51 925	74,772
4	00007976	42113	12,537	21817	97 913	140,990	15427	69 234	99,697
5	00012369	65309	8,084 6	27271	122 39	176,240	19283	86 543	124,620
6	00017704	93478	5,648 3	32725	146 87	211,490	23140	103 85	149,540
7	00023915	1 2627	4,181 5	38179	171 35	246,740	26997	121 16	174,470
8	00031092	1 6416	3,216 3	43633	195 83	281,990	30853	138 47	199,390
9	00039109	2 0681	2 553 1	49087	220 30	317,230	34710	155 78	224,320
10	00047084	2 5336	2,084 0	54542	244 78	352 480	38566	173 09	249,240
11	00057790	3 0513	1,730 4	59996	269 20	387,730	42423	190 39	274,160
12	00068237	3 6029	1 465 5	65450	293 74	422 980	46280	207 70	299,090
13	00079760	4 2084	1,254 6	70904	318 21	458 230	50136	225 01	324,010
14	00091857	4 8500	1 088 7	76359	342 70	493,480	53903	242 32	348 940
15	0010494	5 5409	952 91	81813	367 17	528 730	57850	259 63	373,860

1 6	0011883	6 2741	841 55	87267	391 65	563,980	61706	276 94	398,790
1 7	0013328	7 0374	750 27	92721	416 13	599,230	65563	294 25	423,710
1 8	0014870	7 8513	672 50	98175	440 61	634,470	69419	311 55	448,630
1 9	0016487	8 7050	606 54	1 0363	465 08	669,710	73276	328 86	473,550
2 0	0018179	9 5984	550 09	1 0908	489 56	704,970	77133	346 17	498,480
2 1	0019960	10 539	501 00	1 1454	514 04	740,220	80990	363 48	523,410
2 2	0021825	11 523	458 19	1 1999	538 52	775,460	84846	380 79	548,330
2 3	0023746	12 538	421 12	1 2545	563 00	810,720	88703	398 10	573,260
2 4	0025737	13 589	388 54	1 3090	587 48	845,960	92559	415 40	598,180
2 5	0027798	14 677	359 73	1 3635	611 96	881,210	96416	432 71	623,100
2 6	0029940	15 808	334 00	1 4181	636 43	916,450	1 0027	450 02	648,020
2 7	0032165	16 983	310 90	1 4726	660 91	951,700	1 0413	467 33	672,950
2 8	0034446	18 187	290 31	1 5272	685 39	986,960	1 0799	484 64	697,880
2 9	0036810	19 435	271 67	1 5817	709 87	1,022,200	1 1184	501 95	722,800
3 0	0039223	20 710	254 95	1 6363	734 34	1,057,500	1 1570	519 25	747,720
3 1	0041738	22 037	239 59	1 6908	758 82	1,092,700	1 1956	536 56	772,650
3 2	0044322	23 402	225 62	1 7453	783 30	1,128,000	1 2341	553 87	797,570
3 3	0046972	24 801	212 89	1 7999	807 77	1,163,200	1 2727	571 18	822,490
3 4	0049690	26 236	201 25	1 8544	832 26	1,198,500	1 3113	588 49	847,420
3 5	0052473	27 706	190 57	1 9090	856 74	1,233,700	1 3498	605 80	872,350

TABLE IV—(*Continued*)

$d = 10$ *inches* $= 8333$ *foot*

v	$s = \frac{h}{l}$	$s_m = 5\,280\frac{h}{l}$	$G = \frac{l}{h}$	Q: Cubic Feet per Second	Q: Gallons per Minute	Q: Gallons per Day	Q′: Cubic Feet per Second	Q′: Gallons per Minute	Q′: Gallons per Day
3 6	0055394	29 248	180 53	1 9635	881 21	1,268,900	1 3884	623 10	897,260
3 7	0058386	30 828	171 27	2 0180	905 69	1,304,200	1 4270	640 41	922,190
3 8	0061477	32 459	162 66	2 0726	930 17	1,339,400	1 4655	657 72	947,110
3 9	0064612	34 115	154 77	2 1271	954 64	1,374,700	1 5041	675 03	972,030
4 0	0067820	35 809	147 45	2 1817	979 13	1,409,900	1 5427	692 34	996,970
4 1	0071126	37 554	140 60	2 2362	1,003 6	1,445,200	1 5812	709 64	1 021,900
4 2	0074509	39 341	134 21	2 2908	1,028 1	1,480,400	1 6198	726 96	1,046,800
4 3	0077961	41 163	128 27	2 3453	1,052 6	1,515,700	1 6584	744 27	1,071,700
4 4	0081484	43 023	122 72	2 3998	1,077 0	1,550,900	1 6969	761 57	1,096,700
4 5	0085079	44 921	117 54	2 4544	1,101 5	1,586,200	1 7355	778 88	1,121,600
4 6	0088746	46 858	112 68	2 5089	1,126 0	1,621,400	1 7741	796 20	1,146,500
4 7	0092480	48 829	108 13	2 5635	1,150 5	1,656,700	1 8126	813 50	1,171,400
4 8	0096285	50 838	103 86	2 6180	1,175 0	1,691,900	1 8512	830 81	1,196,400
4 9	010016	52 885	99 839	2 6726	1,199 4	1,727,200	1 8898	848 13	1,221,300
5 0	010410	54 965	96 059	2 7271	1,223 9	1,762,400	1 9283	865 43	1,246,200

5 5	012484	65 913	80 105	2 9998	1,346 3	1,938,700	2 1211	951 96	1,370,800
6 0	014722	77 732	67 925	3 2725	1,468 7	2,114,900	2 3140	1,038 5	1,495,400
6 5	017152	90 561	58 303	3 5452	1,591 1	2,291,100	2 5068	1,125 0	1,620,100
7 0	019746	104 26	50 643	3 8179	1,713 5	2,467,400	2 6997	1,211 6	1,744,700
7 5	022499	118 80	44 446	4 0906	1,835 9	2,643,600	2 8925	1,298 1	1,869,300
8 0	025409	134 16	39 357	4 3633	1,958 3	2,819,900	3 0853	1,384 7	1,993,900
8 5	028495	150 45	35 094	4 6361	2,080 7	2,996,100	3 2782	1,471 2	2,118 600
9 0	031734	167 56	31 512	4 9087	2,203 0	3,172,300	3 4710	1,557 8	2,243,200
9 5	035190	185 80	28 417	5 1814	2,325 4	3,348,600	3 6638	1,644 3	2,367,800
10 0	038805	204 89	25 770	5 4542	2,447 8	3,524,800	3 8566	1,730 9	2,492,400
10 5	042619	225 02	23 464	5 7269	2,570 2	3,701,100	4 0495	1,817 4	2,617,000
11 0	046593	246 01	21 462	5 9996	2 692 6	3,877,300	4 2423	1,903 9	2,741,600
11 5	050728	267 84	19 713	6 2723	2,815 0	4,053,600	4 4352	1,990 5	2,866,300
12 0	055020	290 50	18 175	6 5450	2,937 4	4,229,800	4 6280	2,077 0	2,990,900
12 5	059555	314 45	16 791	6 8177	3,059 8	4,406,100	4 8208	2,163 6	3,115,500
13 0	064255	339 27	15 563	7 0904	3,182 1	4,582,300	5 0136	2,250 1	3,240,100
13 5	069124	364 97	14 467	7 3631	3,304 5	4,758,500	5 2064	2,336 6	3,364,700
14 0	074158	391 55	13 485	7 6359	3,427 0	4,934,800	5 3993	2,423 2	3,489,400
14 5	079354	418 99	12 602	7 9086	3,549 4	5,111,000	5 5922	2,509 7	3,614,000
15 0	084709	447 26	11 805	8 1813	3,671 7	5,287,300	5 7850	2,596 3	3,738,600

TABLE IV—(*Continued*)

$d = 12$ *inches* $= 1$ *foot*

v	$s = \frac{h}{l}$	$s_m = 5\,280 \frac{h}{l}$	$G = \frac{l}{h}$	Q			Q'		
				Cubic Feet per Second	Gallons per Minute	Gallons per Day	Cubic Feet per Second	Gallons per Minute	Gallons per Day
1	0000041604	021967	240,360	078540	35 248	50,757	055535	24 924	35,891
2	000016468	086948	60,725	15708	70 497	101,520	11107	49 848	71,781
3	000036772	19416	27,194	23562	105 75	152,270	16661	74 772	107,670
4	000064875	34254	15,414	31416	140 99	203,030	22214	99 697	143,560
5	00010075	53194	9,925 9	39270	176 24	253,790	27768	124 62	179,450
6	00014373	75889	6,957 4	47124	211 49	304,540	33321	149 54	215,340
7	00019442	1 0265	5,143 6	54978	246 74	355,300	38875	174 47	251,230
8	00025194	1 3302	3,909 3	62832	281 99	406,060	44428	199 39	287 120
9	00031684	1 6729	3,156 2	70685	317 23	456,810	49982	224 32	323 010
10	00038805	2 0489	2,577 0	78540	352 48	507,570	55535	249 24	358 910
11	00046653	2 4633	2,143 5	86393	387 73	558 330	61089	274 16	394,790
12	00055254	2 9174	1,809 8	94248	422 98	609,090	66642	299 09	430 690
13	00064424	3 4016	1,552 2	1 0210	458 23	659,840	72195	324 01	466 570
14	00074353	3 9258	1,344 9	1 0996	493 48	710,610	77750	348 94	502,470
15	00084934	4 4845	1,177 4	1 1781	528 73	761,360	83303	373 86	538,360

1 6	00096000	5 0687	1,041 7	1 2566	563 98	812,120	88857	398 79	574,250
1 7	0010801	5 7031	925 81	1 3352	599 23	862,880	04410	423 71	610,140
1 8	0012069	6 3725	828 55	1 4137	634 47	913,630	99963	448 63	646,030
1 9	0013402	7 0764	746 14	1 4922	669 71	964,380	1 0552	473 55	681,910
2 0	0014751	7 7884	677 92	1 5708	704 97	1,015,100	1 1107	498 48	717,810
2 1	0016208	8 5580	616 96	1 6493	740 22	1,065,900	1 1662	523 41	753,700
2 2	0017728	9 3605	564 07	1 7279	775 46	1,116,700	1 2218	548 33	789,590
2 3	0019295	10 188	518 27	1 8064	810 72	1,167,400	1 2773	573 26	825,490
2 4	0020910	11 041	478 23	1 8850	845 96	1,218,200	1 3328	598 18	861,370
2 5	0022582	11 923	442 82	1 9635	881 21	1,268,900	1 3884	623 10	897,260
2 6	0024319	12 840	411 20	2 0420	916 45	1,319,700	1 4439	648 02	933,150
2 7	0026124	13 793	382 79	2 1206	951 70	1,370,400	1 4994	672 95	969,040
2 8	0027986	14 776	357 32	2 1991	986 96	1,421,200	1 5550	697 88	1,004,900
2 9	0029916	15 796	334 26	2 2777	1,022 2	1,472,000	1 6105	722 80	1,040,800
3 0	0031902	16 844	313 46	2 3562	1,057 5	1,522,700	1 6661	747 72	1,076,700
3 1	0033960	17 931	294 46	2 4347	1,092 7	1,573,500	1 7216	772 65	1,112,600
3 2	0036075	19 048	277 20	2 5133	1,128 0	1,624,200	1 7771	797 57	1,148,500
3 3	0038229	20 185	261 58	2 5918	1,163 2	1,675,000	1 8327	822 49	1,184,400
3 4	0040457	21 361	247 18	2 6704	1,198 5	1,725,800	1 8882	847 42	1,220,300
3 5	0042738	22 565	233 99	2 7489	1,233 7	1,776,500	1 9437	872 35	1,256,200

TABLE IV—(*Continued*)

d = 12 inches = 1 foot

v	$s = \frac{h}{l}$	$s_m = 5\,280 \frac{h}{l}$	$G = \frac{l}{h}$	Q			Q'		
				Cubic Feet per Second	Gallons per Minute	Gallons per Day	Cubic Feet per Second	Gallons per Minute	Gallons per Day
3 6	0045093	23 809	221 76	2 8274	1,268 9	1,827,300	1 9993	897 26	1,292,100
3 7	0047484	25 071	210 60	2 9060	1,304 2	1,878,000	2 0548	922 19	1,327,900
3 8	0049951	26 374	200 20	2 9845	1,339 4	1,928,800	2 1103	947 11	1,363,800
3 9	0052448	27 692	190 66	3 0630	1,374 7	1,979,500	2 1659	972 03	1,399,700
4 0	0055025	29 053	181 74	3 1416	1,409 9	2,030,300	2 2214	996 97	1,435,600
4 1	0057705	30 468	173 30	3 2201	1,445 2	2,081,000	2 2769	1,021 9	1,471,500
4 2	0060445	31 915	165 44	3 2987	1,480 4	2,131,800	2 3325	1,046 8	1,507,400
4 3	0063243	33 392	158 12	3 3772	1,515 7	2,182,600	2 3880	1,071 7	1,543,300
4 4	0066097	34 899	151 29	3 4557	1,550 9	2,233,300	2 4435	1,096 7	1,579,200
4 5	0069010	36 437	144 91	3 5343	1,586 2	2,284,100	2 4991	1,121 6	1,615,100
4 6	0071981	38 006	138 92	3 6129	1,621 4	2,334,900	2 5546	1,146 5	1,651,000
4 7	0075007	39 603	133 32	3 6914	1,656 7	2,385,600	2 6102	1,171 4	1,686,900
4 8	0078089	41 231	128 06	3 7699	1,691 9	2,436,400	2 6657	1,196 4	1,722,700
4 9	0081229	42 888	123 11	3 8485	1,727 2	2,487,100	2 7213	1,221 3	1,758,700
5 0	0084421	44 574	118 45	3 9270	1,762 4	2,537,900	2 7768	1,246 2	1,794,500

5 5	010130	53 487	98 714	4 3197	1,938 7	2,791,600	3 0544	1,370 8	1,974,000
6 0	011955	63 122	83 647	4 7124	2,114 9	3,045,400	3 3321	1,495 4	2,153,400
6 5	013939	73 595	71 743	5 1051	2,291 1	3,299,200	3 6098	1,620 1	2,332,900
7 0	016059	84 791	62 270	5 4978	2,467 4	3,553,000	3 8875	1,744 7	2,512,300
7 5	018313	96 690	54 607	5 8905	2,643 6	3,806,800	4 1651	1,869 3	2,691,800
8 0	020696	109 27	48 318	6 2832	2,819 9	4,060,600	4 4428	1,993 9	2,871,200
8 5	023185	122 41	43 132	6 6759	2,996 1	4,314,400	4 7205	2,118 6	3,050,700
9 0	025791	136 17	38 774	7 0685	3,172 3	4,568,100	4 9982	2,243 2	3,230,100
9 5	028623	151 13	34 937	7 4612	3,348 6	4,821,900	5 2758	2,367 8	3,409,600
10 0	031591	166 80	31 654	7 8540	3,524 8	5,075,700	5 5535	2,492 4	3,589,100
10 5	034693	183 18	28 824	8 2467	3,701 1	5,329,500	5 8312	2,617 0	3,768,500
11 0	037925	200 24	26 368	8 6393	3,877 3	5,583,300	6 1089	2,741 6	3,947,900
11 5	041287	217 99	24 221	9 0321	4,053 6	5,837,100	6 3866	2,866 3	4,127,400
12 0	044775	236 41	22 334	9 4248	4,229 8	6,090,900	6 6642	2,990 9	4,306,900
12 5	048440	255 76	20 644	9 8175	4,406 1	6,344,700	6 9419	3,115 5	4,486,300
13 0	052234	275 79	19 145	10 210	4,582 3	6,598,400	7 2195	3,240 1	4,665,700
13 5	056158	296 51	17 807	10 603	4,758 5	6,852,200	7 4972	3,364 7	4,845,200
14 0	060214	317 93	16 607	10 996	4,934 8	7,106,100	7 7750	3,489 4	5,024,700
14 5	064494	340 53	15 505	11 388	5,111 0	7,359,900	8 0527	3,614 0	5,204,200
15 0	068913	363 86	14 511	11 781	5,287 3	7,613,600	8 3303	3,738 6	5,383,600

TABLE IV—(*Continued*)

d = 14 inches = 1 1667 feet

v	$s = \frac{h}{l}$	$s_m = 5\,280\,\frac{h}{l}$	$G = \frac{l}{h}$	Q			Q		
				Cubic Feet per Second	Gallons per Minute	Gallons per Day	Cubic Feet per Second	Gallons per Minute	Gallons per Day
1	0000034647	018294	288,620	10690	47 977	69,086	075589	33 924	48,851
2	000013731	072500	72,827	21380	95 953	138,170	15118	67 848	97,701
3	000030703	16211	32,570	32070	143 93	207,260	22677	101 77	146,550
4	000054072	28550	18,494	42760	191 91	276,340	30236	135 70	195,400
5	000083954	44327	11,911	53450	239 88	345,430	37795	169 62	244,250
6	00011993	63324	8,338 0	64140	287 86	414,510	45353	203 54	293,100
7	00016220	85641	6,165 2	74831	335 84	483,600	52913	237 47	341,960
8	00021049	1 1114	4,750 8	85520	383 81	552,690	60471	271 39	390,800
9	00026423	1 3951	3,784 5	96210	431 79	621,770	68030	305 32	439,650
1 0	00032462	1 7140	3,080 5	1 0690	479 77	690,860	75589	339 24	488,510
1 1	00039021	2 0603	2,562 7	1 1759	527 74	759,940	83148	373 16	537,350
1 2	00046131	2 4357	2,167 8	1 2828	575 72	829,030	90707	407 09	586,210
1 3	00053869	2 8443	1,856 4	1 3897	623 69	898,110	98265	441 01	635,050
1 4	00061954	3 2711	1,614 1	1 4966	671 68	967,210	1 0583	474 94	683 910
1 5	00070760	3 7361	1,413 2	1 6035	719 65	1,036,300	1 1338	508 86	7[illegible]60

1 6	00080101	4 2293	1,248 4	1 7104	767 63	1,105,400	1 2094	542 79	781,610
1 7	00089964	4 7501	1,111 6	1 8173	815 61	1,174,500	1 2850	576 71	830,460
1 8	0010051	5 3070	994 90	1 9242	863 57	1,243,500	1 3606	610 63	879,310
1 9	0011141	5 8826	897 55	2 0311	911 55	1,312,600	1 4362	644 56	928,150
2 0	0012281	6 4844	814 25	2 1380	959 53	1,381,700	1 5118	678 48	977,010
2 1	0013481	7 1180	741 77	2 2449	1,007 5	1,450,800	1 5874	712 41	1,025,900
2 2	0014738	7 7814	678 53	2 3518	1,055 5	1,519,900	1 6630	746 33	1,074,700
2 3	0016038	8 4678	623 53	2 4587	1,103 5	1,589,000	1 7386	780 26	1,123,600
2 4	0017393	9 1835	574 94	2 5656	1,151 4	1,658,100	1 8141	814 18	1,172,400
2 5	0018790	9 9209	532 21	2 6725	1,199 4	1,727,100	1 8897	848 11	1,221,300
2 6	0020251	10 692	493 81	2 7794	1,247 4	1,796,200	1 9653	882 02	1,270,100
2 7	0021761	11 489	459 55	2 8863	1,295 4	1,865,300	2 0409	915 95	1,319,000
2 8	0023319	12 312	428 83	2 9932	1,343 4	1,934,400	2 1165	949 88	1,367,800
2 9	0024925	13 160	401 21	3 1001	1,391 3	2,003,500	2 1921	983 81	1,416,700
3 0	0026577	14 033	376 26	3 2070	1,439 3	2,072,600	2 2677	1,017 7	1,465,500
3.1	0028302	14 943	353 34	3 3139	1,487 3	2,141,700	2 3433	1,051 6	1,514,400
3 2	0030089	15 887	332 35	3 4208	1,535 3	2,210,800	2 4189	1,085 6	1,563,200
3 3	0031912	16 849	313 36	3 5277	1,583 2	2,279,800	2 4944	1,119 5	1,612,100
3 4	0033799	17 846	295 87	3 6346	1,631 2	2,348,900	2 5700	1,153 4	1,660,900
3 5	0035718	18 859	279 97	3 7415	1,679 2	2,418,000	2 6456	1,187 4	1,709,800

TABLE IV—(*Continued*)

$d = 14$ *inches* $= 1\ 1667$ *feet*

v	$s = \frac{h}{l}$	$s_m = 5\ 280 \frac{h}{l}$	$G = \frac{l}{h}$	Q			Q′		
				Cubic Feet per Second	Gallons per Minute	Gallons per Day	Cubic Feet per Second	Gallons per Minute	Gallons per Day
3 6	0037683	19 897	265 37	3 8484	1,727 1	2,487,100	2 7212	1,221 3	1,758,600
3 7	0039715	20 970	251 79	3 9553	1,775 1	2,556,200	2 7968	1,255 2	1,807,500
3 8	0041775	22 057	239 38	4 0622	1,823 1	2,625,200	2 8724	1,289 1	1,856,300
3 9	0043902	23 180	227 78	4 1691	1,871 1	2,694,300	2 9479	1,323 0	1,905,200
4 0	0046054	24 316	217 14	4 2760	1,919 1	2,763,400	3 0236	1,357 0	1,954,000
4 1	0048296	25 500	207 06	4 3829	1,967 0	2,832,500	3 0991	1,390 9	2,002,900
4 2	0050587	26 710	197 68	4 4898	2,015 0	2,901,600	3 1748	1,424 8	2,051,700
4 3	0052926	27 945	188 94	4 5967	2,063 0	2,970,700	3 2503	1,458 7	2,100,600
4 4	0055312	29 205	180 79	4 7036	2,111 0	3,039,800	3 3259	1,492 7	2,149,400
4 5	0057747	30 490	173 17	4 8105	2,158 9	3,108,900	3 4015	1,526 6	2,198,300
4 6	0060231	31 802	166 03	4 9175	2,206 9	3,178,000	3 4771	1,560 5	2,247,100
4 7	0062761	33 138	159 33	5 0244	2,254 9	3,247,100	3 5527	1,594 4	2,296,000
4 8	0065336	34 497	153 06	5 1312	2,302 9	3,316,100	3 6283	1,628 4	2,344,800
4 9	0067959	35 882	147 15	5 2382	2,350 9	3,385,200	3 7039	1,662 3	2,393,700
5 0	0070629	37 292	141 59	5 3450	2,398 8	3,454,300	3 7795	1,696 2	2,442,500

5 5	0084653	44 696	118 13	5 8795	2,638 7	3,799,700	4 1574	1,865 8	2,686,800
6 0	0099784	52 685	100 22	6 4140	2,878 6	4,145,100	4 5353	2,035 4	2,931,000
6 5	011621	61 356	86 054	6 9485	3,118 5	4,490,600	4 9133	2,205 1	3,175,300
7 0	013373	70 609	74 777	7 4831	3,358 4	4,836,000	5 2913	2,374 7	3,419,600
7 5	015217	80 343	65 717	8 0175	3,598 2	5,181,400	5 6692	2,544 3	3,663,800
8 0	017160	90 602	58 276	8 5520	3,838 1	5,526,900	6 0471	2,713 9	3,908,000
8 5	019275	101 77	51 880	9 0866	4,078 0	5,872,300	6 4251	2,883 6	4,152,300
9 0	021502	113 53	46 508	9 6210	4,317 9	6,217,700	6 8030	3,053 2	4,396,500
9 5	023836	125 85	41 953	10 155	4,557 7	6,563,100	7 1809	3,222 8	4,640,800
10 0	.026279	138 75	38 053	10 690	4,797 7	6,908,600	7 5589	3,392 4	4,885,100
10 5	028884	152 51	34 621	11 225	5,037 6	7,254,000	7 9369	3,562 1	5,129,300
11 0	031604	166 87	31 642	11 759	5,277 4	7,599,400	8 3148	3,731 6	5,373,500
11 5	034437	181 82	29 039	12 291	5,517 3	7,944,900	8 6928	3,901 3	5,617,800
12 0	037381	197 37	26 752	12 828	5,757 2	8,290,300	9 0707	4,070 9	5,862,100
12 5	040457	213 61	24 718	13 363	5,997 1	8,635,700	9 4487	4,240 5	6,106,300
13 0	043645	230 44	22 912	13 897	6,236 9	8,981,100	9 8265	4,410 1	6,350,500
13 5	046945	247 87	21 301	14 431	6,476 8	9,326,500	10 204	4,579 7	6,594,800
14 0	050358	265 89	19 858	14 966	6,716 8	9,672,100	10 583	4,749 4	6,839,100
14 5	.053963	284 92	18 531	15 501	6,956 6	10,018,000	10 960	4,919 0	7,083,400
15 0	057689	304 59	17 334	16 035	7,196 5	10,363,000	11 338	5,088 6	7,327,600

TABLE IV—(*Continued*)

d = 16 inches = 1 3333 feet

v	$s = \frac{h}{l}$	$s_m = 5\ 280 \frac{h}{l}$	$G = \frac{l}{h}$	Q			Q′		
				Cubic Feet per Second	Gallons per Minute	Gallons per Day	Cubic Feet per Second	Gallons per Minute	Gallons per Day
1	0000029058	015342	344,140	13963	62 666	90,238	098732	44 411	63,807
2	000011548	060975	86,592	27926	125 33	180,480	19746	88 622	127,610
3	000025774	13608	38,799	41889	188 00	270,710	29620	132 93	191,420
4	000045596	24075	21,932	55852	250 66	360,950	39493	177 24	255,230
5	000070778	37371	14,129	69815	313 33	451,190	49366	221 55	319,040
6	00010125	53459	9,876 7	83778	375 99	541,430	59239	265 86	382,840
7	00013713	72402	7,292 6	97742	438 66	631,670	69113	310 18	446,650
8	00017821	94091	5,611 5	1 1170	501 33	721,900	78986	354 49	510,460
9	00022403	1 1829	4,463 8	1 2567	563 99	812,140	88859	398 80	574,260
1 0	00027425	1 4480	3,646 3	1 3963	626 66	902,380	98732	443 11	638,070
1 1	00032958	1 7402	3,034 2	1 5359	689 32	992,610	1 0861	487 42	701,880
1 2	00039022	2 0603	2,562 7	1 6756	751 99	1,082,900	1 1848	531 73	765,680
1 3	00045560	2 4055	2,194 9	1 8152	814 65	1,173,100	1 2835	576 04	829,490
1 4	00052565	2 7754	1,902 3	1 9548	877 32	1,263,300	1 3823	620 35	893,310
1 5	00060027	3 1694	1,665 9	2 0945	939 98	1,353,600	1 4810	664 66	957,110

1 6	00067939	3 5872	1,471 9	2 2341	1,002 7	1,443,800	1 5797	708 97	1,020,900
1 7	00076294	4 0283	1,310 7	2 3737	1,065 3	1,534,100	1 6785	753 29	1,084,700
1 8	00085079	4 4921	1,175 4	2 5133	1,128 0	1,624,300	1 7772	797 59	1,148,500
1 9	00094456	4 9872	1,058 7	2 6530	1,190 6	1,714,500	1 8759	841 90	1,212,300
2 0	0010429	5 5064	958 87	2 7926	1,253 3	1,804,800	1 9746	886 22	1,276,100
2 1	0011447	6 0438	873 61	2 9322	1,316 0	1,895,000	2 0734	930 53	1,340,000
2 2	0012512	6 6062	799 25	3 0718	1,378 6	1,985,200	2 1721	974 83	1,403,800
2 3	0013620	7 1912	734 23	3 2115	1,441 3	2,075,500	2 2709	1,019 2	1,467,600
2 4	0014776	7 8015	676 78	3 3511	1,504 0	2,165,700	2 3696	1,063 5	1,531,400
2 5	0015975	8 4345	625 99	3 4908	1,566 6	2,256,000	2 4683	1,107 8	1,595,200
2 6	0017215	9 0893	580 90	3 6304	1,629 3	2,346,200	2 5670	1,152 1	1,659,000
2 7	0018497	9 7661	540 64	3 7700	1,692 0	2,436,400	2 6658	1,196 4	1,722,800
2 8	0019819	10 465	504 56	3 9097	1,754 7	2,526,700	2 7645	1,240 7	1,786,600
2 9	0021182	11 184	472 11	4 0493	1,817 3	2,616,900	2 8633	1,285 0	1,850,400
3 0	0022583	11 924	442 80	4 1889	1,880 0	2,707,100	2 9620	1,329 3	1,914,200
3 1	0024036	12 691	416 04	4 3285	1,942 6	2,797,400	3 0607	1,373 6	1,978,000
3 2	0025528	13 479	391 72	4 4682	2,005 3	2,887,600	3 1594	1,417 9	2,041,800
3 3	0027046	14 280	369 73	4 6078	2,068 0	2,977,800	3 2581	1,462 2	2,105,600
3 4	0028617	15 109	349 45	4 7474	2,130 6	3,068,100	3 3569	1,506 6	2,169,500
3 5	0030225	15 959	330 85	4 8871	2,193 3	3,158,300	3 4557	1,550 9	2,233,300

TABLE IV—(*Continued*)

d = 16 inches = 1 3333 feet

v	$s = \frac{h}{l}$	$s_m = 5\,280\,\frac{h}{l}$	$G = \frac{l}{h}$	Q			Q'		
				Cubic Feet per Second	Gallons per Minute	Gallons per Day	Cubic Feet per Second	Gallons per Minute	Gallons per Day
3 6	0031915	16 851	313 33	5 0267	2,256 0	3,248,600	3 5543	1,595 2	2,297,000
3 7	0033650	17 767	297 18	5 1663	2,318 6	3,338,800	3 6531	1,639 5	2,360,900
3 8	0035426	18 705	282 28	5 3059	2,381 3	3,429,000	3 7518	1,683 8	2,424,700
3 9	0037243	19 664	268 50	5 4455	2,443 9	3,519,200	3 8505	1,728 1	2,488,500
4 0	0039104	20 647	255 73	5 5852	2,506 6	3,609,500	3 9493	1,772 4	2,552,300
4 1	0041004	21 650	243 88	5 7248	2,569 3	3,699,700	4 0480	1,816 7	2,616,100
4 2	0042968	22 687	232 73	5 8645	2,632 0	3,790,000	4 1468	1,861 1	2,679,900
4 3	0044953	23 735	222 45	6 0041	2,694 6	3,880,300	4 2455	1,905 4	2,743,700
4 4	0046999	24 815	212 77	6 1437	2,757 3	3,970,500	4 3442	1,949 7	2,807,500
4 5	0049066	25 907	203 81	6 2833	2,819 9	4,060,700	4 4229	1,994 0	2,871,300
4 6	0051173	27 019	195 42	6 4230	2,882 6	4,151,000	4 5417	2,038 3	2,935,100
4 7	0053345	28 166	187 46	6 5627	2 945 3	4,241,200	4 6404	2,082 6	2,999,000
4 8	0055530	29 320	180 08	6 7022	3,007 9	4,331,400	4 7391	2,126 9	3,062,700
4 9	0057786	30 511	173 05	6 8420	3,070 6	4,421,700	4 8379	2,171 3	3,126,600
5 0	0060051	31 707	166 53	6 9815	3,133 3	4,511,900	4 9366	2,215 5	3,190,400

5 5	0072096	38 066	138 70	7 6796	3,446 6	4,963,100	5 4303	2,437 1	3,509 400
6 0	0085129	44 948	117 47	8 3778	3,759 9	5,414,300	5 9239	2,658 6	3,828,400
0 5	0099218	52 387	100 79	9 0759	4,073 2	5,865,400	6 4176	2,880 2	4,147,400
7 0	011427	60 335	87 510	9 7742	4,386 6	6,316,700	6 9113	3,101 8	4,466,500
7 5	013000	68 637	76 925	10 472	4,699 9	6,767,800	7 4049	3,323 3	4,785 500
8 0	014657	77 386	68 229	11 170	5,013 3	7,219,000	7 8986	3,544 9	5,104,600
8 5	016512	87 185	60 561	11 869	5,326 6	7,670,300	8 3923	3,766 4	5,423,600
9 0	018474	97 542	54 130	12 567	5,639 9	8,121,400	8 8859	3,988 0	5,742,600
9 5	020457	108 01	48 882	13 265	5,953 2	8,572,500	9 3795	4,209 5	6,061,600
10 0	022528	118 95	44 389	13 963	6,266 6	9,023,800	9 8732	4,431 1	6,380,700
10 5	024773	130 80	40 367	14 661	6,579 9	9,475,000	10 367	4,652 6	6,699,800
11 0	027117	143 18	36 877	15 359	6,893 2	9,926,100	10 861	4,874 2	7,018 800
11 5	029562	156 08	33 827	16 058	7,206 6	10,377,000	11 354	5,095 8	7,337,900
12 0	032104	169 51	31 149	16 756	7,519 9	10,829,000	11 848	5,317 3	7,656,800
12 5	034744	183 45	28 782	17 454	7,833 2	11,280,000	12 342	5,538 9	7,975,900
13 0	037480	197 89	26 681	18 152	8,146 5	11,731,000	12 835	5,760 4	8,294,900
13 5	040313	212 85	24 806	18 850	8,459 8	12,182,000	13 329	5,981 9	8,613,900
14 0	043240	228 31	23 127	19 548	8,773 2	12,633,000	13 823	6,203 5	8,933,100
14 5	046311	243 96	21 593	20 247	9,086 6	13,085,000	14 316	6,425 1	9,252,100
15 0	049480	261 25	20 210	20 945	9,399 8	13,536,000	14 810	6,646 6	9,571,100

d = 18 inches = 1 5 feet

v	$s = \frac{h}{l}$	$s_m = 5{,}280\frac{h}{l}$	$G = \frac{l}{h}$	Q: Cubic Feet per Second	Q: Gallons per Minute	Q: Gallons per Day	Q': Cubic Feet per Second	Q': Gallons per Minute	Q': Gallons per Day
1	0000025000	013200	400,000	17671	79 309	114,200	12495	56 079	80,753
2	0000099502	052536	100,500	35343	158 62	228,410	24991	112 16	161,510
3	000022276	11761	44,892	53014	232 92	342,610	37486	168 24	242,260
4	000039270	20734	25,465	70685	317 23	456,810	49982	224 32	323,010
5	000060945	32179	16,408	88357	396 54	571,020	62477	280 39	403,770
6	000087163	46021	11,473	1 0603	475 85	685,220	74972	336 47	484,520
7	00011783	62213	8,486 9	1 2370	555 16	799,430	87468	392 55	565,280
8	00015310	80837	6,531 6	1 4137	634 47	913,630	99963	448 63	646,030
9	00019242	1 0160	5,197 0	1 5904	713 77	1,027,800	1 1246	504 71	726,780
1 0	00023631	1 2477	4,231 7	1 7671	793 09	1,142,000	1 2495	560 79	807 530
1 1	00028494	1 5045	3,509 5	1 9438	872 39	1,256,200	1 3745	616 86	888,280
1 2	00033671	1 7778	2,969 9	2 1206	951 70	1,370,400	1 4994	672 95	969,040
1 3	00039306	2 0753	2,544 1	2 2973	1,031 0	1,484,600	1 6244	729 02	1,049,800
1 4	00045424	2 3984	2,201 5	2 4740	1,110 3	1,598,900	1 7494	785 11	1,130,600
1 5	00051864	2 7384	1,928 1	2 6507	1,189 6	1,713,000	1 8743	841 18	1,211,300

1 6	00058694	3 0990	1,703 8	2 8274	1,268 9	1,827,300	1 9993	897 26	1,292,100
1 7	00066019	3 4858	1,514 7	3 0041	1,348 2	1,941,500	2 1242	953 34	1,372,800
1 8	00073745	3 8937	1,356 0	3 1808	1,427 5	2,055,700	2 2492	1,009 4	1,453,600
1 9	00081717	4 3146	1,223 7	3 3575	1,506 9	2,169,900	2 3741	1,065 5	1,534,300
2 0	00090049	4 7546	1,110 5	3 5343	1,586 2	2,284,100	2 4991	1,121 6	1,615,100
2 1	00098915	5 2226	1,011 0	3 7110	1,665 5	2,398,300	2 6240	1,177 7	1,695,800
2 2	0010816	5 7106	924 59	3 8877	1,744 8	2,512,500	2 7490	1,233 7	1,776,600
2 3	0011777	6 2184	849 08	4 0644	1,824 1	2,626,700	2 8740	1,289 8	1,857,300
2 4	0012776	6 7456	782 73	4 2411	1,903 4	2,740,900	2 9989	1,345 9	1,938,100
2 5	0013811	7 2922	724 05	4 4178	1,982 7	2,855,100	3 1238	1,402 0	2,018,800
2 6	0014882	7 8574	671 97	4 5945	2,062 0	2,969,300	3 2488	1,458 0	2,099,600
2 7	0015988	8 4417	625 46	4 7712	2,141 3	3,083,500	3 3737	1,514 1	2,180,300
2 8	0017130	9 0444	583 78	4 9480	2,220 6	3,197,700	3 4987	1,570 2	2,261,100
2 9	0018305	9 6652	546 29	5 1247	2,300 0	3,311,900	3 6237	1,626 3	2,341,900
3 0	0019515	10 304	512 44	5 3014	2,379 2	3,426,100	3 7486	1,682 4	2,422,600
3 1	0020777	10 970	481 29	5 4781	2,458 6	3,540,300	3 8736	1,738 4	2,503,300
3 2	0022087	11 662	452 76	5 6548	2,537 9	3,654,500	3 9985	1,794 5	2,584,100
3 3	0023421	12 366	426 97	5 8315	2,617 2	3,768,700	4 1234	1,850 6	2,664,800
3 4	0024802	13 095	403 19	6 0083	2,696 5	3,882,900	4 2484	1,906 7	2,745,600
3 5	0026207	13 837	381 58	6 1850	2,775 8	3,997,100	4 3734	1,962 8	2,826,400

TABLE IV—(*Continued*)

$d = 18$ *inches* $= 1\ 5$ *feet*

v	$s = \frac{h}{l}$	$s_m = 5\ 280 \frac{h}{l}$	$G = \frac{l}{h}$	Q: Cubic Feet per Second	Q: Gallons per Minute	Q: Gallons per Day	Q': Cubic Feet per Second	Q': Gallons per Minute	Q': Gallons per Day
3 6	0027671	14 610	361 38	6 3617	2,855 1	4,111,300	4 4983	2,018 8	2,907,100
3 7	0029173	15 403	342 78	6 5384	2,934 4	4,225,500	4 6233	2,074 9	2,987,900
3 8	0030711	16 215	325 61	6 7151	3,013 7	4,339,700	4 7482	2,131 0	3,068,600
3 9	0032286	17 047	309 73	6 8918	3,093 0	4,453,900	4 8732	2,187 1	3,149,300
4 0	0033897	17 897	295 01	7 0685	3,172 3	4,568,100	4 9982	2,243 2	3,230,100
4 1	0035543	18 766	281 35	7 2452	3,251 6	4,682,300	5 1231	2,299 2	3,310,900
4 2	0037225	19 655	268 63	7 4220	3,331 0	4,796,600	5 2481	2,355 3	3,391,600
4 3	0038942	20 561	256 79	7 5987	3,410 3	4,910,800	5 3730	2,411 4	3,472,400
4 4	0040694	21 486	245 74	7 7754	3,489 6	5,024,900	5 4979	2,467 5	3,553,100
4 5	0042481	22 430	235 40	7 9521	3,568 9	5,139,100	5 6229	2,523 5	3,633,900
4 6	0044303	23 392	225 72	8 1289	3,648 2	5,253,400	5 7479	2,579 6	3,714,700
4 7	0046158	24 371	216 65	8 3056	3,727 5	5,367,600	5 8729	2,635 7	3,795,400
4 8	0048047	25 369	208 13	8 4822	3,806 8	5,481,800	5 9978	2,691 8	3,876,100
4 9	0049971	26 385	200 12	8 6590	3,886 2	5,596,000	6 1228	2,747 9	3,956,900
5 0	0051928	27 418	192 58	8 8357	3,965 4	5,710,200	6 2477	2,803 9	4,037,700

5 5	0062455	32 976	160 11	9 7192	4,361 9	6,281,200	6 8724	3,084 3	4,441,400
6 0	0073881	39 009	135 35	10 603	4,758 5	6,852,200	7 4972	3,364 7	4 845,200
6 5	0086181	45 503	116 04	11 486	5,155 0	7,423,200	8 1219	3,645 1	5,248,900
7 0	0099341	52 452	100 66	12 370	5,551 6	7,994,300	8 7468	3,925 5	5 652,800
7 5	011334	59 843	88 231	13 253	5,948 1	8,565,200	9 3715	4 205 9	6,056,500
8 0	012816	67 667	78 028	14 137	6,344 7	9,136,300	9 9963	4,486 3	6,460,300
8 5	014393	75 994	69 478	15 021	6,741 2	9,707,300	10 621	4,766 7	6,864,000
9 0	016052	84 754	62 297	15 904	7,137 7	10,278,000	11 246	5,047 1	7,267,800
9 5	017791	93 938	56 207	16 788	7,534 3	10 849,000	11 871	5,327 5	7,671,500
10 0	019610	103 54	50 994	17 671	7,930 9	11,420,000	12 495	5,607 9	8,075,300
10 5	021563	113 85	46 376	18 555	8,327 4	11,991,000	13 120	5,888 3	8,479,100
11 0	023603	124 62	42 368	19 438	8,723 9	12,562,000	13 745	6,168 6	8,882,800
11 5	025729	135 85	38 867	20 322	9,120 5	13,133,000	14 370	6,449 1	9,286,700
12 0	027940	147 52	35 791	21 206	9,517 0	13,704,000	14 994	6,729 5	9,690,400
12 5	030236	159 64	33 073	22 089	9,913 6	14,275,000	15 619	7 009 9	10,094,000
13 0	032615	172 21	30 661	22 973	10,310	14,846,000	16 244	7,290 2	10,498,000
13 5	035078	185 21	28 508	23 856	10,707	15,417,000	16 869	7,570 6	10,902,000
14 0	037624	198 65	26 579	24 740	11,103	15,989,000	17 494	7,851 1	11,306,000
14 5	040294	212 75	24 818	25 624	11,500	16,560,000	18 118	8,131 5	11,709,000
15 0	043050	227 30	23 229	26 507	11,896	17,130,000	18 743	8,411 8	12,113,000

TABLE IV—(*Continued*)

d = 20 *inches* = 1 6667 *feet*

v	$s = \frac{h}{l}$	$s_m = 5\ 280 \frac{h}{l}$	$G = \frac{l}{h}$	Q			Q′		
				Cubic Feet per Second	Gallons per Minute	Gallons per Day	Cubic Feet per Second	Gallons per Minute	Gallons per Day
1	0000021903	011565	456,560	21817	97 913	140,990	15427	69 234	99,697
2	0000087014	045943	114,920	43633	195 83	281,990	30853	138 47	199,390
3	000019410	10248	51,519	65450	293 74	422,980	46280	207 70	299,090
4	000034268	18093	29,182	87267	391 65	563,980	61706	276 94	398,790
5	000053170	28074	18,807	1 0908	489 56	704,970	77133	346 17	498,480
6	000076162	40213	13,130	1 3090	587 48	845,960	92559	415 40	598,180
7	00010294	54350	9,714 7	1 5272	685 39	986,960	1 0799	484 64	697,880
-8	00013397	70734	7,464 5	1 7453	783 30	1,128,000	1 2341	553 87	797,570
-9	00016864	89043	5,929 7	1 9635	881 21	1,268,900	1 3884	623 10	897,260
1 0	00020709	1 0934	4,828 9	2 1817	979 13	1,409,900	1 5427	692 34	996,970
1 1	00024922	1 3159	4,012 5	2 3998	1,077 0	1,550,900	1 6969	761 57	1,096,700
1 2	00029552	1 5603	3,383 9	2 6180	1,175 0	1,691,900	1 8512	830 81	1,196,400
1 3	00034556	1 8245	2,893 9	2 8362	1,272 9	1,832,900	2 0054	900 04	1,296,000
1 4	00039858	2 1045	2,508 9	3 0544	1,370 8	1,973,900	2 1597	969 28	1,395,800
1 5	00045503	2 4025	2,197 7	3 2725	1,468 7	2,114,900	2 3140	1,038 5	1,495,400

1 6	00051486	2 7184	1,942 3	3 4907	1,566 6	2,255,900	2 4683	1,107 7	1,595,100
1 7	00057799	3 0518	1,730 1	3 7089	1,664 5	2,396,900	2 6225	1,177 0	1,694,800
1 8	00064557	3 4086	1,549 0	3 9270	1,762 4	2,537,900	2 7768	1,246 2	1,794,500
1 9	00071659	3 7836	1,395 5	4 1451	1,860 3	2,678,900	2 9310	1,315 4	1,894,200
2 0	00078955	4 1688	1,266 5	4 3633	1,958 3	2,819,900	3 0853	1,384 7	1,993,900
2 1	00086718	4 5787	1,153 2	4 5815	2,056 2	2,960,900	3 2396	1,453 9	2,093,600
2 2	00094811	5 0060	1,054 7	4 7997	2,154 1	3,101,800	3 3938	1,523 1	2,193,300
2 3	0010318	5 4480	969 15	5 0179	2,252 0	3,242,900	3 5481	1,592 4	2,293,000
2 4	0011187	5 9065	893 92	5 2360	2,349 9	3,383,800	3 7024	1,661 6	2,392,700
2 5	0012080	6 3782	827 81	5 4542	2,447 8	3,524,800	3 8566	1,730 9	2,492,400
2 6	0013015	6 8720	768 33	5 6723	2,545 7	3,665,800	4 0109	1,800 1	2,592,100
2 7	0013981	7 3819	715 25	5 8905	2,643 6	3,806,800	4 1651	1,869 3	2,691,800
2 8	0014985	7 9121	667 33	6 1087	2,741 6	3,947,800	4 3195	1,938 6	2,791,500
2 9	0016020	8 4584	624 22	6 3269	2,839 5	4,088,800	4 4737	2,007 8	2,891,200
3 0	0017093	9 0251	585 03	6 5450	2,937 4	4,229,800	4 6280	2,077 0	2,990,900
3 1	0018198	9 6084	549 52	6 7632	3,035 3	4,370,800	4 7822	2,146 2	3,090,600
3 2	0019343	10 213	516 98	6.9814	3,133 2	4,511,800	4 9365	2,215 5	3,190,300
3 3	0020510	10 829	487 57	7 1995	3,231 1	4,652,800	5 0907	2,284 7	3,290,000
3 4	0021718	11 467	460 45	7 4177	3,329 0	4,793,800	5 2451	2,354 0	3,389,700
3 5	0022946	12 115	435 81	7 6359	3,427 0	4,934,800	5 3993	2,423 2	3,489,400

TABLE IV—(*Continued*)

$d = 20$ *inches* $= 1\ 6667$ *feet*

v	$s = \frac{h}{l}$	$s_m = 5\ 280 \frac{h}{l}$	$G = \frac{l}{h}$	Q			Q'		
				Cubic Feet per Second	Gallons per Minute	Gallons per Day	Cubic Feet per Second	Gallons per Minute	Gallons per Day
3 6	0024239	12 798	412 56	7 8540	3,524 8	5,075,700	5 5535	2,492 4	3,589,100
3 7	0025566	13 499	391 15	8 0722	3,622 8	5,216,700	5 7078	2,561 7	3,688,800
3 8	0026913	14 210	371 57	8 2903	3,720 7	5,357,700	5 8621	2,630 9	3,788,400
3 9	0028305	14 945	353 30	8 5084	3,818 6	5,498,700	6 0163	2,700 1	3,888,100
4 0	0029731	15 698	336 35	8 7267	3,916 5	5,639,800	6 1706	2,769 4	3,987,900
4 1	0031188	16 467	320 63	8 9448	4,014 4	5,780,700	6 3248	2,838 6	4,087,500
4 2	0032680	17 255	306 00	9 1630	4,112 3	5,921,700	6 4792	2,907 8	4,187,300
4 3	0034185	18 050	292 52	9 3812	4,210 3	6,062,800	6 6335	2,977 1	4,287,000
4 4	0035740	18 870	279 80	9 5993	4 308 1	6,203,700	6 7877	3,046 3	4,386,600
4 5	0037326	19 708	267 91	9 8175	4,406 1	6,344,700	6 9419	3,115 5	4,486,300
4 6	0038945	20 563	256 77	10 036	4,504 0	6,485,700	7 0963	3,184 8	4,586,100
4 7	0040595	21 434	246 34	10 254	4,601 9	6,626,700	7 2505	3,254 0	4 685,800
4 8	0042253	22 310	236 67	10 472	4,699 8	6,767,700	7 4047	3,323 2	4,785,400
4 9	0043966	23 214	227 45	10 690	4,797 8	6,908,800	7 5591	3,392 5	4,885,200
5 0	0045709	24 134	218 78	10 908	4,895 6	7,049,700	7 7133	3,461 7	4,984,800

5 5	0054855	28 963	182 30	11 999	5,385 2	7,754,600	8 4846	3,807 9	5,483,300
6 0	0064746	34 185	154 45	13 090	5 847 8	8,459 600	9 2559	4,154 0	5,981,800
6 5	0075513	39 870	132 43	14 181	6,364 3	9,164,500	10 027	4,500 2	6,480,200
7 0	0087030	45 952	114 90	15 272	6,853 9	9,869,600	10 799	4,846 4	6,978,800
7 5	0099275	52 417	100 73	16 363	7,343 4	10,575,000	11 570	5,192 5	7,477,200
8 0	011224	59 261	89 096	17 453	7,833 0	11,280,000	12 341	5,538 7	7 975,700
8 5	012617	66 616	79 259	18 544	8,322 6	11,985,000	13 113	5,884 9	8,474,200
9 0	014084	74 364	71 002	19 635	8,812 1	12,689,000	13 884	6,231 0	8,972,600
9 5	015625	82 499	64 000	20 726	9,301 7	13,394,000	14 655	6,577 2	9,471,100
10 0	017239	91 019	58 010	21 817	9,791 3	14,099,000	15 427	6,923 4	9,969,700
10 5	018954	100 08	52 758	22 908	10,281	14,804,000	16 198	7,269 6	10,468,000
11 0	020746	109 54	48 203	23 998	10,770	15,509,000	16 969	7,615 7	10,967,000
11 5	022613	119 40	44 222	25 089	11,260	16,214,000	17 741	7,962 0	11,465,000
12 0	024555	129 65	40 725	26 180	11,750	16,919,000	18 512	8,308 1	11,964,000
12 5	026571	140 29	37 635	27 271	12,239	17,624,000	19 283	8,654 3	12,462,000
13 0	028660	151 32	34 892	28 362	12,729	18,329,000	20 054	9,000 4	12,960,000
13 5	030822	162 74	32 444	29 452	13,218	19,034,000	20 826	9,346 5	13,459,000
14 0	033057	174 54	30 251	30 544	13,708	19,739,000	21 597	9,692 8	13,958,000
14 5	035421	187 02	28 232	31 634	14,197	20,444,000	22 369	10,039	14,456,000
15 0	037863	199 92	26 411	32 725	14,687	21,149,000	23 140	10,385	14,954,000

TABLE IV—(*Continued*)

$d = 24$ *inches* $= 2$ *feet*

v	$s = \frac{h}{l}$	$s_m = 5\,280\,\frac{h}{l}$	$G = \frac{l}{h}$	Q			Q′		
				Cubic Feet per Second	Gallons per Minute	Gallons per Day	Cubic Feet per Second	Gallons per Minute	Gallons per Day
1	0000017288	0091281	578,430	31416	140 99	203,030	22214	99 697	143 560
2	0000068656	036250	145,650	62832	281 99	406,060	44428	199 39	287,120
3	000015336	080971	65,208	94248	422 98	609,090	66642	299 09	430,690
4	000027065	14290	36,949	1 2566	563 98	812,120	88857	398 79	574,250
5	000041977	22164	23,823	1 5708	704 97	1,015,100	1 1107	498 48	717,810
6	000060110	31738	16,636	1 8850	845 96	1,218,200	1 3328	598 18	861,370
7	000081210	42879	12,314	2 1991	986 96	1,421,200	1 5550	697 88	1 004,900
8	00010567	55793	9,463 5	2 5133	1,128 0	1,624,200	1 7771	797 57	1,148,500
9	00013298	70213	7,519 9	2 8274	1,268 9	1,827,300	1 9993	897 26	1,292,100
1 0	00016325	86193	6,125 8	3 1416	1,409 9	2,030,300	2 2214	996 97	1 435,600
1 1	00019639	1 0370	5,091 8	3 4557	1,550 9	2,233,300	2 4435	1,096 7	1,579,200
1 2	00023283	1 2293	4,295 0	3 7699	1,691 9	2,436,400	2 6657	1,196 4	1,722,700
1 3	00027168	1 4344	3,680 9	4 0840	1,832 9	2,639,400	2 8878	1,296 0	1,866,300
1 4	00031326	1 6540	3,192 3	4 3983	1 973 9	2,842,400	3 1100	1,395 8	2,009,900
1 5	00035820	1 8913	2,791 7	4 7124	2 114 9	3,045,400	3 3321	1,495 4	2,153,400

1 6	00040597	2 1435	2,463 3	5 0266	2,255 9	3,248,500	3 5543	1,595 1	2,297,000
1 7	00045650	2 4103	2,190 6	5 3407	2,396 9	3,451,500	3 7764	1,694 8	2,440,600
1 8	00050976	2 6915	1,961 7	5 6548	2,537 9	3,654,500	3 9985	1,794 5	2,584,100
1 9	00056461	2 9811	1,771 1	5 9690	2,678 9	3,857,500	4 2207	1,894 2	2,727,700
2 0	00062313	3 2901	1,604 8	6 2832	2,819 9	4,060,600	4 4428	1,993 9	2,871,200
2 1	00068495	3 6165	1,460 0	6 5974	2,960 9	4,263 600	4 6650	2,093 6	3,014,800
2 2	00074908	3 9551	1,335 0	6 9115	3,101 8	4,466,600	4 8871	2,193 3	3,158,300
2 3	00081587	4 3077	1,225 7	7 2257	3,242 9	4,669,700	5 1093	2,293 0	3,301,900
2 4	00088477	4 6715	1,130 2	7 5398	3,383 8	4,872,700	5 3314	2,392 7	3,445,500
2 5	00095616	5 0485	1,045 9	7 8540	3,524 8	5,075,700	5 5535	2,492 4	3,589,100
2 6	0010310	5 4436	969 93	8 1681	3,665 8	5,278,700	5 7756	2,592 1	3 732,600
2 7	0011079	5 8495	902 63	8 4822	3,806 8	5,481,800	5 9978	2,691 8	3,876,100
2 8	0011872	6 2684	842 31	8 7965	3,947 8	5,684,900	6 2200	2,791 5	4,019,800
2 9	0012683	6 6965	788 46	9 1107	4,088 8	5,887,900	6 4421	2,891 2	4,163,300
3 0	0013517	7 1367	739 83	9 4248	4,229 8	6,090,900	6 6642	2,990 9	4,306,900
3 1	0014403	7 6047	694 30	9 7389	4,370 8	6,293,900	6 8864	3,090 6	4,450,400
3 2	0015315	8 0865	652 94	10 053	4,511 8	6,497,000	7 1085	3,190 3	4 594,000
3 3	0016253	8 5816	615 26	10 367	4,652 8	6,699,900	7 3306	3,290 0	4,737,500
3 4	0017218	9 0910	580 79	10 681	4,793 8	6,903,000	7 5528	3,389 7	4,881,100
3 5	0018207	9 6135	549 22	10 996	4,934 8	7,106,100	7 7750	3,489 4	5,024,700

TABLE IV—(*Continued*)

$d = 24$ *inches* $= 2$ *feet*

v	$s = \frac{h}{l}$	$s_m = 5\,280\,\frac{h}{l}$	$G = \frac{l}{h}$	Q			Q′		
				Cubic Feet per Second	Gallons per Minute	Gallons per Day	Cubic Feet per Second	Gallons per Minute	Gallons per Day
3 6	0019222	10 149	520 24	11 310	5,075 7	7,309,000	7 9971	3,589 1	5,168,200
3 7	0020262	10 698	493 53	11 624	5,216 7	7,512,100	8 2192	3,688 8	5,311,800
3 8	0021327	11 261	468 89	11 938	5,357 7	7,715,100	8 4413	3,788 4	5,455,300
3 9	0022417	11 836	446 09	12 252	5,498 7	7,918,100	8 6634	3,888 1	5,598,900
4 0	0023532	12 425	424 95	12 566	5,639 8	8,121,200	8 8857	3,987 9	5,742,500
4 1	0024684	13 033	405 13	12 880	5,780 7	8,324,200	9 1077	4,087 5	5,886,000
4 2	0025862	13 655	386 67	13 195	5,921 7	8,527,300	9 3300	4,187 3	6,029,600
4 3	0027079	14 298	369 28	13 509	6,062 8	8,730,300	9 5521	4,287 0	6,173,200
4 4	0028308	14 947	353 26	13 823	6,203 7	8,933,300	9 7742	4,386 6	6,316,700
4 5	0029562	15 609	338 27	14 137	6,344 7	9,136,300	9 9963	4,486 3	6,460,300
4 6	0030842	16 284	324 24	14 451	6,485 7	9,339,400	10 219	4,586 1	6,603,900
4 7	0032146	16 973	311 08	14 766	6,626 7	9,542,500	10 441	4,685 8	6,747,500
4 8	0033492	17 684	298 58	15 080	6,767 7	9,745,400	10 663	4,785 4	6,891,000
4 9	0034847	18 399	286 97	15 394	6,908 8	9,948,600	10 885	4,885 2	7,034,600
5 0	0036225	19 127	276 05	15 708	7,049 7	10,151,000	11 107	4,984 8	7,178,100

5 5	0043549	22 994	229 63	17 279	7,754 6	11,167,000	12 218	5,483 3	7,895,900
6 0	0051492	27 188	194 20	18 850	8,459 6	12,182,000	13 328	5,981 8	8,613,700
6 5	0059971	31 664	166 75	20 420	9,164 5	13,197,000	14 439	6,480 2	9,331,500
7 0	0069021	36 443	144 88	21 991	9,869 6	14,212,000	15 550	6,978 8	10,049,000
7 5	0078970	41 696	126 63	23 562	10,575	15,227,000	16 661	7,477 2	10,767 000
8 0	0089551	47 282	111 67	25 133	11,280	16,242,000	17 771	7,975 7	11,485,000
8 5	010053	53 081	99 470	26 704	11,985	17,258,000	18 882	8,474 2	12,203,000
9 0	011208	59 177	89 224	28 274	12,689	18,273,000	19 993	8,972 6	12,921,000
9 5	012459	65 785	80 260	29 845	13,394	19,288,000	21 103	9,471 1	13 638,000
10 0	013775	72 729	72 597	31 416	14 099	20,303,000	22 214	9,969 7	14,356,000
10 5	015161	80 050	65 958	32 987	14,804	21,318,000	23 325	10,468	15 074,000
11 0	016611	87 704	60 202	34 557	15,509	22,333,000	24 435	10,967	15,792,000
11 5	018125	95 697	55 173	36 129	16,214	23,349,000	25 546	11,465	16 510 000
12 0	019701	104 02	50 759	37 699	16,919	24,364,000	26 657	11,964	17 227,000
12 5	021304	112 49	46 939	39 270	17,624	25,379,000	27 768	12,462	17,945,000
13 0	022964	121 25	43 547	40 840	18 329	26,394,000	28 878	12,960	18,663,000
13 5	024679	130 30	40 520	42 411	19 034	27,409,000	29 989	13,459	19 381,000
14 0	026450	139 66	37 807	43 983	19,739	28,424,000	31 100	13,958	20,099,000
14 5	028341	149 64	35 285	45 553	20,444	29,439,000	32 211	14,456	20,817,000
15 0	030294	159 95	33 010	47 124	21,149	30,454,000	33 321	14,954	21,534,000

TABLE IV—(*Continued*)

$d = 30$ *inches* $= 2\ 5$ *feet*

v	$s = \frac{h}{l}$	$s_m = 5\ 280 \frac{h}{l}$	$G = \frac{l}{h}$	Q			Q'		
				Cubic Feet per Second	Gallons per Minute	Gallons per Day	Cubic Feet per Second	Gallons per Minute	Gallons per Day
1	0000012910	0068166	774,570	49087	220 30	317,230	34710	155 78	224,320
2	0000051343	027109	194 770	98175	440 61	634,470	69419	311 55	448,630
3	000011485	060640	87,070	1 4726	660 91	951,700	1 0413	467 33	672,950
4	000020298	10717	49,265	1 9635	881 21	1,268,900	1 3884	623 10	897,260
5	000031530	16648	31,716	2 4544	1,101 5	1,586,200	1 7355	778 88	1,121,600
6	000045134	23830	22,156	2 9452	1,321 8	1,903,400	2 0826	934 65	1,345,900
7	000061067	32243	16,375	3 4361	1,542 1	2,220,600	2 4297	1,090 4	1,570,200
8	000079283	41861	12,613	3 9270	1 762 4	2,537,900	2 7768	1,246 2	1 794 500
9	000099738	52661	10,026	4 4178	1,982 7	2,855,100	3 1238	1,402 0	2,018,800
1 0	00012239	64620	8,170 7	4 9087	2,203 0	3,172,300	3 4710	1,557 8	2,243,200
1 1	00014749	77872	6,780 3	5 3996	2,423 3	3,489,600	3 8180	1,713 5	2,467,500
1 2	00017480	92295	5,720 7	5 8905	2,643 6	3,800 800	4 1651	1,869 3	2 691 800
1 3	00020431	1 0787	4,894 5	6 3813	2,863 9	4,124,000	4 5122	2,025 1	2,916,100
1 4	00023598	1 2460	4,237 6	6 8723	3,084 3	4,441,300	4 8594	2 180 9	3 140,400
1 5	00026977	1 4244	3,706 8	7 3631	3,304 5	4,758,500	5 2064	2 336 6	3,364,700

1 6	00030567	1 6139	3,271 5	7 8540	3,524 8	5,075,700	5 5535	2,492 4	3,589,100
1 7	00034292	1 8106	2,916 2	8 3449	3,745 2	5,393,000	5 9007	2,648 2	3,813,400
1 8	00038282	2 0213	2,612 2	8 8357	3,965 4	5,710,200	6 2477	2,803 9	4,037,700
1 9	00042475	2 2426	2,354 3	9 3265	4,185 7	6,027,400	6 5948	2,959 7	4,262,000
2 0	00046766	2 4692	2,138 3	9 8175	4,406 1	6,344,700	6 9419	3,115 5	4,486,300
2 1	00051340	2 7108	1,947 8	10 308	4,626 4	6,661 900	7 2890	3,271 3	4,710,600
2 2	00056105	2 9623	1,782 4	10 799	4,846 6	6,979,100	7 6361	3,427 0	4,934,900
2 3	00061059	3 2239	1,637 8	11 290	5,067 0	7,296,400	7 9833	3,582 9	5,159,300
2 4	00066196	3 4951	1,510 7	11 781	5,287 3	7,613,600	8 3303	3,738 6	5,383,600
2 5	00071517	3 7761	1,398 3	12 272	5,507 6	7,930,900	8 6774	3,894 4	5,607,900
2 6	00077099	4 0708	1,297 0	12 763	5,727 8	8,248,000	9 0244	4,050 1	5,832,200
2 7	00082827	4 3732	1,207 3	13 253	5,948 1	8,565,200	9 3715	4,205 9	6,056,500
2 8	00088785	4 6878	1,126 3	13 745	6,168 5	8,882,600	9 7187	4,361 7	6,280,900
2 9	00094875	5 0093	1,054 0	14 235	6,388 8	9,199,800	10 066	4,517 5	6 505,200
3 0	0010119	5 3429	988 21	14 726	6,609 1	9,517,000	10 413	4,673 3	6,729,500
3 1	0010775	5 6893	928 05	15 217	6,829 4	9,834,200	10 760	4,829 0	6,953,800
3 2	0011450	6 0455	873 37	15 708	7,049 7	10,151,000	11 107	4,984 8	7,178,100
3 3	0012149	6 4148	823 10	16 199	7,269 9	10,469 000	11 454	5,140 6	7,402,400
3 4	0012861	6 7906	777 54	16 690	7,490 3	10,786,000	11 801	5,296 4	7,626,800
3 5	0013591	7 1758	735 80	17 181	7,710 6	11,103,000	12 148	5,452 2	7,851,100

TABLE IV—(*Continued*)

d = 30 *inches* = 2 5 *feet*

v	$s = \frac{h}{l}$	$s_m = 5\,280\,\frac{h}{l}$	$G = \frac{l}{h}$	Q: Cubic Feet per Second	Q: Gallons per Minute	Q: Gallons per Day	Q′: Cubic Feet per Second	Q′: Gallons per Minute	Q′: Gallons per Day
3 6	0014346	7 5746	697 06	17 671	7,930 9	11,420,000	12 495	5,607 9	8,075,300
3 7	0015120	7 9833	661 38	18 162	8,151 2	11,738,000	12 843	5,763 7	8,299,700
3 8	0015912	8 4016	628 45	18 653	8,371 4	12,055,000	13 190	5 919 4	8,523,900
3 9	0016723	8 8296	597 98	19 144	8,591 7	12,372,000	13 537	6 075 2	8,748,200
4 0	0017552	9 2674	569 73	19 635	8,812 1	12,689,000	13 884	6,231 0	8,972,600
4 1	0018420	9 7255	542 90	20 126	9 032 3	13,006,000	14 231	6 386 8	9 196 900
4 2	0019307	10 194	517 94	20 617	9,252 7	13,324,000	14 578	6,542 6	9,421,300
4 3	0020215	10 673	494 69	21 108	9,473 1	13,641,000	14 925	6,698 4	9,645,600
4 4	0021141	11 162	473 01	21 598	9,693 3	13,958,000	15 272	6,854 1	9,869,800
4 5	0022088	11 662	452 73	22 089	9 913 6	14,275,000	15 619	7,009 9	10 094,000
4 6	0023055	12 173	433 75	22 580	10,134	14 593,000	15 967	7 165 7	10,319 000
4 7	0024041	12 693	415 96	23 071	10,354	14,910,000	16 314	7,321 5	10 543 000
4 8	0025046	13 224	399 27	23 562	10,575	15,227 000	16 661	7,477 2	10 767,000
4 9	0026071	13 765	383 57	24 053	10,795	15,545,000	17 008	7,633 1	10 992 000
5 0	0027114	14 316	368 81	24 544	11,015	15,862,000	17 355	7,788 8	11,216,000

5 5	0032582	17 203	306 92	26 998	12,117	17,448,000	19 090	8,567 6	12,337,000
6 0	0038507	20 332	259 69	29 452	13,218	19,034,000	20 826	9,346 5	13,459,000
6 5	0045034	23 778	222 05	31 907	14,320	20,620,000	22 561	10,125	14,580 000
7 0	0052048	27 481	192 13	34 361	15,421	22,206,000	24 297	10,904	15,702,000
7 5	0059398	31 362	168 36	36 815	16,523	23,792,000	26 032	11,683	16,824,000
8 0	0067183	35 472	148 85	39 270	17,624	25,379,000	27 768	12,462	17,945,000
8 5	0075485	39 856	132 48	41 724	18,726	26,965,000	29 503	13,241	19,067,000
9 0	0084223	44 469	118 73	44 178	19,827	28,551,000	31 238	14,020	20,188,000
9 5	0093502	49 369	106 95	46 633	20,929	30,137,000	32 974	14,799	21,310,000
10 0	010323	54 507	96 868	49 087	22,030	31,723,000	34 710	15,578	22,432,000
10 5	011361	59 985	88 022	51 542	23,132	33 310,000	36 445	16,356	23,553,000
11 0	012446	65 714	80 347	53 996	24,233	34,896,000	38 180	17,135	24,675,000
11 5	013579	71 695	73 644	56 451	25,335	36,482,000	39 916	17,914	25,796,000
12 0	014758	77 922	67 759	58 905	26,436	38 068,000	41 651	18,693	26,918,000
12 5	015985	84 398	62 560	61 359	27,538	39,654,000	43 387	19,472	28,039,000
13 0	017257	91 115	57 948	63 813	28,639	41,240,000	45 122	20,251	29,161,000
13 5	018576	98 080	53 833	66 267	29,741	42,826,000	46 858	21,030	30,282,000
14 0	019941	105 29	50 148	68 723	30,843	44,413,000	48 594	21,809	31,404,000
14 5	021352	112 74	46 834	71 177	31,944	45,999,000	50 329	22,588	32,526 000
15 0	0122808	120 42	43 845	73 631	33,045	47,585,000	52 064	23,366	33,647 000

TABLE IV—(*Continued*)

$d = 36$ *inches* $= 3$ *feet*

v	$s = \frac{h}{l}$	$s_m = 5\,280 \frac{h}{l}$	$G = \frac{l}{h}$	Q Cubic Feet per Second	Q Gallons per Minute	Q Gallons per Day	Q' Cubic Feet per Second	Q' Gallons per Minute	Q' Gallons per Day
1	0000010054	0053084	994,650	70685	317 23	456,810	49982	224 32	323,010
2	0000039967	021102	250,210	1 4137	634 47	913,630	99963	448 63	646,030
3	0000089366	047185	111,900	2 1206	951 70	1,370,400	1 4994	672 95	969,040
4	000015788	083359	63,340	2 8274	1,268 9	1,827,300	1 9993	897 26	1,292,100
5	000024513	12943	40,795	3 5343	1,586 2	2,284,100	2 4991	1,121 6	1,615,100
6	000035149	18558	28,451	4 2411	1,903 4	2,740,900	2 9989	1,345 9	1,938,100
7	000047639	25153	20,991	4 9480	2,220 6	3,197,700	3 4987	1,570 2	2,261,100
8	000061957	32713	16,140	5 6548	2,537 9	3,654,500	3 9985	1,794 5	2,584,100
9	000077909	41136	12,835	6 3617	2,855 1	4,111,300	4 4983	2,018 8	2,907,100
1 0	000095770	50566	10,442	7 0685	3,172 3	4,568,100	4 9982	2,243 2	3,230,100
1 1	00011513	60787	8,686 0	7 7754	3,489 6	5 024 900	5 4979	2,467 5	3,553,100
1 2	00013642	72028	7,330 4	8 4822	3,806 8	5 481,800	5 9978	2 691 8	3,876,100
1 3	00015940	84161	6,273 6	9 1890	4,124 0	5,938,500	6 4976	2,916 1	4,199,100
1 4	00018406	97181	5 433 1	9 8960	4,441 3	6 395,400	6 9975	3,140 4	4 522,200
1 5	00021035	1 1107	4,753 9	10 603	4,758 5	6,852,200	7 4972	3,364 7	4,845,200

1 6	00023880	1 2609	4,187 6	11 310	5,075 7	7,309,000	7 9971	3,589 1	5,168,200
1 7	00026839	1 4171	3,725 9	12 017	5 393 0	7,765,900	8 4969	3,813 4	5,491,200
1 8	00029954	1 5816	3,338 4	12 723	5,710 2	8 222,600	8 9966	4,037 7	5,814,200
1 9	00033225	1 7543	3,009 7	13 430	6,027 4	8,679,400	9 4964	4,262 0	6 137,200
2 0	00036650	1 9351	2,728 5	14 137	6,344 7	9,136,300	9 9963	4,486 3	6,460,300
2 1	00040224	2 1238	2,486 1	14 844	6,661 9	9,593,100	10 496	4,710 6	6,783,300
2 2	00043944	2 3202	2,275 6	15 551	6,979 1	10,050,000	10 996	4,934 9	7,106,200
2 3	00047812	2 5245	2 091 5	16 258	7,296 4	10,507,000	11 496	5,159 3	7,429,300
2 4	00051820	2 7361	1,929 7	16 964	7,613 6	10,964,000	11 996	5,383 6	7,752,300
2 5	00055969	2 9552	1,786 7	17 671	7,930 9	11,420,000	12 495	5,607 9	8,075,300
2 6	00060325	3 1852	1,657 7	18 378	8,248 0	11,877,000	12 995	5,832 2	8,398,300
2 7	00064790	3 4209	1,543 4	19 085	8,565 2	12,334,000	13 495	6,056 5	8,721,300
2 8	00069437	3 6662	1,440 2	19 792	8,882 6	12,791,000	13 995	6,280 9	9,044,400
2 9	00074181	3 9167	1,348 1	20 499	9,199 8	13 248,000	14 495	6,505 2	9,367,400
3 0	00079104	4 1767	1,264 2	21 206	9,517 0	13,704,000	14 994	6,729 5	9,690,400
3 1	00084266	4 4492	1,186 7	21 912	9,834 2	14,161,000	15 494	6,953 8	10,013,000
3 2	00089578	4 7297	1,116 3	22 619	10,151	14,618,000	15 994	7,178 1	10,336,000
3 3	00095036	5 0179	1,052 2	23 326	10,469	15,075,000	16 494	7,402 4	10,659,000
3 4	0010065	5 3141	993 57	24 033	10,786	15,532,000	16 994	7,626 8	10,982,000
3 5	0010640	5 6178	939 85	24 740	11,103	15,989,000	17 494	7,851 1	11,306,000

TABLE IV—(*Continued*)

$d = 36$ inches $= 3$ feet

v	$s = \frac{h}{l}$	$s_m = 5\,280\frac{h}{l}$	$G = \frac{l}{h}$	Q			Q′		
				Cubic Feet per Second	Gallons per Minute	Gallons per Day	Cubic Feet per Second	Gallons per Minute	Gallons per Day
3 6	0011236	5 9328	889 96	25 447	11,420	16,445 000	17 993	8,075 3	11 628,000
3 7	0011848	6 2558	844 01	26 154	11,738	16,902 000	18 493	8 299 7	11,951,000
3 8	0012467	6 5825	802 12	26 860	12,055	17,359 000	18 993	8,523 9	12,274,000
3 9	0013108	6 9210	762 89	27 567	12,372	17,816,000	19 493	8 748 2	12 597 000
4 0	0013764	7 2676	726 51	28 274	12,689	18 273,000	19 993	8,972 6	12,921,000
4 1	0014443	7 6261	692 36	28 981	13,006	18,729,000	20 492	9,196 9	13,243,000
4 2	0015139	7 9932	660 56	29 688	13,324	19,186,000	20 992	9,421 3	13,567,000
4 3	0015849	8 3684	630 94	30 395	13,641	19,643,000	21 492	9,645 6	13,890,000
4 4	0016574	8 7512	603 34	31 101	13,958	20,100,000	21 992	9,869 8	14,212,000
4 5	0017315	9 1424	577 52	31 808	14,275	20,557,000	22 492	10,094	14,536,000
4 6	0018072	9 5420	553 34	32 515	14,593	21,014,000	22 992	10,319	14,859,000
4 7	0018843	9 9492	530 69	33 222	14 910	21 470,000	23 491	10,543	15,182 000
4 8	0019630	10 364	509 44	33 929	15,227	21,927,000	23 991	10,767	15,505,000
4 9	0020431	10 788	489 44	34 636	15 545	22 384,000	24 491	10,992	15 828 000
5 0	0021248	11 219	470 64	35 343	15,862	22,841,000	24 991	11,216	16,151,000

5 5	0025553	13 492	391 35	38 877	17,448	25,125,000	27 490	12,337	17,766 000
6 0	0030224	15 958	330 86	42 411	19,034	27,409,000	29 989	13,459	19 381 000
6 5	0035296	18 636	283 32	45 945	20,620	29,693,000	32 488	14,580	20,996,000
7 0	0040731	21 506	245 51	49 480	22,206	31,977,000	34 987	15,702	22,611,000
7 5	0046583	24 596	214 67	53 014	23,792	34 261,000	37 486	16,824	24,226 000
8 0	0052802	27 879	189 39	56 548	25,379	36,545,000	39 985	17 945	25,841,000
8 5	0059384	31 355	168 40	60 083	26,965	38,829,000	42 484	19,067	27,456,000
9 0	0066324	35 019	150 78	63 617	28,551	41,113,000	44 983	20,188	29,071,000
9 5	0073616	38 869	135 84	67 151	30,137	43,397,000	47 482	21,310	30,686,000
10 0	0081261	42 905	123 06	70 685	31,723	45,681,000	49 982	22,432	32,301 000
10 5	0089417	47 212	111 84	74 220	33,310	47,966,000	52 481	23,553	33,916,000
11 0	0097947	51 715	102 10	77 754	34,896	50,249,000	54 979	24,675	35,531,000
11 5	010685	56 417	93 588	81 289	36,482	52,534,000	57 479	25,796	37,147,000
12 0	011612	61 310	86 119	84 822	38,068	54,818,000	59 978	26,918	38,761,000
12 5	012575	66 397	79 521	88 357	39,654	57,102,000	62 477	28,039	40,377,000
13 0	013575	71 675	73 665	91 890	41,240	59,385,000	64 976	29,161	41,991,000
13 5	014611	77 145	68 442	95 425	42,826	61,669,000	67 475	30,282	43,606,000
14 0	015683	82 808	63 762	98 960	44,413	63,954,000	69 975	31,404	45,222,000
14 5	016791	88 654	59 557	102 49	45,999	66,238,000	72 474	32,526	46,837,000
15 0	017934	94 689	55 761	106 03	47,585	68,522,000	74 972	33,647	48,452,000

z	$s = \frac{h}{l}$	$s_u = 5\,280\,\frac{h}{l}$	$G = \frac{l}{h}$	Q			Q′		
				Cubic Feet per Second	Gallons per Minute	Gallons per Day	Cubic Feet per Second	Gallons per Minute	Gallons per Day
1	00000078179	0041278	1,279,100	96212	431 80	621,780	68031	305 32	439,660
2	0000031058	016399	321,970	1 9242	863 59	1,243,600	1 3606	610 65	879,330
3	0000069402	036644	144,090	2 8864	1,295 4	1,865,300	2 0409	915 97	1,319,000
4	000012253	064695	81,613	3 8485	1,727 2	2,487 100	2 7213	1,221 3	1,758,700
5	000019012	10038	52,599	4 8106	2 159 0	3,108,900	3 4016	1,526 6	2,198,300
6	000027249	14387	36,699	5 7727	2,590 8	3,730 700	4 0819	1,831 9	2 638,000
7	000036916	19491	27,089	6 7349	3,022 6	4,352,500	4 7622	2,137 3	3,077 700
8	000048045	25368	20,814	7 6970	3,454 4	4,974,300	5 4425	2,442 6	3,517 300
9	000060590	31991	16,504	8 6590	3,886 2	5,596 000	6 1228	2,747 9	3 956,900
1 0	000074537	39355	13,416	9 6212	4,318 0	6,217,800	6 8031	3,053 2	4,396 600
1 1	000089867	47449	11,128	10 583	4,749 7	6,839,600	7 4834	3,358 5	4,836,300
1 2	00010656	56265	9,384 0	11 545	5,181 6	7,461 400	8 1638	3,663 9	5,275 900
1 3	00012461	65796	8 024 7	12 507	5,613 3	8,083,100	8 8440	3,969 2	5,715,600
1 4	00014418	76125	6,935 9	13 470	6,045 2	8,705,000	9 5245	4,274 5	6,155,300
1 5	00016491	87070	6,064 0	14 432	6,476 9	9 326,700	10 205	4,579 8	6 594,900

16	00018718	98828	5,342 6	15 394	6,908 8	9,948,600	10 885	4,885 2	7,034,600
17	00021079	1 1130	4,744 1	16 356	7,340 6	10,570,000	11 565	5,190 5	7,474,300
18	00023574	1 2447	4,242 0	17 318	7,772 3	11,192,000	12 246	5,495 8	7,913,900
19	00026202	1 3834	3,816 5	18 280	8,204 1	11,814,000	12 926	5,801 1	8,353,500
20	00028962	1 5292	3,452 8	19 242	8,635 9	12,436,000	13 606	6,106 5	8,793,300
21	00031852	1 6818	3,139 5	20 205	9,067 8	13,057,000	14 287	6,411 8	9,232,900
22	00034871	1 8412	2,867 7	21 167	9,499 5	13,679,000	14 967	6,717 1	9,672,500
23	00038021	2 0075	2,630 1	22 129	9,931 4	14,301,000	15 647	7,022 5	10,112,000
24	00041270	2 1790	2,423 1	23 091	10,363	14 923,000	16 328	7,327 7	10,552,000
25	00044643	2 3571	2,240 0	24 053	10,795	15,545,000	17 008	7,633 1	10,992,000
26	00048164	2 5430	2,076 3	25 015	11,227	16,166,000	17 688	7,938 3	11,431,000
27	00051811	2 7356	1,930 1	25 977	11,658	16,788,000	18 368	8,243 7	11,871,000
28	00055616	2 9365	1,798 0	26 940	12,090	17,410,000	19 049	8,549 1	12,311,000
29	00059548	3 1441	1,679 3	27 902	12,522	18,032,000	19 729	8,854 4	12,750,000
30	00063644	3 3604	1,571 2	28 864	12,954	18,653,000	20 409	9,159 7	13,190,000
31	00067830	3 5814	1,474 3	29 826	13,386	19,275,000	21 090	9,465 0	13,630,000
32	00072141	3 8090	1,386 2	30 788	13,818	19,897,000	21 770	9,770 3	14,069,000
33	00076526	4 0405	1,306 7	31 750	14,249	20,519,000	22 450	10,076	14,509,000
34	00081081	4 2810	1,233 3	32 712	14,681	21,141,000	23 131	10,381	14,949,000
35	00085759	4 5280	1,166 1	33 674	15,113	21,763,000	23 811	10,686	15,388,000

TABLE IV—(*Continued*)

$d = 42$ *inches* $= 3 5$ *feet*

v	$s = \frac{h}{l}$	$s_m = 5\,280\,\frac{h}{l}$	$G = \frac{l}{h}$	Q: Cubic Feet per Second	Q: Gallons per Minute	Q: Gallons per Day	Q′: Cubic Feet per Second	Q′: Gallons per Minute	Q′: Gallons per Day
3 6	00090611	4 7842	1,103 6	34 636	15,545	22,384,000	24 491	10,992	15,828,000
3 7	00095594	5 0437	1,046 1	35 598	15,976	23,006,000	25 172	11,297	16,267,000
3 8	0010070	5 3170	993 02	36 560	16,408	23,628,000	25 852	11,602	16,707,000
3 9	0010594	5 5935	943 95	37 522	16,840	24,249,000	26 532	11,907	17,147,000
4 0	0011130	5 8765	898 48	38 485	17,272	24,871,000	27 213	12,213	17,587,000
4 1	0011686	6 1699	855 76	39 447	17,704	25,493,000	27 893	12,518	18,026,000
4 2	0012247	6 4665	816 51	40 409	18,136	26,115,000	28 573	12,823	18,466,000
4 3	0012829	6 7738	779 47	41 371	18,567	26,737,000	29 254	13,129	18,906,000
4 4	0013415	7 0832	745 42	42 333	18,999	27,358,000	29 934	13,434	19,345,000
4 5	0014023	7 4042	713 10	43 295	19,431	27,980,000	30 614	13,739	19,785,000
4 6	0014644	7 7321	682 86	44 258	19,863	28,602,000	31 295	14,045	20,225 000
4 7	0015268	8 0616	654 95	45 220	20,295	29,224,000	31 975	14,350	20,664,000
4 8	0015914	8 4027	628 36	46 182	20,726	29,846,000	32 655	14,655	21,104,000
4 9	0016563	8 7454	603 74	47 144	21,158	30,468,000	33 336	14,961	21,544,000
5 0	0017235	9 1000	580 22	48 106	21,590	31 080,000	34 016	15,266	21,983,000

5 5	0020747	10 954	482 00	52 916	23,749	34,198,000	37 417	16,793	24,181,000
6 0	0024562	12 969	407 13	57 727	25,908	37,307,000	40 819	18,319	26,380,000
6 5	0028695	15 151	348 49	62 537	28,067	40,416,000	44 220	19,846	28,578,000
7 0	0033128	17 401	301 86	67 349	30,226	43,525,000	47 622	21,373	30,777,000
7 5	0037879	20 000	264 00	72 159	32,385	46,634,000	51 023	22 899	32 975,000
8 0	0042928	22 666	232 95	76 970	34,544	49,743,000	54 425	24,426	35,173,000
8 5	0048305	25 536	206 76	81 781	36,703	52,852,000	57 827	25,953	37,371,000
9 0	0054114	28 572	184 79	86 590	38,862	55,960,000	61 228	27,479	39,569,000
9 5	0060092	31 729	166 41	91 401	41,020	59,069,000	64 629	29,005	41,768,000
10 0	0066364	35 040	150 69	96 212	43,180	62,178,000	68 031	30,532	43,966,000
10 5	0073020	38 554	136 95	101 02	45,339	65,287,000	71 433	32,059	46,165,000
11 0	0079976	42 227	125 04	105 83	47,497	68,396,000	74 834	33,585	48,363,000
11 5	0087239	46 062	114 63	110 64	49,657	71 506,000	78 237	35,112	50,562,000
12 0	0094796	50 052	105 49	115 45	51,816	74,614,000	81 638	36,639	52,759,000
12 5	010265	54 200	97 416	120 27	53,975	77,723,000	85 039	38,165	54,958,000
13 0	011088	58 542	90 190	125 07	56,133	80,831,000	88 440	39,692	57,156,000
13 5	011933	63 004	83 803	129 89	58,292	83,940,000	91 842	41,218	59,354,000
14 0	012816	67 667	78 028	134 70	60,452	87,050,000	95 245	42,745	61,553,000
14 5	013720	72 439	72 889	139 51	62,611	90,159,000	98 646	44,272	63,751,000
15 0	014652	77 361	68 251	144 32	64,769	93,267,000	102 05	45,798	65,949,000

TABLE IV—(*Continued*)

$d = 48$ *inches* $= 4$ *feet*

v	$s = \frac{h}{l}$	$s_m = 5\,280\,\frac{h}{l}$	$G = \frac{l}{h}$	Q Cubic Feet per Second	Q Gallons per Minute	Q Gallons per Day	Q' Cubic Feet per Second	Q' Gallons per Minute	Q' Gallons per Day
1	0000000062810	0033163	1,592,100	1 2566	563 98	812,120	88857	398 79	574,250
2	0000025062	013233	399,010	2 5133	1,128 0	1,624,200	1 7771	797 57	1,148,500
3	0000056250	029700	177,780	3 7699	1,691 9	2 436,400	2 6657	1,196 4	1,722,700
4	0000099749	052667	100,250	5 0266	2,255 9	3,248,500	3 5543	1,595 1	2,297,000
5	000015547	082088	64,321	6 2832	2,819 9	4,060,600	4 4428	1,993 9	2,871,200
6	000022332	11791	44,780	7 5398	3,383 8	4,872,700	5 3314	2,392 7	3,445,500
7	000030320	16009	32,981	8 7965	3,947 8	5,684,900	6 2200	2,791 5	4,019 800
8	000039502	20857	25,315	10 053	4,501 8	6,497,000	7 1085	3,190 3	4 594 000
9	000049869	26331	20,053	11 310	5,075 7	7,309,000	7 9971	3,589 1	5 168,200
1 0	000061412	32425	16,284	12 566	5,639 8	8,121,200	8 8857	3,987 9	5 742 500
1 1	000074119	39135	13,402	13 823	6,203 7	8,933,300	9 7742	4,386 6	6 3[illegible] 6 700
1 2	000087983	46455	11,366	15 080	6 767 7	9,745,400	10 663	4,785 4	6 891 000
1 3	00010300	54381	9,709 1	16 336	7,331 6	10,557,000	11 551	5,184 2	7,465,200
1 4	00011915	62910	8,392 9	17 593	7 895 7	11,370,000	12 440	5,583 0	8 039,500
1 5	00013642	72031	7 330 1	18 850	8,459 6	12 182 000	13 328	5,981 8	8,613,700

1 6	00015443	81536	6,475 6	20 106	9,023 6	12,994,000	14 217	6,380 6	9,188,000
1 7	00017388	91810	5,751 0	21 363	9,587 6	13,806,000	15 106	6,779 4	9,762,300
1 8	00019444	1 0266	5,143 0	22 619	10,151	14,618,000	15 994	7,178 1	10,336,000
1 9	00021608	1 1409	4,628 0	23 876	10 715	15,430,000	16 883	7,576 9	10,911,000
2 0	00023880	1 2609	4,187 6	25 133	11,280	16,242,000	17 771	7,975 7	11,485,000
2 1	00026277	1 3874	3,805 7	26 389	11,843	17,055,000	18 660	8,374 5	12,059,000
2 2	00028782	1 5197	3,474 4	27 646	12,407	17,867,000	19 548	8,773 2	12,633,000
2 3	00031376	1 6566	3,187 1	28 903	12,971	18,679,000	20 437	9,172 1	13,208,000
2 4	00034097	1 8003	2,932 9	30 159	13,535	19,491,000	21 326	9,570 8	13,782,000
2 5	00036924	1 9496	2,708 3	31 416	14,099	20,303,000	22 214	9,969 7	14,356,000
2 6	00039858	2 1045	2,508 9	32 672	14,663	21,115,000	23 103	10,368	14,930,000
2 7	00042898	2 2650	2,331 1	33 929	15,227	21,927,000	23 991	10,767	15,505,000
2 8	00046014	2 4295	2 173 3	35 186	15,791	22,739,000	24 880	11,166	16 079,000
2 9	00049261	2 6009	2,030 0	36 443	16,355	23,552,000	25 769	11,565	16,653,000
3 0	00052611	2 7779	1,900 7	37 699	16,919	24,364,000	26 657	11,964	17,227,000
3 1	00056102	2 9622	1,782 5	38 956	17,483	25,176,000	27 545	12 362	17,802,000
3 2	00059661	3 1501	1,676 1	40 212	18,047	25,988,000	28 434	12,761	18,376,000
3 3	00063362	3 3455	1,578 2	41 469	18,611	26,800,000	29 322	13,160	18 950,000
3 4	00067127	3 5443	1,489 7	42 726	19,175	27,612,000	30 211	13,559	19 525,000
3 5	00071040	3 7509	1,407 7	43 983	19 739	28 424,000	31 100	13,958	20,099,000

TABLE IV—(*Continued*)

d = *48 inches* = *4 feet*

v	$s = \frac{h}{l}$	$s_m = 5\ 280 \frac{h}{l}$	$G = \frac{l}{h}$	Q			Q′		
				Cubic Feet per Second	Gallons per Minute	Gallons per Day	Cubic Feet per Second	Gallons per Minute	Gallons per Day
3 6	00075055	3 9629	1,332 4	45 239	20,303	29,236,000	31 988	14,356	20,673,000
3 7	00079123	4 1776	1,263 9	46 495	20,867	30,048 000	32 877	14,755	21,247,000
3 8	00083345	4 4006	1,199 8	47 752	21,431	30,860,000	33 765	15,154	21,821,000
3 9	00087611	4 6258	1,141 4	49 008	21,995	31,672,000	34 654	15,552	22,395,000
4 0	00092039	4 8596	1,086 5	50 266	22,559	32,485,000	35 543	15,951	22,970,000
4 1	00096565	5 0986	1,035 6	51 522	23,123	33,297,000	36 431	16,350	23,544,000
4 2	0010127	5 3469	987 48	52 779	23,687	34,109 000	37 320	16,749	24,118,000
4 3	0010600	5 5969	943 37	54 036	24,251	34,921,000	38 209	17,148	24,693,000
4 4	0011091	5 8563	901 59	55 292	24,815	35,733,000	39 097	17,546	25,267,000
4 5	0011586	6 1172	863 14	56 548	25 379	36,545,000	39 985	17,945	25 841,000
4 6	0012090	6 3835	827 12	57 806	25,943	37,358,000	40 874	18,344	26 416,000
4 7	0012613	6 6595	792 85	59 062	26,507	38,170,000	41 763	18,743	26 990,000
4 8	0013137	6 9363	761 20	60 318	27,071	38,982,000	42 651	19,142	27,564,000
4 9	0013681	7 2235	730 94	61 576	27,635	39,794,000	43 540	19,541	28,138,000
5 0	0014226	7 5110	702 96	62 832	28,199	40,606,000	44 428	19,939	28,712,000

5 5	0017142	9 0511	583 35	69 115	31,018	44,666,000	48 871	21,933	31,583,000
6 0	0020317	10 727	492 20	75 398	33,838	48,727,000	53 314	23,927	34,455,000
6 5	0023778	12 555	420 55	81 681	36,658	52,787,000	57 756	25,921	37,325,000
7 0	0027502	14 521	363 61	87 965	39,478	56,849,000	62 200	27,915	40,198,000
7 5	0031439	16 600	318 08	94 248	42,298	60,909,000	66 642	29,909	43,069,000
8 0	0035621	18 808	280 73	100 53	45,118	64,970,000	71 085	31,903	45,940,000
8 5	0040045	21 144	249 72	106 81	47,938	69,030,000	75 528	33,897	48,811,000
9 0	0044705	23 604	223 69	113 10	50,757	73,090,000	79 971	35,891	51,682,000
9 5	0049670	26 225	201 33	119 38	53,577	77,151,000	84 413	37,884	54,553,000
10 0	0054881	28 977	182 21	125 66	56,398	81,212,000	88 857	39,879	57,425,000
10 5	0060421	31 902	165 50	131 95	59,217	85,273,000	93 300	41,873	60,296,000
11 0	0066217	34 962	151 02	138 23	62,037	89,333,000	97 742	43,866	63,167,000
11 5	0072274	38 160	138 36	144 51	64,857	93,394,000	102 19	45,861	66,039,000
12 0	0078581	41 491	127 26	150 80	67,677	97,454,000	106 63	47,854	68,910,000
12 5	0085206	44 988	117 36	157 08	70,497	101,510,000	111 07	49,848	71,781,000
13 0	0092092	48 624	108 59	163 36	73,316	105,570,000	115 51	51,842	74,652,000
13 5	0099241	52 399	100 77	169 64	76 136	109,640 000	119 96	53,836	77,523,000
14 0	010665	56 313	93 761	175 93	78,957	113,700,000	124 40	55,830	80,395,000
14 5	011416	60 278	87 593	182 21	81,777	117,760,000	128 84	57,824	83,266,000
15 0	012191	64 367	82 029	188 50	84,596	121,820,000	133 28	59,818	86,137,000

TABLE IV—(*Continued*)

$d = 54$ *inches* $= 4\ 5$ *feet*

v	$s = \frac{h}{l}$	$s_m = 5\ 280\ \frac{h}{l}$	$G = \frac{l}{h}$	Q: Cubic Feet per Second	Q: Gallons per Minute	Q: Gallons per Day	Q′: Cubic Feet per Second	Q′: Gallons per Minute	Q′: Gallons per Day
1	00000052791	0027873	1,894,300	1 5904	713 77	1,027,800	1 1246	504 71	726,780
2	0000021061	011120	474,810	3 1808	1,427 5	2,055,700	2 2492	1,009 4	1,453,600
3	0000047263	024955	211,580	4 7712	2,141 3	3,083,500	3 3737	1,514 1	2,180,300
4	0000083803	044248	119,330	6 3617	2,855 1	4,111,300	4 4983	2,018 8	2,907,100
5	000013060	068954	76,572	7 9521	3,568 9	5 139,100	5 6229	2 523 5	3,633,900
6	000018756	099031	53,316	9 5425	4,282 6	6,166,900	6 7475	3,028 2	4,360,600
7	000025462	13444	39,274	11 133	4,996 4	7,194,800	7 8721	3,533 0	5,087,400
8	000033167	.17512	30,150	12 723	5,710 2	8,222,600	8 9966	4,037 7	5,814,200
9	000041865	22104	23,886	14 314	6,423 9	9,250 400	10 121	4,542 3	6 540,900
1 0	000051548	27217	19,399	15 904	7,137 7	10,278,000	11 246	5,047 1	7,267,800
1 1	000062204	32844	16,076	17 494	7,851 5	11,306,000	12 370	5,551 8	7,994,500
1 2	000073830	38982	13,545	19 085	8,565 2	12 334,000	13 495	6,056 5	8,721,300
1 3	000086413	45626	11,572	20 675	9,279 0	13,362,000	14 619	6,561 2	9,448,000
1 4	000099952	52774	10,005	22 266	9 992 9	14 390,000	15 744	7,065 9	10,175,000
1 5	00011443	60417	8,739 2	23 856	10,707	15,417,000	16 869	7 570 0	10,902,000

1 6	00013019	68742	7,680 9	25 447	11,420	16,445,000	17 993	8,075 3	11,628,000
1 7	00014658	77393	6,822 3	27 037	12,134	17,473,000	19 118	8,580 1	12,355,000
1 8	00016388	86527	6,102 1	28 627	12,848	18,501,000	20 242	9,084 7	13,082,000
1 9	00018209	96144	5,491 7	30 218	13,562	19,529,000	21 367	9,589 4	13,809,000
2 0	00020121	1 0624	4,969 8	31 808	14,275	20,557,000	22 492	10,094 0	14,536,000
2 1	00022139	1 1689	4,517 0	33 399	14,989	21,584,000	23 616	10,599	15,262,000
2 2	00024247	1 2802	4,124 3	34 989	15,703	22,612,000	24 741	11,104	15,989,000
2 3	00026428	1 3954	3,783 8	36 580	16,417	23,640,000	25 865	11,608	16,716,000
2 4	00028716	1 5162	3,482 3	38 170	17,130	24,668,000	26 990	12,113	17,443,000
2 5	00031094	1 6418	3,216 0	39 760	17,844	25,696,000	28 114	12,618	18,169,000
2 6	00033561	1 7720	2,979 6	41 350	18,558	26,723,000	29 239	13,122	18,896,000
2 7	00036117	1 9070	2,768 8	42 941	19,272	27,751,000	30 363	13,627	19,623,000
2 8	00038761	2 0466	2,579 9	44 532	19,986	28,779,000	31 488	14,132	20,350,000
2 9	00041551	2 1939	2,406 7	46 122	20,700	29,807,000	32 613	14,637	21,077,000
3 0	00044403	2 3444	2,252 1	47 712	21,413	30,835,000	33 737	15,141	21,803,000
3 1	00047312	2 4980	2,113 6	49 303	22,127	31,863,000	34 862	15,646	22,530,000
3 2	00050308	2 6563	1,987 7	50 893	22,841	32,890,000	35 987	16,151	23,257,000
3 3	00053351	2 8169	1,874 4	52 483	23,554	33,918,000	37 111	16,655	23,983,000
3 4	00056515	2 9839	1,769 5	54 074	24,268	34,946,000	38 236	17,160	24,710,000
3 5	00059760	3 1553	1,673 4	55 665	24,982	35,974,000	39 360	17,665	25,437 000

TABLE IV—(*Continued*)

$d = 54$ *inches* $= 4\ 5$ *feet*

v	$s = \frac{h}{l}$	$s_m = 5\ 280\ \frac{h}{l}$	$G = \frac{l}{h}$	Q			Q'		
				Cubic Feet per Second	Gallons per Minute	Gallons per Day	Cubic Feet per Second	Gallons per Minute	Gallons per Day
3 6	00063134	3 3334	1,583 9	57 255	25,696	37,002,000	40 485	18,169	26,164,000
3 7	00066547	3 5137	1,502 7	58 845	26,409	38,029,000	41 609	18,674	26,891,000
3 8	00070094	3 7009	1,426 7	60 435	27,123	39,057,000	42 734	19,179	27,617,000
3 9	00073673	3 8899	1,357 3	62 025	27,837	40,085,000	43 858	19,683	28,344,000
4 0	00077391	4 0862	1,292 1	63 617	28,551	41,113,000	44 983	20,188	29,071,000
4 1	c0081191	4 2869	1 231 7	65 206	29,264	42,141,000	46 107	20,693	29,798,000
4 2	00085080	4 4922	1,175 4	66 797	29,979	43,159,000	47 232	21,198	30,525,000
4 3	00089051	4 7019	1,122 9	68 388	30,692	44,197,000	48 357	21,703	31,251,000
4 4	00093107	4 9160	1,074 0	69 978	31,406	45,224,000	49 481	22,207	31,978,000
4 5	00097246	5 1345	1,028 3	71 568	32,120	46,252,000	50 606	22,712	32,705,000
4 6	0010147	5 3577	985 49	73 159	32,834	47,280,000	51 731	23,217	33,432,000
4 7	0010578	5 5851	945 37	74 750	33,547	48,308,000	52 855	23,721	34,159,000
4 8	0011017	5 8169	907 70	76 340	34,261	49,336,000	53 980	24,226	34,885,000
4 9	0011464	6 0531	872 27	77 931	34,975	50,364,000	55 105	24,731	35,612,000
5 0	0011920	6 2935	838 96	79 521	35,689	51,391,000	56 229	25,235	36,339,000

5 5	0014318	7 5598	698 43	87 472	39,257	56,530,000	61 851	27,759	39,972,000
6 0	0016915	8 9312	591 18	95 425	42,826	61,669,000	67 475	30,282	43,606,000
6 5	0019793	10 451	505 22	103 38	46,395	66,808,000	73 097	32,806	47,240,000
7 0	0022889	12 085	436 90	111 33	49,964	71,948,000	78 721	35,330	50,874,000
7 5	0026158	13 811	382 29	119 28	53,533	77,087,000	84 343	37,853	54,508,000
8 0	0029629	15 644	337 50	127 23	57,102	82,226,000	89 966	40,377	58,142,000
8 5	0033300	17 582	300 30	135 19	60,671	87,366,000	95 589	42 900	61,776,000
9 0	0037164	19 622	269 08	143 14	64,239	92,504,000	101 21	45,423	65,409,000
9 5	0041345	21 830	241 87	151 09	67,808	97,643,000	106 83	47,947	69,043,000
10 0	0045744	24 152	218 61	159 04	71,377	102,780,000	112 46	50,471	72,678,000
10 5	0050394	26 608	198 44	166 99	74,946	107,920,000	118 08	52,994	76,312,000
11 0	0055265	29 180	180 95	174 94	78,515	113,060,000	123 70	55,518	79,945,000
11 5	0060359	31 869	165 68	182 90	82,084	118,200,000	129 33	58,042	83,580,000
12 0	0065670	34 674	152 28	190 85	85,652	123,340,000	134 95	60,565	87,213,000
12 5	0071203	37 595	140 44	198 80	89,222	128,480,000	140 57	63,088	90,847,000
13 0	0076956	40 632	129 95	206 75	92,790	133,620,000	146 19	65,612	94,480,000
13 5	0082926	43 784	120 59	214 70	96,358	138,760,000	151 82	68,135	98,114,000
14 0	0089117	47 053	112 21	222 66	99,929	143,900,000	157 44	70,659	101,750,000
14 5	0095523	50 436	104 69	230 61	103,500	149,040,000	163 06	73,183	105,380,000
15 0	010215	53 932	97 899	238 56	107,070	154,170,000	168 69	75,706	109,020,000

TABLE IV—(*Continued*)

$a = 60$ *inches* $= 5$ *feet*

v	$s = \frac{h}{l}$	$s_m = 5\,280\,\frac{h}{l}$	$G = \frac{l}{h}$	Q Cubic Feet per Second	Q Gallons per Minute	Q Gallons per Day	Q' Cubic Feet per Second	Q' Gallons per Minute	Q' Gallons per Day
1	00000045025	0023773	2,221,000	1 9635	881 21	1,268,900	1 3884	623 10	897,260
2	0000017960	0094829	556,790	3 9270	1 762 4	2,537,900	2 7768	1,246 2	1 794,500
3	0000040298	021277	248,150	5 8905	2,643 6	3,806,800	4 1651	1 869 3	2,691,800
4	0000071441	037721	139,970	7 8540	3,524 8	5,075,700	5 5535	2 492 4	3,589,100
5	000011132	058775	89,834	9 8175	4,406 1	6,344,700	6 9419	3,115 5	4,486,300
6	000015985	084400	62,559	11 781	5,287 3	7,613,600	8 3303	3 738 6	5,383,600
7	000021697	11456	46,090	13 745	6,168 5	8,882,600	9 7187	4,361 7	6,280,900
8	000028259	14920	35,388	15 708	7,049 7	10,151,000	11 107	4,984 8	7,178,100
9	000035663	18830	28,040	17 671	7,930 9	11,420,000	12 495	5,607 9	8,075,300
1 0	000043905	23181	22,777	19 635	8,812 1	12,689,000	13 884	6,231 0	8,972,600
1 1	000052974	27970	18,877	21 598	9,693 3	13,958,000	15 272	6,854 1	9,869,800
1 2	000062865	33193	15 907	23 562	10,575	15 227,000	16 661	7,477 2	10,767,000
1 3	000073568	38844	13,593	25 525	11,456	16,496,000	18 049	8,100 3	11,664,000
1 4	000085324	45051	11,720	27 489	12,337	17,765,000	19 437	8,723 5	12 562,000
1 5	000097667	51568	10,239	29 452	13,218	19,034,000	20 826	9,346 5	13,459,000

1 6	00011081	58505	9,024 9	31 416	14,099	20,303,000	22 214	9,969 7	14,356 000
1 7	00012473	65857	8,017 3	33 380	14,981	21,572,000	23 603	10,593	15 254,000
1 8	00013983	73831	7,151 4	35 343	15,862	22,841,000	24 991	11,216	16,151,000
1 9	00015535	82026	6,436 9	37 306	16,743	24,110,000	26 379	11,839	17 048 000
2 0	00017214	90889	5,809 2	39 270	17,624	25,379,000	27 768	12,462	17,945,000
2 1	00018937	99986	5,280 7	41 233	18,505	26,648,000	29 156	13,085	18,843,000
2 2	00020738	1 0950	4,822 0	43 197	19,387	27,916,000	30 544	13,708	19,740,000
2 3	00022601	1 1973	4,424 6	45 161	20,268	29,186,000	31 933	14,331	20,637,000
2 4	00024555	1 2965	4,072 5	47 124	21,149	30,454 000	33 321	14,954	21,534,000
2 5	00026586	1 4037	3,761 4	49 087	22,030	31,723,000	34 710	15,578	22,432,000
2 6	00028712	1 5160	3,482 8	51 051	22,911	32,992,000	36 098	16,201	23,329,000
2 7	00030941	1 6337	3,232 0	53 014	23,792	34,261,000	37 486	16,824	24,226,000
2 8	00033203	1 7531	3,011 8	54 978	24,674	35,530 000	38 875	17,447	25,123,000
2 9	00035539	1 8764	2,813 8	56 942	25,555	36,799,000	40 263	18,070	26,021,000
3 0	00037947	2 0036	2,635 2	58 905	26,436	38,068,000	41 651	18,693	26,918,000
3 1	00040459	2 1362	2,471 6	60 868	27,317	39,337,000	43 040	19,316	27,815,000
3 2	00043080	2 2746	2,321 2	62 832	28,199	40,606,000	44 428	19,939	28,712,000
3 3	00045747	2 4154	2,185 9	64 795	29,080	41,875,000	45 816	20,562	29,609,000
3 4	00048526	2 5621	2,060 8	66 759	29,961	43,144,000	47 205	21,186	30,507,000
3 5	00051346	2 7111	1,947 6	68 723	30,843	44,413,000	48 594	21,809	31,404,000

TABLE IV—(*Continued*)

d = 60 inches = 5 feet

v	$s = \frac{h}{l}$	$s_m = 5{,}280 \frac{h}{l}$	$G = \frac{l}{h}$	Q: Cubic Feet per Second	Q: Gallons per Minute	Q: Gallons per Day	Q′: Cubic Feet per Second	Q′: Gallons per Minute	Q′: Gallons per Day
3 6	00054200	2 8617	1,845 0	70 685	31,723	45,681,000	49 982	22,432	32,301,000
3 7	00057126	3 0162	1,750 5	72 649	32,605	46,950,000	51 370	23,055	33,199,000
3 8	00060076	3 1720	1,664 6	74 612	33,486	48,219,000	52 758	23,678	34 096,000
3 9	00063136	3 3336	1,583 9	76 576	34,367	49,488,000	54 146	24,301	34,993,000
4 0	00066267	3 4989	1,509 0	78 540	35,248	50,757,000	55 535	24,924	35,891,000
4 1	00069517	3 6705	1,438 5	80 503	36,129	52,026,000	56 923	25,547	36,787,000
4 2	00072895	3 8488	1,371 8	82 467	37,011	53,295,000	58 312	26,170	37,685,000
4 3	00076294	4 0283	1,310 7	84 431	37,892	54,564,000	59 701	26,794	38,582,000
4 4	00079822	4 2145	1,252 8	86 393	38,773	55,833,000	61 089	27,416	39,479,000
4 5	00083366	4 4017	1,199 5	88 357	39,654	57,102,000	62 477	28,039	40,377,000
4 6	00086982	4 5920	1,149 7	90 321	40,536	58,371,000	63 866	28,663	41,274,000
4 7	00090730	4 7908	1,102 1	92 285	41,417	59,640,000	65 254	29,286	42,172,000
4 8	00094493	4 9892	1,058 3	94 248	42,298	60,909 000	66 642	29,909	43,069,000
4 9	00098401	5 1955	1,016 2	96 212	43,180	62 178,000	68 031	30,532	43 966,000
5 0	0010230	5 4014	977 51	98 175	44,061	63,447,000	69 419	31,155	44,863,000

5 5	0012284	6 4859	814 07	107 99	48,466	69,791,000	76 361	34,270	49,349,000
6 0	0014507	7 6598	689 30	117 81	52,873	76,136,000	83 303	37,386	53,836,000
6 5	0016973	8 9617	589 17	127 63	57,278	82,480,000	90 244	40,501	58,322,000
7 0	0019625	10 362	509 57	137 45	61,685	88,826,000	97 187	43,617	62,809,000
7 5	0022458	11 858	445 28	147 26	66,091	95,170,000	104 13	46,733	67,295,000
8 0	0025472	13 449	392 58	157 08	70,497	101,510 000	111 07	49,848	71,781,000
8 5	0028666	15 136	348 84	166 90	74,903	107,860,000	118 01	52,964	76,268,000
9 0	0032037	16 915	312 14	176 71	79,309	114,200,000	124 95	56,079	80,753,000
9 5	0035583	18 788	281 03	186 53	83,714	120,550,000	131 90	59,194	85,239,000
10 0	0039303	20 752	254 43	196 35	88,121	126,890,000	138 84	62,310	89,726,000
10 5	0043229	22 825	231 32	206 17	92,527	133,240,000	145 78	65,426	94 213,000
11 0	0047330	24 990	211 28	215 98	96,933	139,580,000	152 72	68,541	98,698,000
11 5	0051608	27 249	193 77	225 80	101,340	145,930,000	159 67	71,657	103,190,000
12 0	0056058	29 599	178 39	235 62	105,750	152,270,000	166 61	74,772	107,670,000
12 5	0060780	32 091	164 53	245 44	110,150	158,620,000	173 55	77,888	112,160,000
13 0	0065686	34 682	152 24	255 25	114,560	164,960,000	180 49	81,003	116,640,000
13 5	0070778	37 371	141 29	265 07	118,960	171,300,000	187 43	84,118	121,130,000
14 0	0076059	40 159	131 48	274 89	123,370	177,650,000	194 37	87,235	125,620,000
14 5	0081525	43 045	122 66	284 71	127,780	184,000,000	201 32	90,350	130,100,000
15 0	0087173	46 027	114 71	294 52	132,180	190,340,000	208 26	93,465	134,590,000

TABLE IV—(*Continued*)

d = 72 inches = 6 feet

v	$s = \frac{h}{l}$	$s_m = 5\ 280 \frac{h}{l}$	$G = \frac{l}{h}$	Q Cubic Feet per Second	Q Gallons per Minute	Q Gallons per Day	Q' Cubic Feet per Second	Q' Gallons per Minute	Q' Gallons per Day
1	00000035551	0018771	2,812,900	2 8274	1,268 9	1,827,300	1 9993	897 26	1,292,100
2	0000014179	0074865	705,260	5 6548	2,537 9	3,654,500	3 9985	1,794 5	2,584,100
3	0000031809	016795	314,380	8 4822	3,806 8	5,481,800	5 9978	2,691 8	3,876,100
4	0000056385	029771	177,350	11 310	5,075 7	7,309,000	7 9971	3,589 1	5,168,200
5	0000087842	046380	113,840	14 137	6,344 7	9,136,300	9 9963	4,486 3	6,460,300
6	000012612	066590	79,290	16 964	7,613 6	10,964,000	11 996	5,383 6	7,752,300
7	000017116	090369	58,427	19 792	8,882 6	12,791,000	13 995	6,280 9	9,044,400
8	000022288	11768	44,866	22 619	10,151	14,618,000	15 994	7,178 1	10,336,000
9	000028124	14849	35,557	25 447	11,420	16,445,000	17 993	8,075 3	11,628,000
10	000034619	18278	28,886	28 274	12,689	18,273,000	19 993	8,972 6	12,921,000
11	000041762	22050	23,945	31 101	13,958	20,100,000	21 992	9,869 8	14,212,000
12	000049552	26163	20,181	33 929	15,227	21,927,000	23 991	10,767	15,505,000
13	000058154	30705	17,196	36 756	16,496	23,754,000	25 990	11,664	16 797,000
14	000067243	35504	14,871	39 584	17,765	25,582,000	27 990	12,562	18,089,000
15	000077192	40757	12,955	42 411	19,034	27,409,000	29 989	13,459	19,381,000

16	000087561	46232	11,421	45 239	20,303	29,236,000	31 988	14 356	20,673,000
17	000098551	52034	10,147	48 066	21,572	31,063,000	33 988	15,254	21,965,000
18	00011048	58335	9,051 1	50 893	22,841	32,890 000	35 987	16,151	23,257,000
19	00012272	64798	8,148 4	53 720	24,110	34,718,000	37 986	17,048	24,549,000
20	00013598	71799	7,353 8	56 548	25 379	36 545 000	39 985	17,945	25,841,000
21	00014970	79039	6,680 2	59 376	26,648	38,372,000	41 985	18,843	27,133,000
22	00016416	86678	6,091 4	62 203	27,916	40,109,000	43 984	19,740	28,425,000
23	00017902	94521	5,586 0	65 031	29,186	42,027,000	45 983	20,637	29,717,000
24	00019447	1 0268	5,142 1	67 858	30,454	43 854,000	47 982	21,534	31,009,000
25	00021053	1 1116	4,749 9	70 685	31,723	45,681,000	49 982	22,432	32,301,000
26	00022719	1 1995	4,401 7	73 512	32,992	47,508,000	51 980	23,329	33,593,000
27	00024443	1 2906	4,091 2	76 340	34,261	49,336,000	53 980	24,226	34,885,000
28	00026207	1 3837	3,815 8	79 168	35,530	51,163,000	55 980	25,123	36,178,000
29	00028047	1 4808	3,565 5	81 995	36,799	52,991,000	57 979	26,021	37,470,000
30	00029944	1 5810	3,339 6	84 822	38,068	54,818,000	59 978	26,918	38,761,000
31	00031948	1 6869	3,130 0	87 650	39,337	56,645,000	61 977	27,815	40,053,000
32	00034017	1 7961	2,939 7	90 477	40,606	58 472,000	63 976	28 712	41,346,000
33	00036175	1 9100	2,764 3	93 304	41,875	60,299,000	65 975	29,609	42,637,000
34	00038372	2 0260	2,606 1	96 132	43,144	62,127,000	67 975	30,507	43,930,000
35	00040630	2 1453	2,461 2	98 960	44,413	63,954,000	69 975	31,404	45,222,000

TABLE IV—(*Continued*)

$d = 72$ *inches* $= 6$ *feet*

v	$s = \frac{h}{l}$	$s_m = 5\,280 \frac{h}{l}$	$G = \frac{l}{h}$	Q Cubic Feet per Second	Q Gallons per Minute	Q Gallons per Day	Q' Cubic Feet per Second	Q' Gallons per Minute	Q' Gallons per Day
3 6	00042917	2 2660	2,330 1	101 79	45,681	65,781,000	71 973	32,301	46,514,000
3 7	00045299	2 3918	2,207 5	104 61	46,950	67,608,000	73 972	33,199	47,806,000
3 8	00047706	2 5188	2,096 2	107 44	48,219	69 435,000	75 971	34,096	49,098,000
3 9	00050210	2 6511	1,991 6	110 27	49,488	71,262,000	77 970	34,993	50,389,000
4 0	00052736	2 7845	1,896 2	113 10	50 757	73 090 000	79 971	35 891	51,682,000
4 1	00055317	2 9207	1,807 8	115 92	52 026	74 917 000	81 969	36,787	52 974,000
4 2	00057959	3 0602	1,725 4	118 75	53,295	76,745,000	83 969	37,685	54,266,000
4 3	00060655	3 2026	1,648 7	121 58	54,564	78,572,000	85 960	38,582	55,558,000
4 4	00063409	3 3480	1 577 1	124 41	55,833	80,399,000	87 967	39,479	56,850 000
4 5	00066219	3 4963	1 510 1	127 23	57,102	82,226 000	89 966	40,377	58,142,000
4 6	00069086	3 6477	1,447 5	130 06	58,371	84,054,000	91 967	41,274	59,435,000
4 7	00072008	3 8020	1,388 7	132 89	59 640	85 882 000	93 966	42,172	60,727,000
4 8	00074984	3 9591	1,333 6	135 72	60,909	87 708 000	95 964	43,069	62 018,000
4 9	00078019	4 1194	1 281 7	138 54	62,178	89,536,000	97 965	43,966	63,311,000
5 0	00081104	4 2822	1,233 0	141 37	63,447	91,363,000	99 963	44,863	64,603,000

5 5	00097665	5 1567	1,023 9	155 51	69 791	100,500 000	109 96	49,349	71,062 000
6 0	0011567	6 1073	864 53	169 64	76 136	109,640,000	119 96	53,836	77 523 000
6 5	0013531	7 1445	739 03	183 78	82 480	118,770,000	129 95	58,322	83,983,000
7 0	0015643	8 2592	639 28	197 92	88,826	127,910,000	139 95	62,809	90 444 000
7 5	0017840	9 4195	560 53	212 06	95,170	137,040,000	149 94	67,295	96 904,000
8 0	0020166	10 647	495 90	226 19	101,510	146,180,000	159 94	71,781	103,360,000
8 5	0022690	11 980	440 72	240 33	107,860	155,320,000	169 94	76,268	109 820 000
9 0	0025354	13 387	394 41	254 47	114,200	164,450,000	179 93	80,753	116,280,000
9 5	0028156	14 866	355 16	268 60	120,550	173,590,000	189 93	85,239	122,740 000
10 0	0031094	16 418	321 60	282 74	126,890	182,730,000	199 93	89,726	129,210,000
10 5	0034253	18 085	291 94	296 88	133,240	191,860,000	209 92	94,213	135,670,000
11 0	0037561	19 832	266 23	311 01	139,580	201,000,000	219 92	98,698	142,120,000
11 5	0041019	21 658	243 78	325 15	145,930	210,140,000	229 92	103,190	148,590,000
12 0	0044626	23 562	224 08	339 29	152,270	219,270,000	239 91	107,670	155,050,000
12 5	0048383	25 546	206 69	353 43	158,620	228,410,000	249 91	112,160	161,510,000
13 0	0052285	27 606	191 26	367 56	164,960	237,540,000	259 90	116,640	167,970,000
13 5	0056338	29 746	177 50	381 70	171,300	246,680,000	269 90	121,130	174,430,000
14 0	0060540	31 965	165 18	395 84	177,650	255,820,000	279 90	125,620	180,890,000
14 5	0064886	34 259	154 12	409 98	184,000	264,950,000	289 89	130,100	187,350,000
15 0	0069379	36 632	144 14	424 11	190,340	274,090,000	299 89	134,590	193,810,000

HYDRAULICS

(PART 3)

FLOW OF WATER IN CONDUITS AND CHANNELS

DEFINITIONS

1. The term **channel** is applied in hydraulics to the bed of any long body of water flowing under the action of gravity, not, as in water-supply pipes, under pressure An artificial channel dug in the ground for the conveyance of water, and whose bed is formed by the natural soil, is called a **canal.** A canal of small dimensions is usually called a **ditch.**

2. A **conduit** differs from a canal in having an artificial bed Flumes and sewer pipes are examples of conduits

3. The **slope** of a channel is the ratio of the fall to the length in which the fall occurs If s is the slope, h the fall, and l the length in which the fall h occurs, then,

$$s = \frac{h}{l}$$

EXAMPLE —If a canal has a fall of $2\frac{1}{8}$ inches in 500 feet, what is the slope?

SOLUTION —The fall $2\frac{1}{8}$ in $=$ 177 ft , hence,

$$s = \frac{177}{500} = 000354 \quad \text{Ans}$$

4. The **wetted perimeter** of the cross-section of a channel is the part of the boundary in contact with the water If, for example, a circular conduit whose diameter

is 4 feet is half full, its wetted perimeter is equal to one-half its circumference, or $\frac{1}{2} \times 3.1416 \times 4 = 6.2832$ feet

5. The **hydraulic radius** of a channel is the ratio of the area of the cross-section of the water in the channel to the wetted perimeter If the wetted perimeter is denoted by p, the area of cross-section by F, and the hydraulic radius by r, we have

$$r = \frac{F}{p}$$

The hydraulic radius is sometimes called the **hydraulic mean depth.**

EXAMPLE —What is the hydraulic radius of a circular conduit 4 feet in diameter and half full of water?

SOLUTION —Here $F = \frac{1}{2} \times .7854 \times 4^2 = 6.2832$ sq ft, and

$$p = \frac{1}{2} \times 3.1416 \times 4 = 6.2832 \text{ ft}$$

Therefore, $$r = \frac{F}{p} = \frac{6.2832}{6.2832} = 1 \quad \text{Ans}$$

6. The hydraulic radius for a circular cross-section filled with water is $\frac{1}{4} d$, denoting the diameter by d For

$$F = \frac{1}{4} \pi d^2, \text{ and } p = \text{circumference} = \pi d,$$

hence, from the formula in Art **5**,

$$r = \frac{F}{p} = \frac{\frac{1}{4} \pi d^2}{\pi d} = \frac{1}{4} d$$

7. Permanent Flow.—When the quantity of water that passes through any cross-section in any and every interval of time is the same as that which passes through every other cross-section, the flow is said to be **permanent** or **steady.** As shown in *Hydraulics*, Part 1, $Q = F_1 v_1 = F_2 v_2 = F_3 v_3$, etc, and it was there shown that mean velocities at different sections are inversely as the sectional areas

8. Uniform Flow—When the channel has a uniform cross-section, $F_1 = F_2 = F_3$, etc, and $v_1 = v_2 = v_3$, etc, that is, the velocity is constant Under these conditions, the flow is said to be **uniform**

9. The **mean velocity** is the average velocity of flow for the whole cross-section of the water in the channel

Owing to the friction along the sides and bottom, the water filaments next the walls move most slowly, and the velocity is different in the various parts of a cross-section

10. The **discharge** is the amount of water flowing through any section in a unit of time, and is equal to the product of the area of the water cross-section and the mean velocity at that cross-section If Q denotes the discharge, F, the area of the cross-section of the water, and v, the mean velocity, we have, as in the case of orifices and pipes,

$$Q = Fv$$

VELOCITY AND DISCHARGE

11. General Formula for Velocity —The velocity depends on the inclination or slope, the form and dimensions, and the smoothness of the channel In the design of channels, *velocity* is the key to the solution of all problems When this is known for different sizes, different forms, different materials, and different depths of flow, it is a simple matter to determine the size and calculate the discharge for any particular case

Experience shows that formulas for velocities in channels cannot be derived solely by mathematical investigations, but must be based both on experiments and on mathematical reasoning The following general formula, known as **Chezy's formula**, is the basis of all formulas for the flow of water in channels

$$v = c\sqrt{rs}$$

in which c is a variable coefficient, depending both on the character and conditions of the bed and on the values of the hydraulic radius r and the slope s

12. Kutter's Formula —A general expression for the value of the coefficient c has been deduced from the investigations of Ganguillet and Kutter, and is known as **Kutter's formula** This formula is now very extensively used, having been experimentally proved more accurate than any of the many others found in hydraulic literature It has

proved applicable to streams of all sizes, from sewers to large rivers Kutter's formula is as follows

$$c = \frac{23 + \frac{1}{n} + \frac{00155}{s}}{5521 + \left(23 + \frac{00155}{s}\right)\frac{n}{\sqrt{r}}}$$

In this formula, n is a coefficient, called the **coefficient of roughness,** whose value depends on the character and condition of the bed

Table I, at the end of this Section, gives the values that may be used under the conditions most often met with in practice

EXAMPLE 1 —What is the value of c for a rough plank sluice 24 inches wide, when the depth of water in the sluice is 15 inches, and the fall 3 inches in 100 feet?

SOLUTION —The slope $s = 25 - 100 = 0025$, the wetted perimeter $p = 2 + (2 \times 1\ 25) = 4\ 5$ ft , and the area of the water cross-section $F = 2 \times 1\ 25 = 2\ 5$ sq ft The hydraulic radius is, therefore, $r = 2\ 5 - 4\ 5 = 5556$ From the table, the value of n for unplaned timber is found to be 012, therefore,

$$c = \frac{23 + \frac{1}{012} + \frac{00155}{0025}}{5521 + \left(23 + \frac{00155}{0025}\right) \times \frac{012}{\sqrt{5556}}} = 114\ 7 \quad \text{Ans}$$

EXAMPLE 2 —(*a*) What is the velocity in example 1? (*b*) What is the discharge?

SOLUTION —(*a*) Substituting the value found for c in the formula $v = c\sqrt{rs}$ (Art 11)

$$v = 114\ 7\sqrt{5556 \times 0025} = 4\ 27 \text{ ft per sec} \quad \text{Ans}$$

(*b*) Substituting the values of F and v in the formula $Q = Fv$,

$$Q = 2\ 5 \times 4\ 27 = 10\ 675 \text{ cu ft per sec} \quad \text{Ans}$$

13. Thrupp's Formula for Flow of Water —The following formula, proposed by Thrupp, represents with fair accuracy the results of a wide range of experiments It applies to uniform flow in open channels or to flow under pressure in pipes

As in previous formulas, r is the hydraulic radius, and s the slope Thrupp's formula is

$$v = m\,r^x s^y$$

The values of m, x, and y for different conditions are given in Table II, at the end of this Section.

This formula is very useful not only for finding the value of v directly, but for determining the values of other quantities to be used as approximations in applying Kutter's formula, as illustrated in some of the following examples.

EXAMPLE 1.—Compute by Thrupp's formula the velocity of flow in example 2 of Art. **12**.

SOLUTION.—For unplaned plank, $m = 118.33$, $x = .615$, and $y = .50$. Substituting in the formula,

$$v = 118.33 \times .5556^{.615} \times .0025^{.50} = 4.12 \text{ ft. per sec.} \quad \text{Ans.}$$

EXAMPLE 2.—A canal or ditch having the cross-section shown in Fig. 1 is to deliver 100 cubic feet of water per second. What must be the fall per 1,000 feet of length to give this discharge?

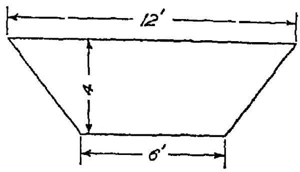

FIG. 1

SOLUTION.—The area of the water cross-section is

$$F = 4 \times \tfrac{1}{2} \times (6 + 12) = 36 \text{ sq. ft.},$$

and the mean velocity is, therefore,

$$v = \frac{Q}{F} = \frac{100}{36} = 2.778 \text{ ft. per sec.}$$

The wetted perimeter is

$$p = 6 + 2\sqrt{4^2 + 3^2} = 16 \text{ ft.},$$

and the hydraulic radius is, therefore,

$$r = \frac{F}{p} = \frac{36}{16} = 2.25 \text{ ft.}$$

First, from Thrupp's formula,

$$s^y = \frac{v}{m r^x}, \text{ whence } s = \sqrt[y]{\frac{v}{m r^x}}$$

Substituting the values for earth,

$$s = \sqrt[.5]{\frac{2.778}{65.1 \times 2.25^{.7}}},$$

or,

$$s = .00056645$$

This value of s may be taken as a first approximation. Now, using Kutter's formula, with $n = .0225$,

$$c = \frac{23 + \dfrac{1}{.0225} + \dfrac{.00155}{.00056645}}{1 + \left(23 + \dfrac{.00155}{.00056645}\right) \times \dfrac{.0225}{\sqrt{2.25}}} = 74.81$$

From the formula in Art. **11**,

$$s = \frac{v^2}{c^2 r} = \frac{2.778^2}{74.81^2 \times 2.25} = .000613$$

The fall per 1,000 ft is, therefore,

$$000613 \times 1{,}000 = \ 613 \text{ ft } = 7\tfrac{3}{8} \text{ in , nearly} \quad \text{Ans}$$

EXAMPLE 3 —In a stream of fairly uniform cross-section, the fall is 1 foot per 1,000 feet, the surface width is 30 feet, the wetted perimeter is 36 feet, and the average depth is 4 feet The bed is somewhat obstructed by stones and weeds Calculate approximately the volume of water flowing, both by Kutter's and by Thrupp's formula

SOLUTION — $F = 30 \times 4 = 120$ sq ft , approximately $r = \frac{F}{p} = \frac{120}{36} = 3\frac{1}{3}$ ft $s = 001$ For n, the value 03 may be taken From Kutter's formula, Art **12**,

$$c = \frac{23 + \frac{1}{03} + \frac{00155}{001}}{5521 + \left(23 + \frac{00155}{001}\right) \times \frac{03}{\sqrt{\frac{10}{3}}}} = 60\ 6$$

Substituting in the formula of Art **11**,

$$v = 60\ 6\sqrt{3\tfrac{1}{3} \times\ 001} = 3\ 5 \text{ ft per sec}$$

$$Q = Fv = 120 \times 3\ 5 = 420 \text{ cu ft per sec} \quad \text{Ans}$$

By Thrupp's formula,

$$v = 46\ 64 \times (3\tfrac{1}{3})^{\ 78} \times\ 001^{\ 5} = 3\ 77 \text{ ft per sec}$$

and $$Q = 120 \times 3\ 77 = 450 \text{ cu ft per sec , nearly}$$

Neither result can be regarded as more than a rough approximation

EXAMPLE 4 —A circular brick sewer laid with a grade of 2 feet in 1,000 is to discharge 70 cubic feet per second when running full Find the diameter required for this discharge, using constants for smooth brickwork

SOLUTION —From the formula of Art **10** we have, $v = \frac{Q}{7854\, d^2}$ Writing $25\, d$ instead of r in Thrupp's formula, we have

$$v = m(\ 25\, d)^x\, s^y$$

Placing these two values of v equal to each other,

$$m(\ 25\, d)^x\, s^y = \frac{Q}{7854\, d^2},$$

whence $$7854 \times (\ 25)^x m s^y d^{2+x} = Q,$$

and, therefore, $$d = \sqrt[2+x]{\frac{Q}{7854 \times (\ 25)^x\, m s^y}}$$

Substituting in this equation the values of Q and s, together with those of x, m, and y, taken from the table,

$$d = \sqrt[2+\ 61]{\frac{70}{7854 \times (\ 25)^{\ 61} \times 129\ 1 \times\ 002^{\ 5}}} = 3\ 945 \text{ ft}$$

Then, $$r = 3\ 945 \div 4 = \ 986 \text{ ft}$$

This value is to be used as a first approximation in Kutter's formula The value of n is 015

Substituting in Kutter's formula,

$$c = \frac{23 + \frac{1}{015} + \frac{00155}{002}}{5521 + \left(23 + \frac{00155}{002}\right) \times \frac{015}{\sqrt{986}}} = 99\ 25$$

Equating the value of $v = \frac{Q}{7854\, d^2}$ to that given by the formula of Art **11**, and writing $25\, d$ instead of r,

$$\frac{Q}{7854\, d^2} = c\sqrt{25\, d\, s}$$

Squaring, $$Q^2 = 25 \times 7854^2\, c^2\, s\, d^5,$$

whence, $$d = \sqrt[5]{\frac{Q^2}{25 \times 7854^2\, c^2\, s}}$$

Substituting numerical values in this equation,

$$d = \sqrt[5]{\frac{70^2}{25 \times 7854^2 \times 99\ 25^2 \times 002}} = 4\ 380 \text{ ft}$$

From this value of d,

$$r = 4\ 380 - 4 = 1\ 095 \text{ ft}$$

Using this value of r in Kutter's formula,

$$c = \frac{23 + \frac{1}{015} + \frac{00155}{002}}{5521 + \left(23 + \frac{00155}{002}\right) \times \frac{015}{\sqrt{1\ 095}}} = 101\ 3$$

Substituting this value in the equation for d,

$$d = \sqrt[5]{\frac{70^2}{25 \times 7854^2 \times 101\ 3^2 \times 002}} = 4\ 34 \text{ ft} \quad \text{Ans}$$

EXAMPLES FOR PRACTICE

1 A circular brick sewer 3 feet in diameter falls 3 75 feet in a length of 2,500 feet What is the slope? Ans 0015

2 The sewer in example 1 flows half full Calculate (*a*) its wetted perimeter, (*b*) its hydraulic radius, (*c*) the value of c from Kutter's formula, using the value of n for fouled sewers, (*d*) the mean velocity of flow, (*e*) the discharge in cubic feet per second

Ans $\begin{cases} (a)\ 4\ 7124 \text{ ft} \\ (b)\ \ 75 \\ (c)\ 80\ 9 \\ (d)\ 2\ 71 \text{ ft per sec} \\ (e)\ 9\ 58 \text{ cu ft per sec} \end{cases}$

3 A flume is $4\frac{1}{2}$ feet wide and of rectangular cross-section What must be the slope for a discharge of 56 cubic feet per second, when the

depth is 2 8 feet? The walls are of unplaned plank Use Thrupp's formula

Ans $\begin{cases} 00108 \text{ or} \\ 1 \text{ in } 926 \end{cases}$

4 Calculate, by Kutter's formula, the diameter of a rough brick sewer to discharge 35 cubic feet per second with a fall of 1 2 feet per 1,000 Use n = 017 Ans 3 92 ft

GAUGING STREAMS AND RIVERS

14. The hydraulic engineer is called on to measure the volume of flowing water in making tests of hydraulic machinery, in determining the discharge of sewers or of water-supply pipes, in designing water-power plants and irrigation works, in investigations concerning the supply obtainable from streams, in connection with river improvements, etc

The quantity of water to be measured determines the method of measurement A very small stream may be measured, or **gauged,** by permitting it to flow into a tank of known dimensions, or into a tank resting on a set of scales where the water may be accurately weighed Larger quantities involve the use of a Pitot tube, meter, or weir, while for very large streams and rivers, floats and current meters must be employed

MEASUREMENT OF DISCHARGE BY WEIRS

DEFINITIONS AND GENERAL DESCRIPTION

15. Weirs —A **weir** is a dam or obstruction placed across a stream for the purpose of diverting the water and causing it to flow through a channel of known dimensions, which channel may be a notch or opening in the obstruction itself When properly constructed and carefully managed, a weir forms one of the most convenient and accurate devices for measuring the discharge of streams The notch is usually rectangular in form

Many careful experiments have been made to determine the quantity of water that will flow over different forms of

weirs under varying conditions As the result of these experiments, two classes of weirs, those with and those without *end contractions*, have come into general use

16. Weir With End Contractions—In a **weir with end contractions**, the notch is narrower and shallower than the channel through which the water flows, as shown in Fig 2 (*a*) This causes a contraction at the bottom and

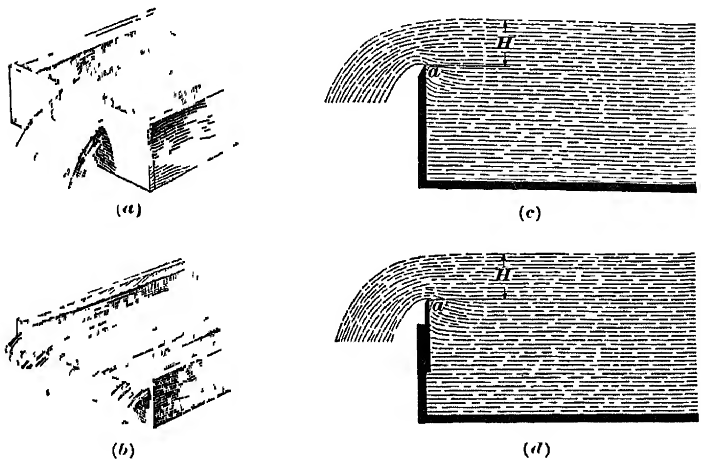

FIG 2

at the two ends (sides) of the issuing stream The end contractions of a weir are said to be complete when the distance from the end of the notch to the side of the channel at each end of the weir is not less than three times the depth of the water on the crest of the weir

17. Weir Without End Contractions.—A **weir without end contractions** is also known as a **weir with end contractions suppressed**, and is commonly called a **suppressed weir**. In a weir of this class, the notch is as wide as the channel leading to it, as shown in Fig 2 (*b*), and, consequently, the issuing stream is not contracted at the sides, but at the bottom only The sides of such a weir should be smooth and straight, and should project a slight distance beyond the crest Means for admitting

air under the falling sheet of water must be made, otherwise, a partial vacuum is formed tending to increase the discharge.

18. Crest of the Weir.—The edge of the notch over which the water flows, as shown in cross-section at *a*, Fig 2 (*c*) and (*d*), is called the **crest** of the weir In all weirs, the inner edge of the crest is made sharp, so that, in passing over it, the water touches only along a line The same statement applies to the inner edge of both the top and the ends of the notch in weirs with end contractions For very accurate work, the edges of the notch should be made with a thin plate of metal having a sharp inner edge, as shown in Fig 2 (*c*), but for ordinary work the edges of the board in which the notch is cut may be chamfered off to an angle of about 30°, as shown at (*d*) The top edge of the notch must be straight and set perfectly level, and the sides must be set carefully at right angles to the top

19. Head.—The head that produces the flow over a weir is the vertical distance from the crest of the weir to the surface of the water, as represented by H in Fig 2 (*c*) and (*d*) It must be measured to a point in the surface of the water so far up-stream that the curve assumed by the flowing water as it approaches the weir will not affect the measurement This will usually be at a distance of from 2 or 3 feet for small weirs to 6 or 8 feet for very large ones

20. Standard Dimensions.—The distance from the crest of the weir to the bottom of the feeding canal or reservoir should be at least three times the head, and, with a weir having end contractions, the distance from the vertical edges to the sides of the canal should also be at least three times the head The water must approach the weir quietly and with little velocity, theoretically, it should have no velocity It is often necessary to place one or more sets of baffle boards or planks across the stream at right angles to the flow, and at varying depths from the surface, to reduce the velocity of the water as it approaches the weir

MEASURING THE HEAD

21. Approximate Method.—Fig 3 shows a simple form of weir placed in a small stream at right angles to the flow, with its face in a vertical plane A plank dam is constructed across the stream at a convenient point, care being taken to prevent any leakage under or around the dam The length of the notch has been calculated to provide for the flow with a head of between 02 and 2 feet A stake *b* is driven firmly into the ground at a point about 6 feet

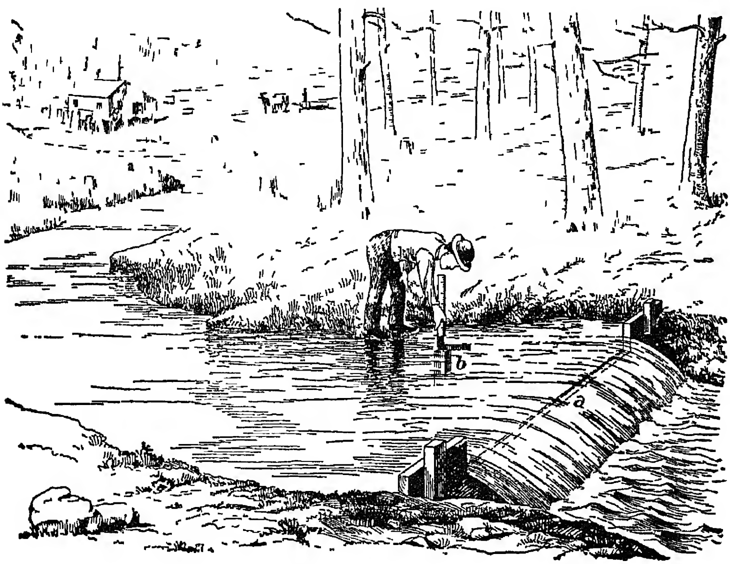

FIG 3

up-stream from the weir and near the bank, as shown The stake is driven until its top is at exactly the same level as the crest *a* The head is then the vertical distance from the top of this stake to the surface of the water, and may be measured by an ordinary square or 2-foot rule, as shown in the figure This is a very simple way of measuring the head, but it does not give an exact measurement, owing to the fact that it is impossible to observe the exact height of the water surface on the side of the square

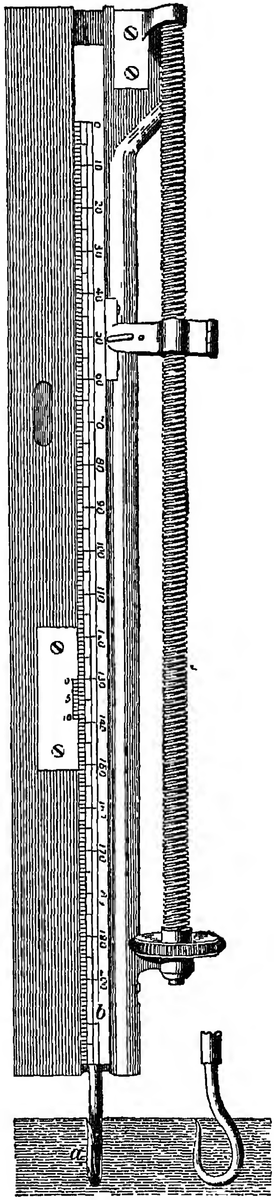

Fig 4

22. The Hook Gauge.—For accurate weir measurements, such as are made in testing the efficiency of waterwheels, the development of water supply, and gauging sewers, the head on the crest is measured with an instrument called a **hook gauge.** In this instrument, which is shown in Fig 4, a hook *a* is attached to the lower end of a sliding scale *b* The scale is graduated to hundredths of a foot, and is provided with a vernier, by means of which it can be read to thousandths of a foot The scale and hook can be raised or lowered slowly by means of the screw *s* The instrument is fastened securely to some solid and substantial object, as a beam or piece of masonry, at a point over the water a few feet up stream from the weir, and where the surface of the water is quiet and protected from wind or eddies. The gauge is so set that the scale will read zero when the point of the hook is at the same level as the crest of the weir When the point of the hook is raised to the surface of the water, it lifts the surface slightly before breaking through To use the gauge, start with the hook below the surface of the water and raise it slowly until the slight elevation caused by the lifting of the surface appears over the point, the reading of the scale for this position of the hook gives the head on the crest

The greatest error that is likely to occur in determining the head on the crest is in setting the point of the hook at the level of the crest, but this can be done accurately by means of an engineers' level

23. An Improvised Hook Gauge.—When for ordinary water measurement it is not necessary to measure the head with the greatest possible accuracy, and the method described in Art 21 is used, a reasonably close reading of the depth of the water above the stake can be obtained by means of a substitute for a hook gauge, improvised from a small piece of tin or metal, bent so as to form a slide on the square, as illustrated in Fig 5 The slide, shown at (*a*) in the figure, has a **V**-shaped notch cut out of its upper part, so as to leave

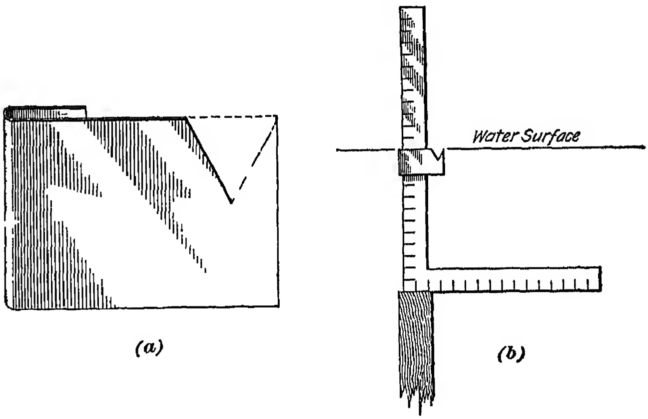

FIG 5

a point on the end of the upper edge, and is so made that when in position on the square its upper edge is horizontal when the square is vertical In measuring the head, the square is held vertically on top of the stake, in the position shown at (*b*), and the height of the water surface is measured by means of the slide used in the manner of a hook gauge Having placed the slide on the square a short distance below the surface, the observer raises it gently until its upper edge just reaches the surface This will be indicated by a slight rounding up of the surface of the water

immediately over the point of the slide, since the point will always lift the surface slightly before breaking through. The head on the crest of the weir will then be shown by the top edge of the slide in contact with the square.

DISCHARGE OF WEIRS

24. Theoretical Discharge.—As shown in *Hydraulics*, Part 1, the theoretical discharge Q for a rectangular orifice with its upper edge at the liquid level is given by the following general formula:

$$Q = \tfrac{2}{3} b \sqrt{2g}\, H^{\frac{3}{2}} = \tfrac{2}{3} \times 8.02\, b\, H^{\frac{3}{2}} = 5.347\, b\, H^{\frac{3}{2}} \qquad (1)$$

Evidently, a weir is such a rectangular orifice, and this formula, therefore, gives the theoretical discharge of the ordinary weir or rectangular notch. The actual discharge Q_0 is obtained by introducing the coefficient of discharge c_3 in the right-hand member; thus,

$$Q_0 = \tfrac{2}{3} \times 8.02\, c_3\, b\, H^{\frac{3}{2}},$$

or,

$$Q_0 = 5.347\, c_3\, b\, H^{\frac{3}{2}} \qquad (2)$$

As usual, b denotes the width of the orifice, that is, the length of the weir in feet, and H, also in feet, is the head, as shown in Fig. 2 (*c*) and (*d*).

25. Effective Head.—Formula **2**, Art. **24**, holds good whenever the velocity with which the water approaches the weir is inappreciable. This velocity is called the **velocity of approach.** If, however, this velocity is considerable, there is an appreciable velocity head at the point at which H is measured, and this must be added to the head H. Let v denote the mean velocity with which the water moves in the channel, and let $h = \frac{v^2}{2g} = .01555\, v^2$ be the corresponding velocity head; then, the velocity v may be considered as resulting from a fall h, Fig. 6, and the *effective head* is $H + h$.

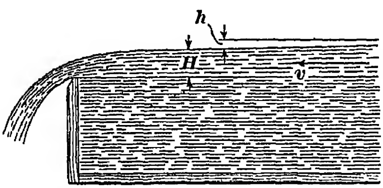

Fig. 6

instead of H alone. The theoretical discharge is, therefore,

$$Q = \tfrac{2}{3}\, b \sqrt{2g}\,(H + h)^{\frac{3}{2}} = 5.347\, b\,(H + h)^{\frac{3}{2}} \qquad (1)$$

On account of the contraction of the stream due to the resistances of the edges of the weir and to their number, an empirical constant factor n is introduced before h, and the effective head is given by the expression

$$H + n\,h$$

The final formula for the actual discharge becomes, therefore,

$$Q_a = \tfrac{2}{3}\, c_3\, b \sqrt{2g}\,(H + n\,h)^{\frac{3}{2}},$$

or,

$$Q_a = 5.347\, c_3\, b\,(H + n\,h)^{\frac{3}{2}} \qquad (2)$$

According to Hamilton Smith, the value of n is $\frac{4}{3}$ for weirs with end contractions, and 1.4 for weirs without end contractions.

26. Calculation of Discharge.—Formula **2**, Art. **25**, may be used when the velocity of approach is taken into account. If, however, that velocity is inappreciable, $h = 0$, and formula **2**, Art. **25**, reduces to formula **2**, Art. **24**. In the calculation of a discharge, first neglect v and calculate the discharge by formula **2**, Art. **24**. This approximate value of Q divided by the area A of the cross-section of the whole channel gives the velocity of approach approximately, that is, $v = \frac{Q}{A}$. Knowing v, h can be computed from the formula $h = .01555\, v^2$, and the effective head $H + n\,h$ determined. A more exact value of Q can then be found by using formula **2**, Art. **25**.

Two tables of the values of c_3 for weirs are given at the end of this Section. Table III applies to weirs with end contractions, and Table V to weirs without end contractions. Values of c_3 for intermediate values of H and b can be obtained by interpolation.

Weirs with end contractions are more often used than those without, though the latter are, in many cases, considered preferable.

27. Francis's Formulas.—According to the theoretical investigations of Prof. James Thompson, the formula for

the discharge of a weir of rectangular section should have the following form, when there is no velocity of approach

$$Q = m\,(b - c\,H)\,H^{\frac{3}{2}}$$

in which m and c are constants to be determined by experiment, and Q, b, and H have the same significance as in the formulas of Art **24.** J B Francis deduced from his experiments at Lowell, Massachusetts, the following formula, which has the same form as that proposed by Professor Thompson, and has become standard

$$Q = 3.33\left(b - \frac{n}{10}H\right)H^{\frac{3}{2}} \qquad (1)$$

or, when $n = o$,
$$Q = 3.33\,b\,H^{\frac{3}{2}} \qquad (2)$$

The constant n denotes the number of end contractions, hence,

for a weir with two end contractions, $n = 2$
for a weir with one end contraction, $n = 1$
for a weir with no end contractions, $n = 0$

When the velocity of approach is taken into account, the formula becomes

$$Q = 3.33\left(b - \frac{n}{10}H\right)\left[(H + h)^{\frac{3}{2}} - h^{\frac{3}{2}}\right] \qquad (3)$$

in which $h = .01555\,v^2$, as in Art **25.**

Table IV, given at the end of this Section, is a very convenient table by means of which the discharge for a given head can be at once obtained

28. Triangular Weir.—A notch of triangular form, Fig 7, was first proposed by Prof James Thompson It may be conveniently used for small flows where the head lies between the limits of .02 and 1 foot It has been proved experimentally that the coefficient c does not vary with the head as much as with rectangular weirs, and a mean value has been determined

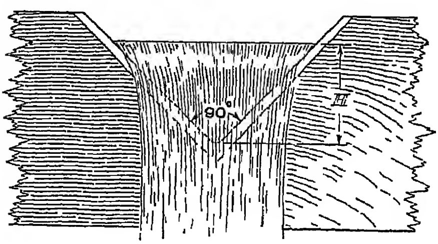

FIG 7

Within the limits of H, as just stated, a right-angled triangular weir with sharp inner edges has the following expression for discharge in cubic feet per second, when H is expressed in feet

$$Q = 2\ 54\ H^{\frac{5}{2}}$$

29. Cippoletti's Trapezoidal Weir —A form of weir devised by the Italian engineer Cippoletti is shown in Fig 8 The sides, instead of being vertical, are inclined, the slope being 4 to 1, as shown It is claimed for this form of weir that the extra flow through the triangular spaces at the ends makes up for the end contractions for all heads within the range of the weir, and in consequence the coefficient c remains constant The formula for the discharge is

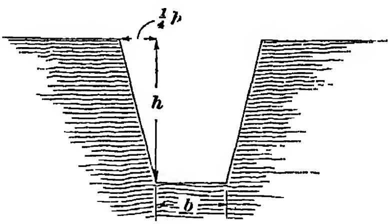

Fig 8

$$Q = \frac{101}{30}\ b\ H^{\frac{3}{2}}$$

in which b = width of weir at crest,

H = head, which may be less than h in Fig 8

EXAMPLE 1 —A weir with end contractions is 5 feet long and the measured head is 872 foot Calculate the discharge on the assumption that the velocity of approach is negligible

SOLUTION —For the given length, c_3 is 604 for a head of 80 ft and 603 for a head of 90 ft Hence, we may take c_3 = 603 Using formula **2**, Art **24**,

$$Q_0 = 5\ 347 \times 603 \times 5 \times 872^{\frac{3}{2}} = 13\ 13 \text{ cu ft per sec} \quad \text{Ans}$$

EXAMPLE 2 —Calculate the discharge in example 1 by Francis's formula

SOLUTION —Substituting the given values in formula **1**, Art **27**,

$$Q = 3\ 33 \times (5 - \tfrac{2}{10} \times 872) \times 872^{\frac{3}{2}} = 13\ 085 \text{ cu ft per sec} \quad \text{Ans}$$

EXAMPLE 3 —In example 1, the channel leading to the weir is 8 feet wide, and the bottom is 2 5 feet below the crest of the weir Calculate the velocity of approach and the effective head

SOLUTION —This example is solved in the manner described in Art **26**. The total depth of the channel is 2 5 + 872 = 3 372 ft

Hence, the area of the cross-section is 3 372 × 8 = 26 976 sq ft, and the mean velocity is

$$v = \frac{Q}{A} = \frac{13\ 13}{26\ 976} = \ 487 \text{ ft per sec} \quad \text{Ans}$$

The equivalent head is h = 01555 × 487² = 0037 ft The effective head is 872 + 0037 = 8757 ft Ans

EXAMPLE 4 —Calculate the discharge for the weir of example 1 (*a*) by formula **2**, Art **25**; (*b*) by formula **3**, Art **27**, using the value of h given in example 3

SOLUTION —(*a*) Substituting the given value in formula **2**, Art **25**,

$Q = 5\ 347 \times\ 603 \times 5 \times (\ 872 + \frac{4}{3} \times\ 0037)^{\frac{3}{2}} = 13\ 24$ cu ft per sec Ans

(*b*) Substituting the given values in formula **3**, Art **27**,

$Q = 3\ 33 \times (5 - \frac{2}{10} \times\ 872) \times (\ 8757^{\frac{3}{2}} -\ 0037^{\frac{3}{2}}) = 13\ 17$ cu ft per sec Ans

NOTE —Examples 1 and 4 show that Francis's formulas give results agreeing closely with those obtained from the formulas in Arts **24** and **25**

EXAMPLE 5 —Calculate the discharge of a triangular weir whose effective head is 9 inches, or 75 foot

SOLUTION —Substituting the given values in the formula of Art **28**,

$$Q = 2\ 54 \times\ 75^{\frac{5}{2}} = 1\ 24 \text{ cu ft per sec} \quad \text{Ans}$$

EXAMPLES FOR PRACTICE

1 A weir with end contractions is 5 feet long, and the measured head is 55 foot, if the water approaches the weir with the velocity of $1\frac{1}{2}$ feet per second, what is the discharge?

Ans 7 49 cu ft per sec

2 A weir without end contractions is 6 feet long, and the head is 25 foot Calculate the discharge, neglecting the velocity of approach

Ans 2 54 cu ft per sec

3 Calculate the discharge in example 2 by Francis's formula

Ans 2 5 cu ft per sec

4 Calculate the discharge from a right-angled triangular weir with a head of 8 inches Ans 92 cu ft per sec

5 Calculate the discharge from a trapezoidal weir 2 feet wide at the crest and with a head of 7 foot Ans 3 94 cu ft per sec

30. Determination of Dimensions —As a practical illustration of the use of the preceding formulas, suppose that it is desired to gauge the small stream whose cross-section is shown in Fig 9 Suppose that, by measuring the

depth of the stream at different points, such as *a*, *b*, *c*, *d*, the area of the cross-section is found to be about 10 2 square feet

Now, if a surface float, such as a block of wood, is found to pass down stream a distance of 30 feet in, say, 20 seconds, the approximate velocity of the stream is 1 5 feet per second, and the discharge Q is

$$10\ 2 \times 1\ 5 = 15\ 3 \text{ cubic feet per second}$$

The next thing is to determine the size of the notch in a weir with end contractions that will permit the flow of this amount of water, the head on the weir not being less than 2 nor more than 24 inches For the purpose of determining the length b of the weir, the end contractions may be neglected, and formula **2**, Art **27**, used This simplifies the operations, and gives a sufficiently close result Solving that formula for b, we have

$$b = \frac{Q}{3\ 33\ H^{\frac{3}{2}}}$$

Any head on the weir may now be assumed Let it be 1 foot, then, by substituting values in the preceding equation,

$$b = \frac{15\ 3}{3\ 33 \times 1^{\frac{3}{2}}} = 4\ 6 \text{ feet}$$

Thus, the length of the notch is established, and its height will be 1 foot plus, say, 6 inches, to provide for an ordinary rise in the stream

Two other conditions remain to be satisfied for a weir with end contractions First, the depth below the crest must be at least three times the head on the weir As the head assumed was 1 foot, this depth must be at least 3 feet The second condition is, that the distance from the vertical edges of the notch to the sides of the stream must be equal to at least three times the head on the weir This will add 3 feet to each end of the notch, making the total length of the weir proper 10 6 feet It will be seen that, by the construction of the necessary planking across the stream, the water level will rise above the weir until it forms a pondage that reduces the velocity of approach to a point where it may ordinarily be disregarded

The hook gauge should be located some 5 or 6 feet up stream, out of the way of the current, and in still water. In all but the most accurate work, reading the gauge at this distance up stream will eliminate all necessity for calculating the velocity of approach, as the reading will be a little higher than the head on the weir, which difference will closely approximate the value of h.

In Fig. 9, the fulfilled conditions for the weir in question are outlined in the cross-section $l m n o$. The weir planking is now extended and firmly embedded in the bottom and

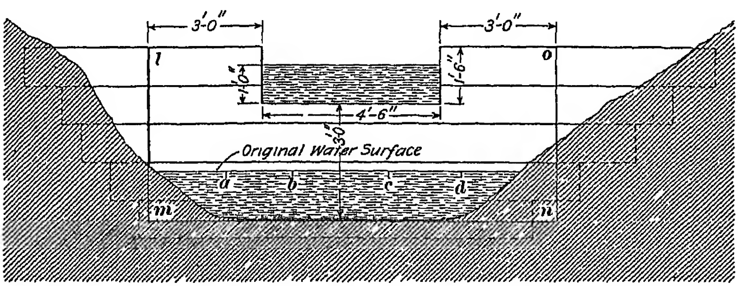

FIG. 9

banks of the stream, making the construction perfectly tight, so that no water can find its way around the ends of the planking or under the bottom. Gauge readings may now be taken.

In Fig. 9, the water level of the original stream is shown, as well as the constructed weir, the shaded portions showing where the planks are embedded in the earth.

31. In cases where larger volumes of water are to be measured by a weir—such as in irrigation work, discharge of sewage into a stream, discharge of a pumping engine into a reservoir, and measurement of the discharge of driven or artesian wells—a rectangular flume is usually constructed 50 feet or more long, with the weir placed at the end where the discharge is measured. The dimensions of the flume or channel depend on the quantity of water to be measured, and the length of the weir is usually the width of the flume, that is, the weir has no end contractions. In this class of

work, the head on the weir is assumed, and the width of the flume is determined by the formula

$$b = \frac{Q}{3.33 H^{\frac{3}{2}}}$$

For example, let it be required to determine with exactness the number of gallons of water pumped daily from a battery of driven wells into a reservoir. The amount is roughly estimated at 5,000,000 gallons.

Now, 5,000,000 gallons per 24 hours is about 7.7 cubic feet per second, and if we assume a head of 1 foot on the weir and substitute in the foregoing equation, we have

$$b = \frac{7.7}{3.33 \times 1^{\frac{3}{2}}} = 2.31 \text{ feet, or, say, 2 feet 4 inches}$$

As the depth below the crest must be at least three times the head on the weir, we have 3 feet for the height of the weir crest above the bottom. But to determine the height of the flume, we must add to this the head on the weir, 1 foot, and, say, 6 inches of running board, to provide for any excess of the estimated 5,000,000 gallons and for fluctuations in the water level. The height of the flume, 4 feet 6 inches, is thus determined.

In flume measurements, the hook gauge is placed in a box about 18 inches square, outside the flume, and 3 or 4 feet back of the weir. A small auger hole permits the water to enter the box, where, the water level being quiet, the gauge can be read to $\frac{1}{1000}$ foot.

32. General Remarks—Other Weir Formulas.—A carefully constructed weir of proper dimensions and favorably located gives more accurate results than any other measuring device now in use. When all necessary precautions are taken, the discharge given by the weir formulas will be within 1 per cent. of the actual discharge.

When any formula is used for the discharge over weirs, care should be taken that the conditions are as nearly as possible identical with those from which the formula was deduced. This refers to length of the crest, head on the weir, velocity of approach, and general dimensions. It may

be well to state the conditions obtaining when three of the most popular formulas were constructed

The many and careful experiments made by J B Francis in 1852, from which his standard formula was derived, were under the following conditions His weir was 10 feet wide, the measuring tank was a canal lock, which contained 12,138 cubic feet of water when filled to a depth of 9 5 feet The head on the weir was measured by two hook gauges, 6 feet up the stream, and the head varied between 5 and 19 inches, with a width of channel of about 14 feet

33. Fteley and Stearns made some experiments in 1879 over weirs where the length of the crest was 5 and 19 feet and the head varied from 1 to 1 foot A section of the Sudbury conduit was used to measure the discharge, and had a capacity of 300,000 cubic feet for an increase of 3 feet in depth Their formula for a standard suppressed weir is as follows

$$Q = 3\ 31\, b\left(H + 1\ 5\frac{v^2}{2g}\right)^{\frac{3}{2}} + \ 007\, b$$

where v is the velocity of approach, and the other letters have the same signification as in Francis's formula, all dimensions being in feet

34. Bazin published, in 1888, the results of his numerous experiments of discharge over weirs having a length of crest varying from 1 5 to 6 feet, and a head ranging from about 2 to 22 inches His formula for weirs with no end contractions, which is here given, takes into account both the velocity of approach and the distance p from the bottom of the channel to the crest

$$Q = \left(\ 405 + \frac{\ 00984}{H}\right)\left[1 + \ 55\left(\frac{H}{p + H}\right)^2\right] b\, H\sqrt{2gH}$$

In this formula, all dimensions are supposed to be expressed in feet

EXAMPLE —Calculate the discharge over a weir 8 feet long, if the head on the crest is 6 inches (*a*) by Fteley and Stearns s formula the velocity of approach being 5 foot per second, (*b*) by Bazin's formula, the distance from the bottom of the channel to the crest of the weir being 1 5 feet

SOLUTION —(a) In this case, $H = \frac{6}{12} = 5$ ft Substituting the given values in the formula of Art 33,

$$Q = 3\ 31 \times 8 \times \left(5 + 1\ 5 \times \frac{5^2}{64\ 32}\right)^{\frac{3}{2}} + 007 \times 8 = 9\ 58 \text{ cu ft per sec}$$

Ans

(b) To apply the formula of Art 34 we have $p = 1\ 5$, the other values being the same as in the preceding example Substituting in the formula,

$$Q = \left(405 + \frac{00984}{5}\right) \times \left[1 + 55 \times \left(\frac{5}{1\ 5 + 5}\right)^2\right] \times 8 \times 5 \times \sqrt{64\ 32 \times 5}$$

$= 9\ 97$ cu ft per sec Ans

EXAMPLES FOR PRACTICE

1 A weir with end contractions is 6 feet long and the head on the crest is 1 foot Assuming that the velocity of approach is negligible, calculate the discharge, by Francis's formula

Ans 19 3 cu ft per sec

2 A weir without end contractions is 5 feet long, and the measured head is 9 inches Assuming the velocity of approach to be 1 5 feet per second, calculate the discharge, using Francis's formula

Ans 11 47 cu ft per sec

3 The length of a weir without end contractions is 10 feet, and the measured head is 1 5 feet If the velocity of approach is 2 feet per second, calculate the discharge (a) by Francis's formula, (b) by Fteley and Stearns's formula, (c) by Bazin's formula, the distance from the crest of the weir to the bottom of the channel being 5 feet

Ans $\begin{cases} (a)\ 64\ 49 \text{ cu ft per sec} \\ (b)\ 66\ 64 \text{ cu ft per sec} \\ (c)\ 62\ 42 \text{ cu ft per sec} \end{cases}$

4 A weir without end contractions is 12 feet long and the head is 1 foot The velocity of approach is 1 25 feet per second, and the distance from the crest of the weir to the bottom of the channel is 8 feet Calculate the discharge (a) by Fteley and Stearns's formula, (b) by Bazin's formula

Ans $\begin{cases} (a)\ 42 \text{ cu ft per sec} \\ (b)\ 40\ 19 \text{ cu ft per sec} \end{cases}$

5 A weir is 3 feet long and the head is 6 inches The velocity of approach is 75 foot per second, and the distance from the crest to the bottom of the channel is 2 feet Calculate the discharge (a) by Fteley and Stearns's formula, (b) by Bazin's formula

Ans $\begin{cases} (a)\ 3\ 67 \text{ cu ft per sec} \\ (b)\ 3\ 69 \text{ cu ft per sec} \end{cases}$

MEASUREMENT OF DISCHARGE BY THE CURRENT METER

DESCRIPTION OF INSTRUMENT

35. Introduction.—The discharge of large streams and rivers is usually determined by first measuring the mean velocity at a cross-section of the flowing water, and then multiplying the velocity as thus determined by the area of that cross-section The velocity can be ascertained either by means of floats, or by the use of special instruments Of these instruments, the current meter, to be described presently, is the most convenient and the one most commonly employed The method by floats and by the Pitot tube will be explained further on

36. General Description of the Current Meter. There are several types of current meter They all work on the same general plan, which consists in immersing a wheel in the stream whose velocity is to be determined, and counting, or otherwise ascertaining, the number of revolutions in a certain time The velocity is then found from a previously established relation or table giving the velocity of flow corresponding to any number of revolutions of the wheel

The form of instrument most commonly used is that illustrated in Fig 10 Its main part is a wheel on the circumference of which are placed four or five conical buckets b, b, and whose axis, which is a vertical rod, revolves in bearings o, o', enclosed in small air chambers or boxes The end o' of the axis is of such form that at every revolution of the wheel it comes in contact with a spring and closes an electric circuit in the wires w w, by which a current is sent to the box e, where, by means of an electromagnetic arrangement a, the number of revolutions is recorded on the dials m and n One of these dials records single revolutions, and the other, hundreds, somewhat like a gas meter If m is the single-revolution dial, and n the hundred-revolution dial, the

number of revolutions indicated by the instrument shown in the figure will be ascertained as follows The pointer on *n* points to number *55*, which means 5,500 revolutions, the

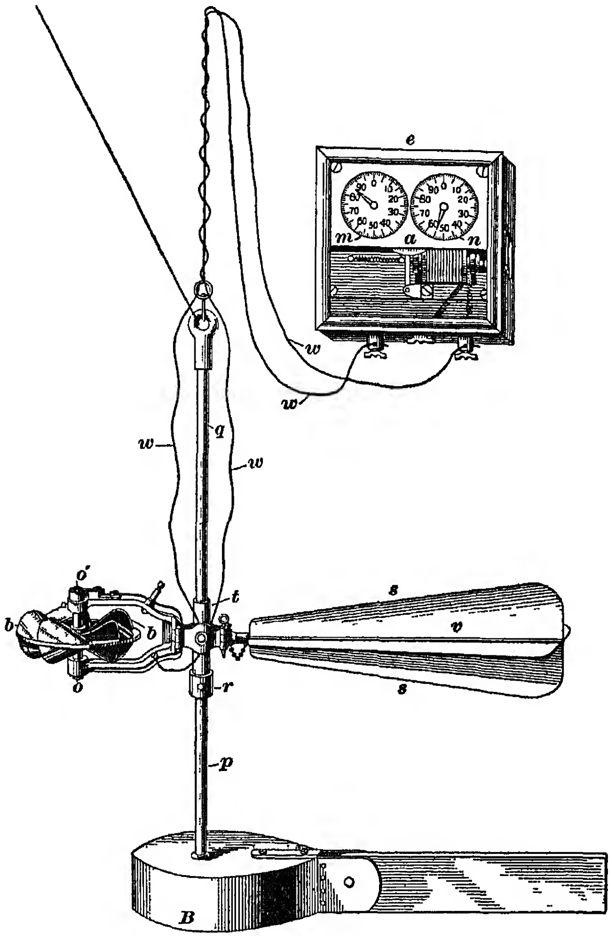

FIG 10

pointer on *m* points to *86*, which means 86 revolutions, the total number of revolutions recorded is, therefore, 5,500 + 86, or 5,586 This assumes that the dials were

both at zero when the count or observations were begun Usually, however, it is not necessary to set the dials at 0 it is sufficient to take a reading before and one after the observations, and to take the difference between them, which will give the number of revolutions during the observations Some instruments have a third dial, reading thousands of revolutions

The frame carrying the wheel *b b* is pivoted to a cylindrical piece *t*, called the **trunnion**, which fits loosely on the rod *p q*, on which it can both turn and slide To the trunnion is also pivoted the rudder *s s*, consisting of a rod that is in line with the longitudinal axis of the wheel frame, and carries four vanes *v* in two planes at right angles to each other The purpose of the rudder is to keep the instrument in the direct line of the current it acts just like the vanes of a windmill The trunnion and the parts attached to it can be kept at any desired height on the rod by means of a sliding ring *r*, which can be set anywhere on the rod The weight *B* with its rudder of wood, weighing about 60 pounds, is used only in deep rivers where velocities are high, otherwise, the meter is simply suspended by a brass rod and lowered to any point of the stream where the velocity is required

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RATING THE INSTRUMENT

37. General Description of Method —In order to determine the velocity of a current from meter observations giving revolutions per second, it is necessary to know the relation between the revolutions per second of the wheel and the velocity of the current in feet per second The determination of this relation is called **rating** the meter This may be done by holding the instrument in a current of known velocity, or by moving it through still water at a uniform speed and noting the time and number of revolutions for a given distance Few opportunities are had to apply the former method, and meters are usually rated by observations in still water A course from 100 to 200 feet in length is measured off and a boat is started at a sufficient

distance from the course to acquire the desired rate of speed before entering the course The instrument is attached to the bow of the boat, as shown in Fig 11, and immersed to a

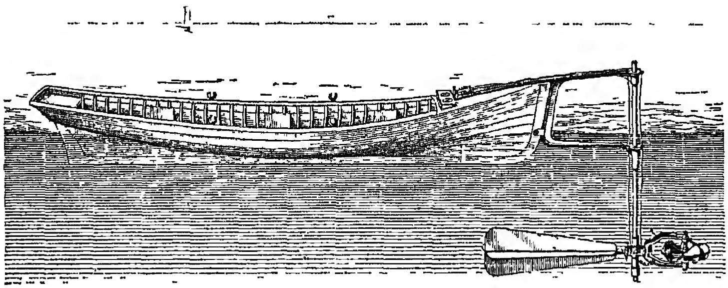

Fig 11

depth of about 3 feet The boat should have no rudder, so that the rudder of the instrument will control its direction An assistant with a stop-watch notes the exact time of entering on and leaving the course The observer reads the number of revolutions for this distance from the dial From ten to forty observations are made, giving the boat speeds that will approximate the highest, lowest, and intermediate current velocities for which the instrument is likely to be used

The distance traversed divided by the time gives the velocity of the boat, and the number of revolutions divided by the time gives the rate of revolution of the wheel It will be found that the number of revolutions per second of the meter wheel is not exactly proportional to the velocity, but bears a relation to it that can be expressed by an algebraic equation From data thus secured, a table is prepared that will give at a glance velocities in feet per second for any number of revolutions of the wheel

38. Field Notes—The field notes of the observations for an actual meter rating are shown here, two pages of a field book being represented

The first five columns on the left-hand page record the field operations They show the number of observations the direction taken by the boat, the dial readings, the number

OBSERVATIONS FOR RATING CURRENT METER NO 1012

Easton, Pa, Aug 10, 1900

W L McNeil, Observer — J H Davis, Assistant

1	2	3	4	5	6	7	
Number of Observation	Direction	Dial Reading	Number of Revolutions	Time Seconds	x = Revolutions per Second	y = Velocity Feet per Second	Remarks
		220					Length of course, 200 feet Upper basin of Morris Canal used for rating station Meter 5 feet in front of bow of boat and 2 feet under water Boat drawn by line from tow path Light wind from S W causing surface motion of water Meter wheel revolved freely E indicates boat going east W indicates boat going west
1	E	322	102	63	1 619	3 175	
2	W	424	102	54	1 889	3 704	
3	E	527	103	40	2 575	5 000	
4	W	625	98	120	817	1 667	
5	E	720	95	153	621	1 307	
6	W	809	89	190	468	1 053	
7	E	899	90	182	495	1 099	
8	W	1,003	104	31	3 355	6 452	
9	E	1,105	102	52	1 962	3 846	
10	W	1,205	100	72	1 389	2 778	
11	E	1,309	104	28	3 714	7 143	
					11)18 904	37 224	
					x_0 = 1 718	3 384 = y_0	

of revolutions of the wheel, and the elapsed time in seconds for each observation. The sixth and seventh columns may be computed later. The locality, date, and the names of the observer and the assistant should be noted, and under the head of Remarks should be stated on the right-hand page the length of the course, the wind velocity, which should be very low, and any other details relating to the local physical conditions that may be deemed important.

The sixth column shows the number of revolutions per second during each observation. This is determined by dividing the number of revolutions during the observation, as given in the fourth column, by the number of seconds taken to make the observation, as given in the fifth column, and is designated by x. The seventh column contains the velocity, in feet per second, during each observation. This is determined by dividing the length of the course, in feet, by the time, in seconds, taken to make each observation, and is denoted by y.

The average values of x and y are denoted by x_0 and y_0, respectively. The value of x_0 is obtained by adding all the values of x, as given in the sixth column, and dividing by the number of observations. Likewise, the value of y_0 is obtained by adding all the values of y, as given in the seventh column, and dividing by the number of observations.

39. Reducing the Results by the Graphic Method. The results of the observations having been tabulated, a reduction table should be made from which the relation between the number of revolutions per second and the velocity in feet per second is determined. Two general methods may be used for calculating this relation—the graphic and the algebraic method. The graphic method is shorter, simpler, and more convenient, and for practical purposes is sufficiently exact, but it is not so accurate as the algebraic method.

In the graphic method, the observations are plotted by coordinates measured from rectangular axes, taking x, the number of revolutions per second, as the abscissa, and y,

the velocity in feet per second, as the ordinate, in each case Cross-section paper is the most convenient for plotting the observations The observations given in the table of Art **38** are thus shown plotted in Fig 12, each observation being indicated by its number In each case, the horizontal distance from the vertical axis corresponds to the number of revolutions per second as given in the sixth column, and the vertical distance above the horizontal axis represents the velocity in feet per second as given in the seventh column, considering each large square of the cross-section paper to represent one unit

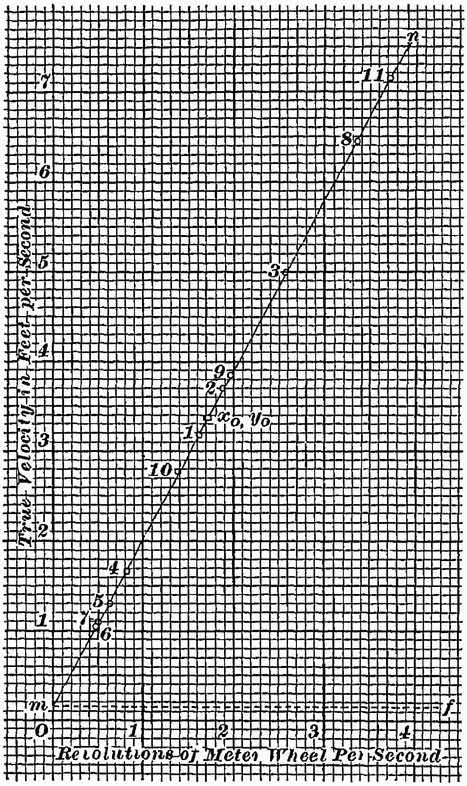

Fig 12

It will be observed that the points thus plotted all lie nearly in a straight line, though in a set of observations as commonly taken in practice it will usually be found that some of the plotted points will deviate more from a straight line than is shown in the figure Having plotted the values of x and y for all the observations, including the mean values x_0 and y_0, the problem is to draw the most probable straight line, that is, the straight line on which all the points may be assumed to lie with the least probable error Such a line may be called a **rating line.**

Considerable assistance in locating the position of the rating line is derived from the fact that it must pass through the point x_0, y_0 whose position is determined by x_0 and y_0, which are the mean values of x and y for all the plotted points, and consequently, the most probable single value of each quantity A fine thread may be stretched through this point and swung carefully in either direction until it occupies a mean position through and near the other points, as shown by the line $m\,n$ In many cases it will be found that a number of the points lie in a straight line that includes the point x_0, y_0, and that those points which do not lie in the straight line lie as much on one side as on the other When such a line can be drawn, it can be taken as the most probable straight line

When the line is drawn in its most probable position, it will be found that it does not pass through the origin O, but intersects the vertical axis a short distance above the horizontal axis, as at the point m The short length $O\,m$ intercepted on the vertical axis represents the effect of friction, which is slight and is assumed to be constant for any number of revolutions per second, as indicated by the line $m\,f$ The equation of the rating line $m\,n$ may be written

$$y = a\,x + b$$

In this equation, x and y are the abscissa and the ordinate, respectively, corresponding to the observed values of x and y, and a and b are constants for the given meter The constant b represents the effect of friction, or $O\,m$, and a is the ratio of $y - b$ to x, as is evident from the figure, or by writing the equation in the form $\frac{y - b}{x} = a$

40. Determination of Constants —When the most probable position of the rating line $m\,n$ has been found, the values of the constants can easily be determined The approximate value of the constant b can be read at once from the plot, since it is represented by the intercept $O\,m$ By substituting in the equation this value for b, and the values of y_0 and x_0 for y and x, respectively, the value of a can at once be determined

EXAMPLE —For the values of x_0 and y_0 as determined from the table in Art **38** and the rating line as shown in Fig 12, what are the values of b and a?

SOLUTION —From Fig 12, it is seen at once that the intercept Om, which represents the value of b, extends across about 1 5 small squares, and, since the width of each small square represents 1 unit, the value of b is equal to

$$1 \times 1\ 5 = 15, \text{ closely} \quad \text{Ans}$$

From the table, the value of y_0 is found to be 3 384 ft per sec and the value of x_0 to be 1 718 rev per sec By substituting these values and the value of b in the equation of Art **39**, it becomes

$$3\ 384 = 1\ 718\,a + 15,$$

whence a gives $a = 1\ 882$ Ans

41. In order to verify the values of a and b as thus obtained, the equation of Art **39** can be applied to the point x_0, y_0, and the values of x_0 and y_0 substituted, and also to any other observation for which the plotted point lies exactly in the line mn—preferably one of the highest or lowest points—and the values of the coordinates x and y for that point substituted This gives two equations, which can readily be solved for the two unknown quantities a and b

EXAMPLE —What are the values of a and b as calculated from the mean values x_0 and y_0 and the values as determined by observation number 11?

SOLUTION —The mean values x_0 and y_0 give the equation

$$3\ 384 = 1\ 718\,a + b \qquad (1)$$

Observation number 11 gives

$$7\ 143 = 3\ 714\,a + b \qquad (2)$$

Subtracting equation (1) from equation (2),

$$3\ 759 = 1\ 996\,a,$$

whence $a = 1\ 883$ Ans

Substituting this value in equation (1), and solving for b,

$$b = 149 \quad \text{Ans}$$

42. The Algebraic Method —If the observed results for a meter rating are tabulated, the values of x and y calculated from each observation, and the mean values x_0 and y_0 derived therefrom, as in the table given in Art **38**, the values of the constants a and b can be determined algebraically The operations for determining the constants are shown in the table on page 33, and may be described in detail as follows

REDUCTION OF OBSERVATIONS FOR RATING CURRENT METER NO 1012

Made at Easton, Pa, Aug 10, 1900

W L McNeil, Observer — J H Davis, Assistant

1 Number	2 Number of Revolutions	3 Time Seconds	4 r = Revolutions per Second	5 v = Velocity Feet per Second	6 $u = x - x_0$	7 $z = y - y_0$	8 $u^2 = (x - x_0)^2$	9 $uz = (x - x_0) \times (y - y_0)$	Remarks
1	102	63	1 619	3 175	− 099	− 209	+ 010	+ 021	Length of base = 200 ft
2	102	54	1 889	3 704	+ 171	+ 320	+ 029	+ 055	
3	103	40	2 575	5 000	+ 857	+ 1 616	+ 734	+ 1 385	
4	98	120	817	1 667	− 901	− 1 717	+ 812	+ 1 547	Meter 5 feet in front of bow of boat and 2 feet under water
5	95	153	621	1 307	− 1 097	− 2 077	+ 1 203	+ 2 278	
6	89	190	468	1 053	− 1 250	− 2 331	+ 1 563	+ 2 914	
7	90	182	495	1 099	− 1 223	− 2 285	+ 1 496	+ 2 795	
8	104	31	3 355	6 452	+ 1 637	+ 3 068	+ 2 680	+ 5 022	
9	102	52	1 962	3 846	+ 244	+ 462	+ 060	+ 113	
10	100	72	1 389	2 778	− 329	− 606	+ 108	+ 199	$a = \frac{23\ 832}{12\ 679} = 1\ 880$
11	104	28	3 714	7 143	+ 1 996	+ 3 759	+ 3 984	+ 7 503	
		11)	18 904	37 224			12 679	23 832	
		$x_0 =$	1 718	3 384 $= y_0$					

The first five columns of this table are the same as in the table of Art **38**, and from the values of x and y in the fourth and fifth columns the mean values x_0 and y_0 are calculated, the same as in the graphic method The value of $x - x_0$ is then calculated for each observation and written in the column following the column of velocities, as shown in the sixth column These values are calculated by subtracting the value of x_0 from the value of x as given by each observation, having due regard for the sign of the remainder Thus, if the value of x_0 is less than that of x, the remainder is positive, but if x_0 is greater than x, the remainder is negative Likewise, the value of $y - y_0$ is calculated for each observation and written in the seventh column of the table The value of $x - x_0$ for each observation is then squared and the square written in the eighth column of the table Also, the values of $x - x_0$ and $y - y_0$ for each observation are multiplied together and the product written in the last column of the table Finally, the values of $(x - x_0)^2$ for all observations, as written in the eighth column, are added together, as are also the values of $(x - x_0)(y - y_0)$, as written in the ninth column, and the sum of the latter is divided by the sum of the former The quotient is the value of the constant a

In order to express, briefly, by formula the operations thus described in detail, let the values of $x - x_0$ be denoted by u, and the values of $y - y_0$ be denoted by z Then,

$$a = \frac{\Sigma(uz)}{\Sigma u^2}$$

Having determined the value of a by this formula, the value of b can readily be determined by substituting this value of a in the formula of Art **39.** In rating a meter by any method, however, it is always advantageous to plot the observations on cross-section paper, as in the graphic method, in order to make an intelligent study of the results

EXAMPLE —For the series of observations shown in the table of Art **38**, what are the values of a and b as determined algebraically?

SOLUTION —The values of $x - x_0$, $y - y_0$, $(x - x_0)^2$, and $(x - x_0) \times (y - y_0)$, as calculated for each observation, are shown in the table of

REDUCTION TABLE FOR CURRENT METER NO 1012

Rated Aug 10, 1900

Revolutions per Second	Velocity Feet per Second	Revolutions per Second	Velocity Feet per Second	Revolutions per Second	Velocity Feet per Second	Revolutions per Second	Velocity Feet per Second
00	154	1 00	2 034	2 00	3 914	3 00	5 794
02	192	1 02	2 072	2 02	3 952	3 02	5 832
04	229	1 04	2 109	2 04	3 989	3 04	5 869
06	267	1 06	2 147	2 06	4 027	3 06	5 907
08	304	1 08	2 184	2 08	4 064	3 08	5 944
10	342	1 10	2 222	2 10	4 102	3 10	5 982
12	380	1 12	2 260	2 12	4 140	3 12	6 020
14	417	1 14	2 297	2 14	4 177	3 14	6 057
16	455	1 16	2 335	2 16	4 215	3 16	6 095
18	492	1 18	2 372	2 18	4 252	3 18	6 132
20	530	1 20	2 410	2 20	4 290	3 20	6 170

this article, and the sums of the last two sets of values are found. The sum of the values of $(x - x_0)^2 = u^2$, as thus found, is 12 679, and the sum of the values $(x - x_0)(y - y_0) = u z$, as also thus found, is 23 832. Hence, by applying the formula, the value of a is found to be

$$23\,832 \div 12\,679 = 1\,880 \quad \text{Ans.}$$

By writing the formula of Art. **39** in the form $b = y - a x$, and substituting in this equation the value of a, and also the values x_0 and y_0 as found in the table of this article, for x and y, respectively, the value of b is found to be

$$b = 3\,384 - (1\,880 \times 1\,718) = 154 \quad \text{Ans.}$$

43. Reduction Table.—By referring to the results obtained in the example solved in the preceding article, it is seen that the values of the constants a and b, as obtained by the more rigid algebraic method, are 1 880 and 154, respectively, which values vary but slightly from those obtained in Arts. **39** and **40** by the less laborious graphic method, which is usually preferred. By substituting the values of these constants in the equation of Art. **39**, which is the fundamental equation for meter rating, it becomes

$$y = 1\,880\,x + 154$$

This is the equation to be used for rating the meter with which the observations were taken. By means of this equation, a reduction table can be made that will show the velocity y of the current for any observed rate of revolution x of the meter wheel within the limits of the tabulated values. The table on page 35 shows part of a reduction table for the meter whose rating has been described, as calculated by this equation. Since this table is given merely for the purpose of showing the form of a reduction table, and is of no value except for computing velocities from observations made with this particular meter, only the upper one-fifth of each column is shown. The velocities given correspond to every two-hundredth part of a revolution per second of the meter wheel. For smaller fractions of a revolution, the corresponding velocities can be found by interpolation. Any desired values within the limits of the observations can be computed from the equation of Art. **39** and tabulated in this manner.

USE OF THE INSTRUMENT FOR DETERMINING VELOCITY AND DISCHARGE

44. To Determine Velocity by the Current Meter. The velocity of the water at any *point* below the surface in the cross-section of a stream can be determined by holding the meter at that point and observing the number of revolutions during a given interval of time. The mean velocity in any *vertical* line of the cross-section can be determined directly by moving the meter vertically at a uniform rate from the surface of the water to the bottom, then back to the surface, and observing the reading of the register before the meter leaves the surface and when it returns again, and the interval of time that elapses between the two surface positions. If the registering mechanism is above water, and is operated by means of electricity, the bottom reading can also be observed and timed. The mean velocity thus obtained will not be strictly accurate, since the meter cannot be run very close to the bottom, but if the observation is made carefully and the meter is lowered and raised at a uniform rate, the results should be reasonably satisfactory, and will be valuable for comparing with the results obtained by mid-depth observations.

If the rates of lowering and raising the meter have been exactly uniform, the number of revolutions registered during the descent should be equal to those registered during the ascent. Then the number of revolutions registered during the descent or the ascent, divided by the time in seconds taken to lower or raise the meter, will give the mean number of revolutions per second for the vertical section of the stream. The number of revolutions registered during the descent and ascent will not usually be exactly equal, however, and the total number of revolutions registered during the descent and ascent, divided by the total time in seconds taken to lower and raise the meter, is taken as the mean rate of revolution.

The mean velocity of the water in a stream can be determined by passing the meter at a slow and uniform rate over

all parts of the vertical cross-section of the stream. It is usually best to make more than one observation for a given cross-section. A good way to use a current meter for determining the mean velocity is to move it slowly across from one side of the stream to the other, holding it submerged in a vertical position, and moving it up and down so as to subject it to the action of the current at all parts of the cross-section. This operation is repeated by moving the meter in the same manner back to the starting point. The number of revolutions and the time in seconds for each observation are noted. If the results of the two observations are reasonably close, the mean is taken; if there is much difference between them, a third observation should be made.

In making the observations, the observer may stand on a bridge that crosses the stream with a clear span, that is, without obstructing piers, if such a bridge is available in a suitable position. If no bridge is available and the stream is not large, a temporary platform may be constructed over it from which to make the meter observations. If the stream is shallow, the observer can make the observations by wading across with the meter. This method is not to be commended, however, when accurate results are required, since the observer's body will offer some obstruction to the current, and will somewhat affect the registration of the meter.

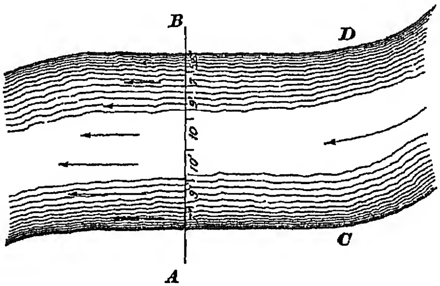

Fig. 13

The ordinary and perhaps the most satisfactory method is as follows:

A straight reach* of uniform cross-section is selected in which to locate the discharge section, and a range is laid off

* The term "reach" is used to describe a straight section of a river, as *A C* or *B D*, Fig. 13.

across the stream at right angles to its axis, as shown at *A B* in Fig 13 The range is located near the lower end of the reach, because the flow is more uniform at such a place than it is just below a bend in the stream, and there is less liability of cross-currents, eddies, or other local disturbances in the current

The range may be marked in any way that is most convenient, but for small streams the most satisfactory method is by means of a wire stretched across the stream The range is then divided into any desired number of parts, and the points of division are marked by means of tags or otherwise Soundings are then taken along the points of division,

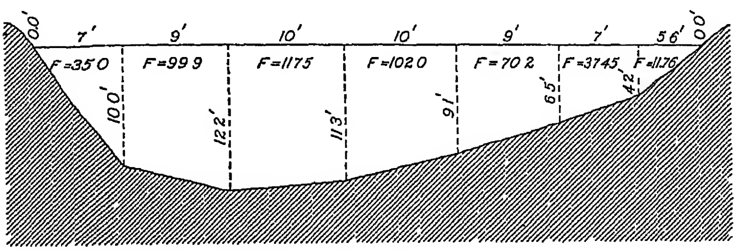

FIG 14

from which a cross-section of the stream is plotted, as illustrated in Fig 14 The area *F* of each division is calculated by multiplying its width by its mean depth

45. To Determine the Discharge.—Having thus determined the dimensions and area of each division of the cross-section, the mean velocity of each division is found by means of time observations with the current meter In making the observations, the meter is held in each division of the cross-section and moved at a uniform rate from the surface to the bottom and again to the surface, and the number of revolutions per second determined in the manner that has been described The velocity in feet per second, corresponding to the number of revolutions per second, is taken from the rating table or determined by calculation, as already explained The mean velocity of flow in each division of the cross-section is thus found and recorded The discharge

in each division of the cross-section is then determined by substituting the values of the area and mean velocity for that division in the formula for discharge The sum of the partial discharges thus obtained is the total discharge of the stream

EXAMPLE —If the current meter for which the table of Art **43** was prepared recorded 2 15 revolutions per second, what was the velocity of the stream?

SOLUTION —By reference to the table it is found that 2 15 falls between 2 14 and 2 16 The corresponding velocities are 4 177 and 4 215 The difference in velocity corresponding to a difference of 2 15 − 2 14, or 01 revolution per second is

$$\frac{4\ 215 - 4\ 177}{02} \times 01 = 019$$

The velocity corresponding to 2 15 revolutions per second is therefore,

$$4\ 177 + 019 = 4\ 196 \text{ ft per sec} \quad \text{Ans}$$

EXAMPLES FOR PRACTICE

1 The values of x and y given in the accompanying table were deduced from a record of a series of observations made for the purpose of rating a current meter Determine the values of a and b to be used in the formula of Art **39.**

Ans $\begin{cases} a = 2\ 884 \\ b = 185 \end{cases}$

Number of Observation	Revolutions per Second x	Velocity per Second y
1	904	2 793
2	850	2 637
3	990	3 040
4	769	2 404
5	1 089	3 325
6	1 188	3 610

2 Determine the velocities corresponding to 1 06 and 1 08 revolutions per second, respectively, of the instrument referred to in example 1

Ans $\begin{cases} 3\ 242 \text{ ft per sec} \\ 3\ 300 \text{ ft per sec} \end{cases}$

3 Determine, by interpolation, from the results obtained in the preceding example, the velocity corresponding to 1 07 revolutions per second Ans 3 271 ft per sec

4 Calculate, by the formula, the velocity corresponding to 3 38 revolutions per second Ans 9 933 ft per sec

MEASUREMENT OF VELOCITY BY FLOATS

46. Surface Floats.—The measurement of velocity in open channels may be effected by the use of floats, which, traversing a known distance in a certain time, indicate more or less accurately the velocity of the water. Surface floats give but roughly approximate results, as they are easily affected by winds, eddies, and cross-currents. A block of wood, a tin can weighted with sand, a long-necked bottle partly filled with water and corked, or any object whose specific gravity can be made nearly equal to that of water, but which exposes a surface easily seen, makes a good surface float.

47. Making the Observations.—For making the observations, a base line is laid off parallel with the axis of the stream. It should not be less than 100 feet in length, and, except for streams of less width than this, need not exceed the width of the stream; for very wide streams, its length may be less than the width of the stream, and a length of 400 feet is probably sufficient in any case. At each end of the base line, and perpendicular to it, a range line is laid off across the stream, as shown in Fig. 15 (*a*). Each range line is thus, as nearly as possible, perpendicular to the general direction of the current. If the stream is not too wide, a wire should be stretched across the stream on each range. At convenient intervals, tags of tin or pasteboard should be attached to the wire, each tag bearing a number that shows its distance from the left-hand bank. Instead of numbered tags, pieces of cloth may be tied to the wire and the distances indicated by different colors. Thus, at a distance of 10 feet from the bank, the wire may be marked with a strip of red cloth, at 20 feet with white, at 30 feet with blue, etc.

The float should be put in the stream at some distance, say 15 to 20 feet, above the upper range, so that it will attain its full velocity before it crosses this range. The time required for the float to traverse the distance between the ranges can be determined by two observers, one at each range, with

stop-watches, or by an observer with a watch at one range, preferably the lower, and an assistant at the other range, who

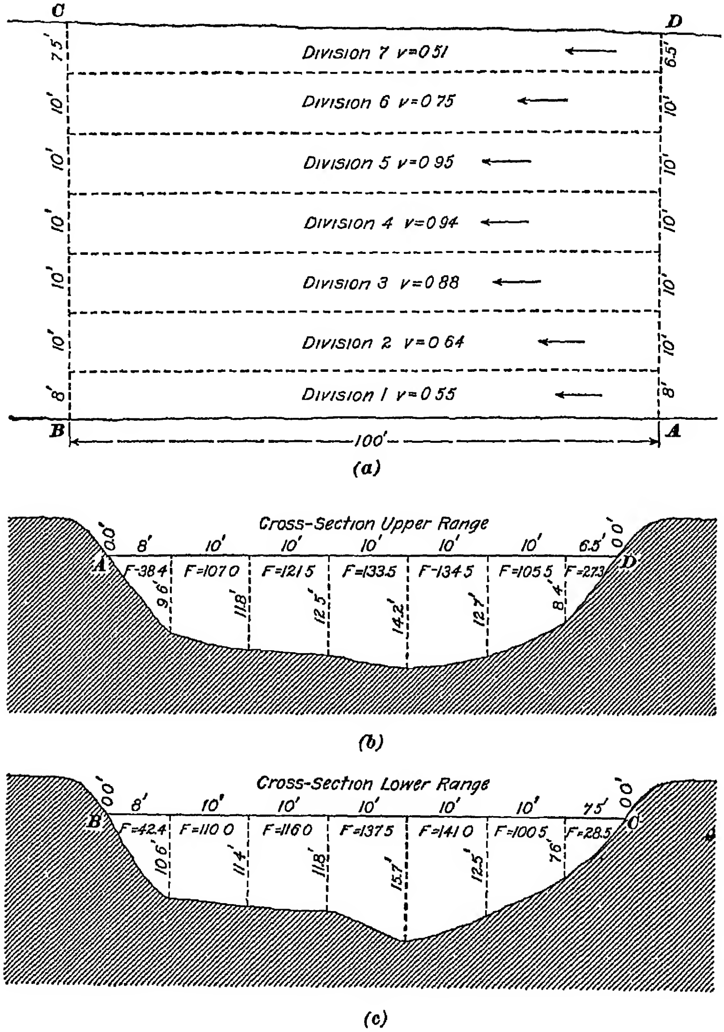

FIG 15

signals the moment each float crosses it, or the observer may start his watch when a float passes the upper range then

walk quickly to the lower range, and note the time when the float passes A stop-watch is preferable for timing the observations, but an ordinary watch may be used

48. Another method is to have a transit on each range with an observer at each transit, who notes the instant each float crosses the vertical cross-wire This method is to be preferred, especially for large rivers, when the two instruments are available, since the transits can be used not only to observe when the float crosses each range, but also to locate the exact position where it crosses each range The method of observation is as follows

Let *A B*, Fig 16, be the measured base, and *A D* and *B C* the upper and lower range lines, respectively, and assume that one transit is set up at *A* and another at *B* The vernier

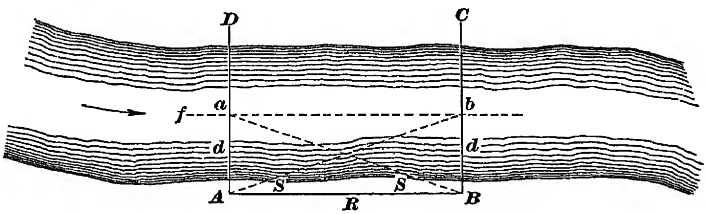

Fig 16

of the transit at *A* is set at zero, the telescope is directed to *B*, the opposite end of the base line, and the instrument is clamped in this position An angle of 90° is then turned off on the horizontal circle, and the telescope is directed along the range *A D* ready to observe when the float crosses this range The vernier of the transit at *B* is set at zero, the telescope directed to *A* and the instrument clamped, and the vernier then unclamped The float is then put into the water above the range *A D*, the telescope of the transit at *B* is directed toward it, and the line of sight is kept on the float, as the latter approaches the range, by carefully turning the transit in azimuth The transitman at *A* observes and calls out or signals the instant the float crosses the upper range *A D* At that instant the transitman at *B* stops turning the transit and reads the angle, which is recorded by a recorder, who

also notes the time. This transitman then directs his telescope along the range $B\,C$, and clamps the instrument in this position, in readiness to observe the instant the float crosses this range. Meanwhile, the transitman at A unclamps the vernier, sights again at B, and as a check notes if the vernier still reads zero, then, with the vernier unclamped, he directs the telescope toward the float, and by turning the transit carefully in azimuth keeps the line of sight on the float as it approaches the lower range. The transitman at B observes and calls out or signals the instant the float crosses the lower range $B\,C$, when the transitman at A ceases turning the transit and reads the angle, which is recorded with the time.

Suppose that the float is put in the water at f, some point above the upper range, and that the dotted line $f\,a\,b$ represents its path as it floats down to the lower range. As it crosses the upper range at a it is observed by both transits, the transit at A is sighted on the range $A\,D$, the transit at B is sighted on the line $B\,a$ and measures the angle $A\,B\,a$. This fully locates the point a, for the distance $A\,a$ is equal to the length of the base $A\,B$ multiplied by the tangent of the angle $A\,B\,a$. The float is likewise observed by both transits as it crosses the lower range $B\,C$. In this case, the transit at B is directed along the range and the transit at A measures the angle $B\,A\,b$.

Let S denote the measured angle at either end of the base, and R the length of the base, as shown in the figure. Then, the distance d along the range from the other end of the base to the point where the float crosses the range is given by the following trigonometric formula:

$$d = R \tan S$$

It should be observed that, although the same notation is used for the triangles $A\,B\,b$ and $B\,A\,a$, they are not necessarily equal, that is, the angles denoted by S and the distances denoted by d may not be equal.

Two boats are usually required in making the observations, one above the upper range to put the floats in the water, and one below to recover the floats after they have passed the lower range.

The area of the cross-section on each range is determined by taking soundings at the points of division, as explained in *Hydrographic Surveying*. These soundings and distances on each range are plotted, thus determining the form of the cross-sections on the upper and lower ranges, as shown in Fig. 15 (*b*) and (*c*). The area of each division of the cross-section is then computed by multiplying its breadth by its mean depth as determined by the soundings. The mean depth of a division is equal to one-half the sum of the depths of the soundings at the two points of division between which it lies. The area of each division is marked on the division in the plot of the cross-section, as shown in Fig. 15 (*b*) and (*c*).

If the division points between corresponding divisions in the upper and lower ranges are joined by straight lines, as shown in Fig. 15 (*a*), the course, or part of the stream over which the measurements are to be taken, will be divided into a number of divisions corresponding to those of the cross-sections on the upper and lower ranges. The mean of the areas of the two corresponding divisions of the cross-sections on the upper and lower ranges is then taken as the mean cross-sectional area of the division of the course in which they are situated.

The observations are repeated as many times as may be considered necessary, and the average of the results observed in each division is taken as the true time required for the float to traverse that division. The velocity of flow, in feet per second, is computed for each division by dividing the length, in feet, of the course, by the time, in seconds, required for the float to traverse the course. In Fig. 15 (*a*), the relative velocities are represented graphically by means of arrowheads, which in the several divisions are located at distances from the upper range proportional to the observed velocities.

Having found the velocity of float in each division of the course, a coefficient of reduction is applied to determine the mean velocity of the division, as will be explained further on. The discharge for each division of the course is then calculated by substituting in the formula for discharge $Q = Fv$

the values obtained for the area and the mean velocity of that division The total discharge of the stream is the sum of the partial discharge thus found

49. Rod Floats —Rod floats consist of wooden rods or hollow tin cylinders of uniform size, as shown in Fig 17, weighted at the lower end so as to float nearly vertical The rod should float with its lower end as near the bottom as possible, without touching at any point, and if it is to be used in deep water it should be of adjustable length or arranged so that it can be spliced It is best to use a number of tin tubes, about 2 inches in diameter, and of such different lengths that each tube will be of suitable length to float in some division of the cross-section whose velocity is to be measured, with just enough above the water surface to be plainly seen The lower ends of the tubes are filled with sand or shot until they float at the required depth with their lower ends only a short distance above the bottom, as shown in Fig 17 The immersion of each rod should be at least nine-tenths the depth of the water in the division in which it floats The velocities of the rod floats are determined in the same manner as described for surface floats

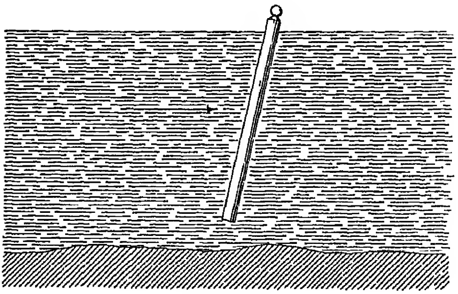

Fig 17

The velocity of a rod float is approximately the mean velocity of the vertical section of the stream in which the float moves The closeness with which the observed results approach the actual mean velocities will depend largely on the smoothness and regularity of the channel and on how nearly the immersed length of each tube approximates the full average depth of the water in the division in which it floats, that is, how near to the bottom of the stream its lower end floats

50. Subsurface, or Double, Floats.—The velocities of streams are sometimes observed by means of double floats, each of which consists of a submerged float connected to a surface float by a cord Such floats are called **subsurface, or double, floats** The submerged float should be heavy enough to sink and at the same time present as large a surface to the water, in proportion to weight, as possible It is maintained at the required depth by means of a fine cord, preferably of woven silk, attached to a surface float that should be of minimum surface and resistance Such a combination is shown in Fig 18 The submerged float *A* consists of two sheets of tin or galvanized sheet metal, fastened together at right angles, as shown in plan at *C* A small weight *D* is attached to the bottom to assist in keeping the float in a vertical position, and in some cases the sheets have cylindrical air cavities along their upper edges

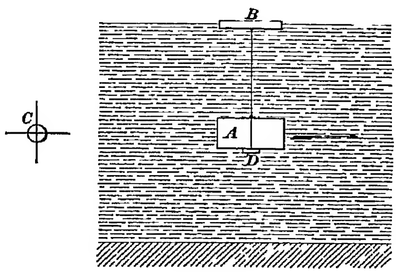

FIG 18

The velocity of the current at any depth can be determined by sinking the lower float to that depth and observing the time required for it to pass over a measured course This determination will be only approximate, however, since the velocity of the lower or submerged float will be to some extent affected by that of the upper or surface float At considerable depths the cord will present some surface to the action of the current, and will in some degree affect the velocity of the lower float

51. The Coefficient of Reduction —It was stated in Art 48 that the observed velocities in a stream, as determined by floats, are not the mean velocities of the sections in which the floats move To determine the mean velocity of any section from the observed velocity of a float in that section, it is necessary to apply a **coefficient of reduction.**

Let v = mean velocity of any longitudinal section,
v' = observed velocity of float in that section,
c = coefficient of reduction

Then, the formula for determining the mean velocity from the observed velocity is

$$v = c\,v'$$

For surface floats, c = 80
For double, or subsurface, floats, c = 90
For rod floats, c = 98

EXAMPLE —Two ranges on a stream are 200 feet apart It is found that a subsurface float traverses the distance between the two ranges in 2 minutes and 35 seconds What is the mean velocity of flow in that part of the stream?

SOLUTION —The time, expressed in seconds, is $2 \times 60 + 35 = 155$ sec The observed velocity is found to be $200 \div 155 = 1\ 29$ ft per sec, closely Hence, according to the formula, the mean velocity is equal to

$9 \times 1\ 29 = 1\ 161$ ft per sec Ans

EXAMPLES FOR PRACTICE

1 A surface float traverses the distance between two ranges that are 150 feet apart in 1 minute and 40 seconds What is the mean velocity of flow in that part of the stream? Ans 1 20 ft per sec

2 A rod float, placed in mid-stream, traverses the distance between two ranges 300 feet apart in 2 minutes and 50 seconds What is the mean velocity of flow in that part of the stream?
Ans 1 72 ft per sec

3 The velocity of a division of a stream, as determined by a subsurface float, is 2 03 feet per second What is the mean velocity of flow for that part of the stream? Ans 1 83 ft per sec

4 The velocity of a division of a stream as determined by a surface float is 1 78 feet per second If the mean cross-section is 107 2 square feet, what is the discharge for that part of the stream?
Ans 152 6 cu ft per sec

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52. Recording Observations —The accompanying field notes, No 1002, are a record of the soundings on the ranges AD and BC of Fig 15 (a), Art **47.** In the first column is shown the number of each sounding, in the

second column is given the distance from the water's edge to each division point of the cross-section, as measured from the bank of the stream adjacent to the base line, in the third column the distances between adjacent division points

FIELD NOTES No 1002

SOUNDINGS ON UPPER RANGE *A D* BEAR CREEK

Number	Distance From Edge of Water	Distance From Preceding Sounding	Depth	Remarks
1	0.0	0.0	0.0	Edge of water, left bank
2	8.0	8.0	9.6	Measurements are in feet and tenths
3	18.0	10.0	11.8	
4	28.0	10.0	12.5	
5	38.0	10.0	14.2	
6	48.0	10.0	12.7	
7	58.0	10.0	8.4	
8	64.5	6.5	0.0	Edge of water, right bank

SOUNDINGS ON LOWER RANGE *B C* BEAR CREEK

Number	Distance From Edge of Water	Distance From Preceding Sounding	Depth	Remarks
1	0.0	0.0	0.0	Edge of water, left bank
2	8.0	8.0	10.6	
3	18.0	10.0	11.4	
4	28.0	10.0	11.8	
5	38.0	10.0	15.7	
6	48.0	10.0	12.5	
7	58.0	10.0	7.6	
8	65.5	7.5	0.0	Edge of water, right bank

are given, and in the fourth column the depths of soundings are shown The distances to the division points are made

the same on each range, and, consequently, the corresponding divisions on each range are of the same length, except the last division, adjacent to the farther bank In this case, it is assumed that rod floats are used

The accompanying field notes, No 1003, are a record of the float observations For each observation, the number of the division of the cross-section is given in the first

FIELD NOTES No 1003

FLOAT OBSERVATIONS, BEAR CREEK

Number of Division	Time of Passage Seconds	Velocity of Float Feet per Second	Mean Velocity Feet per Second	Remarks
1	182	55	54	Lower range 1,500 feet above iron bridge
2	155¾	64	63	Ranges 100 feet apart
3	113½	88	86	Floats used, tin cylinders, 2 inches in diameter
4	106½	94	92	Velocities taken to hundredths
5	105¼	95	93	
6	132¾	75	74	Coefficient of reduction for mean velocity = 98
7	195	51	50	Weather cloudy, no wind

column, the time taken for the float to pass over the distance between the ranges in the second column, the velocity of the float in each division, in feet per second, in the third column, and the mean velocity, in feet per second, in the fourth column Since, as has been stated, a rod float was used in these observations, for which the coefficient of reduction is 98, the mean velocities in the fourth column are obtained by multiplying those in the third column by 98 The velocities are carried only to two decimal places, as this is considered to be as close as is justified by observations made with floats

Discharge Measurement

Bear Creek, Carbon Co, Pa, Aug 14, 1902

Number of Division	Upper Range			Lower Range			Mean Area Square Feet	Mean Velocity Feet per Second	Discharge Cubic Feet per Second	Remarks
	Mean Depth Feet	Width Feet	Area Square Feet	Mean Depth Feet	Width Feet	Area Square Feet				
1	4 8	8	38 4	5 3	8	42 4	40 4	54	21 82	Lower range 1,500 feet
2	10 7	10	107 0	11 0	10	110 0	108 5	63	68 36	above iron bridge
3	12 15	10	121 5	11 6	10	116 0	118 75	86	102 13	Ranges 100 feet apart
4	13 35	10	133 5	13 75	10	137 5	135 5	92	124 66	Floats used, tin cylinders
5	13 45	10	134 5	14 1	10	141 0	137 75	93	128 11	2 inches in diameter
6	10 55	10	105 5	10 05	10	100 5	103 0	74	76 22	Coefficient of reduction
7	4 2	6 5	27 3	3 8	7 5	28 5	27 9	50	13 95	for mean velocity = .98
									535 25	

53. Calculating the Discharge.—Having determined the area of any part or division of the cross-section of a stream, and the mean velocity of flow in that division, the discharge can be calculated by means of the general formula

$$Q = Fv$$

This formula may be applied to the total area and mean velocity of the entire stream, if these can be determined, for the purpose of determining the total discharge. But, under the usual conditions of discharge measurement, it is more accurate to apply it to the area and velocity as observed for each separate division of the cross-section, in which case the total discharge of the stream is equal to the sum of the discharges of the several divisions.

From the field notes of the soundings and of the float observations, the values are tabulated, and the discharge in each division of the cross-section is calculated as shown in the table on page 51, which will be readily understood from the explanations that have been given. The values written in the tenth column are the discharges for the several divisions of the cross-section, and their sum is the total discharge of the stream, in cubic feet per second.

THE PITOT TUBE

54. General Description.—The **Pitot tube**, invented by Pitot in 1730, and later modified by other hydraulicians, is an instrument used for determining the velocity of rivers and other streams. Although not adapted to very accurate work, it is very convenient, owing to its simplicity and handiness, for rapid approximate determinations. A rough sketch of the instrument is shown in Fig. 19. It consists of two communicating glass tubes A and B, one of which, B, is straight and of uniform diameter, while the other, A, is bent at its lower end to form a right angle, and is drawn to a fine point, as shown at P. Both tubes are open at their lower extremities, and at their common upper end they communicate with a short tube J, which can be opened or closed by a cock S. Another cock S_1 works two valves, one in each

tube, which can be opened or closed simultaneously Both tubes are graduated in inches and fractions, or in tenths and hundredths of a foot, or in any other convenient units, the 0 of the graduations being near the lower ends of the tubes, as shown Movable verniers V are used in connection with these graduations

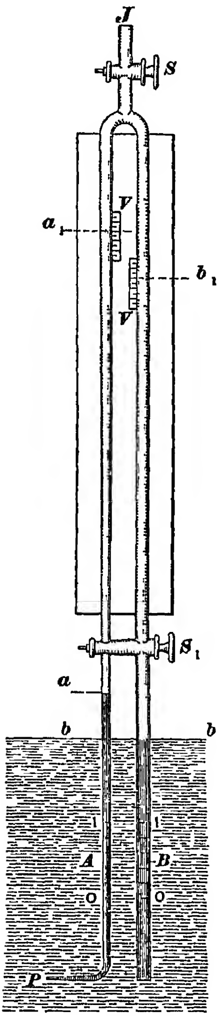

FIG. 19

55. Operation and Theory.—To determine the velocity at any point of a stream, the valves S and S_1 are opened, and the instrument, which is attached to a frame, is immersed in the water so that the straight tube B will be vertical, and the point P of the bent tube will face the current The water will then rise in the straight tube to a height that is approximately that of the water in the stream, that is, the surface of the water in that tube will be practically on a level with the water $b\,b$ in the stream outside In the bent tube, the water will rise to a point whose height depends on the velocity of flow Theoretically, the height $b\,a$ is the head due to the velocity v, so that, if there were no resistances, the velocity would be equal to $\sqrt{2g \times b\,a}$ In order that the difference $b\,a$ between the elevations of the water in the two tubes may be conveniently read, the operator sucks out some air from the tubes A and B by applying his mouth at the small tube J, he then closes the cock S Owing to the formation of a partial vacuum, the water rises to a_1 and b_1 in the two tubes, respectively, the distance $b_1\,a_1$ being equal to $b\,a$ The valves S_1 are then closed, the instrument is taken out of the water, and the heights of the columns in the two tubes

are read The difference between the reading of the tube A and that of the tube B gives the height due to the velocity

Owing to slight variations of velocity in the stream, the columns of water in the tubes A and B will fluctuate noticeably Before closing the valves S_1, the operator observes the tube A until the water reaches its maximum height, he then closes the valves S_1, takes the instrument out, and reads both tubes After this, he again immerses the instrument, opens the valves S_1, and observes the water in the tube A until it reaches its least height, when he closes the valves S_1 and takes the readings as before

If a_1 and a_1' represent, respectively, the greatest and the least height of the water in the tube A, and b_1 and b_1' represent the corresponding heights of the water in the tube B, and d_1 and d_1' represent the respective differences in the readings of the two tubes, then,

$$d_1 = a_1 - b_1, \text{ and } d_1' = a_1' - b_1'$$

The mean difference of reading, or velocity head h_1, is given by the equation

$$h_1 = \frac{d_1 + d_1'}{2}$$

Substituting the values of d_1 and d_1',

$$h_1 = \frac{a_1 - b_1 + a_1' - b_1'}{2} = \frac{a_1 + a_1' - (b_1 + b_1')}{2} \qquad (1)$$

That is, the value of the mean head h_1 is equal to one-hal the difference between the sum of the readings in the tube A and the sum of the readings in the tube B

56. For greater accuracy, several observations at differ ent times should be made at any point for which the velocit is to be determined Two readings should be taken for eac tube at every observation, and the mean of the resultin differences taken as the value of h to be substituted in th formula for velocity

If the greatest and the least readings for the tube A, for number of observations made at different times, are respec ively represented by a_1, a_2, a_3, etc., and a_1', a_2', a_3', etc

the corresponding readings of the tube B by b_1, b_2, b_3, etc., and b_1', b_2', b_3', etc., and the mean heads for the observations by h_1, h_2, h_3, etc., then, from equation (1), Art. **55**,

$$h_1 = \frac{a_1 + a_1' - (b_1 + b_1')}{2}$$

$$h_2 = \frac{a_2 + a_2' - (b_2 + b_2')}{2}$$

$$h_3 = \frac{a_3 + a_3' - (b_3 + b_3')}{2}$$

Then, if the number of observations is denoted by n, the average head h for all the observations is given by the equation

$$h = \frac{h_1 + h_2 + h_3 + \ldots\ldots}{n}$$

Substituting the values of h_1, h_2, etc. just found,

$$h = \left[\frac{a_1 + a_1' - (b_1 + b_1')}{2} + \frac{a_2 + a_2' - (b_2 + b_2')}{2} + \frac{a_3 + a_3' - (b_3 + b_3')}{2} + \ldots\ldots\right] \div n$$

$$= [a_1 + a_1' + a_2 + a_2' + a_3 + a_3' + \ldots - (b_1 + b_1' + b_2 + b_2' + b_3 + b_3' + \ldots)] \div 2n$$

That is, the mean value for the head h, to be used in the formula for velocity, is equal to the sum of all the readings of the tube A minus the sum of all the readings of the tube B, divided by twice the number of observations.

If the sum of the readings of the tube A is denoted by Σa, and the sum of the readings of the tube B by Σb, and the velocity head by h, as usual, then,

$$h = \frac{\Sigma a - \Sigma b}{2n}$$

57. Formula for Velocity.—As previously stated, the theoretical velocity is given by the equation $v = \sqrt{2gh}$, but, owing to friction and other resistances, this does not give the true velocity, the quantity $\sqrt{2gh}$ must be multiplied by a constant coefficient c whose value must be determined experimentally for every instrument. The formula for velocity is, then,

$$v = c\sqrt{2gh} \qquad \textbf{(1)}$$

or, substituting the value of h from the preceding article,

$$v = c\sqrt{2\,g \times \frac{\Sigma a - \Sigma b}{2\,n}} = c\sqrt{\frac{g}{n}(\Sigma a - \Sigma b)} \qquad (2)$$

58. Rating the Instrument.—As in the case of a current meter, the operation of determining the constant c for any instrument is called **rating** the instrument. To accomplish this, readings are taken, in the manner already described, in a stream whose velocity is known, or in still water, the tube being moved at a given velocity.

The velocity v being known, and the value of h determined by substituting the values of the readings in the equation of Art. **56**, the value of c is readily determined from formula **1**, Art. **57**, which solved for c gives

$$c = \frac{v}{\sqrt{2\,g\,h}}$$

EXAMPLE 1.—Readings on the tubes A and B of a Pitot tube held in a stream whose velocity is known to be 3 2 feet per second were taken and recorded as shown in the accompanying table. The number of the observations is recorded in column n. In column a

n	a Feet	a' Feet	b Feet	b' Feet
1	65	61	49	53
2	68	62	47	52
3	69	66	45	48
	2 02	1 89	1 41	1 53
		2 02		1 41
		3 91		2 94

are recorded the values a_1, a_2, etc., in column a', the values a_1', a_2', etc., and in columns b and b', the corresponding readings of the tube B. It is required to find the constant c for that tube.

SOLUTION.—The equation of Art. **56** is applied in finding h. Here $\Sigma a = 65 + 68 + 69 + 61 + 62 + 66 = 3\ 91$, $\Sigma b = 49 + 47 + 45 + 53 + 52 + 48 = 2\ 94$, and $n = 3$. Then,

$$h = \frac{3\ 91 - 2\ 94}{2 \times 3} = 162$$

To determine c, the formula of Art 58 is applied In this case, $v = 3\ 2$, $h = 162$, and $g = 32\ 16$ Then,

$$c = \frac{3\ 2}{\sqrt{2 \times 32\ 16 \times\ 162}} = 98, \text{ approximately}$$

EXAMPLE 2 —The coefficient c of a Pitot tube is 98, and the readings of the tubes A and B, taken at a certain point in a stream, are given in the accompanying table It is required to determine the velocity of the stream

n	a Feet	a' Feet	b Feet	b' Feet
1	52	50	38	40
2	57	55	32	33
3	56	53	30	34
	1 65	1 58	1 00	1 07
		1 65		1 00
		3 23		2 07

SOLUTION —The velocity v is found by substituting known values in formula 2, Art 57 In this case, $c = 98$, $g = 32\ 16$, $\Sigma a = 52 + 57 + 56 + 50 + 55 + 53 = 3\ 23$, $\Sigma b = 38 + 32 + 30 + 40 + 33 + 34 = 2\ 07$, and $n = 3$ Then,

$$v = 98\sqrt{\frac{32\ 16}{3} \times (3\ 23 - 2\ 07)} = 3\ 46 \text{ ft per sec} \quad \text{Ans}$$

EXAMPLES FOR PRACTICE

1 A Pitot tube was held in a stream whose velocity was 2 8 feet per second Determine the constant c for this instrument, the readings on the tubes A and B being as given in the accompanying table

n	a Inches	a' Inches	b Inches	b' Inches
1	13 0	12 5	11 3	11 7
2	13 1	13 0	11 4	11 9
3	13 3	12 7	10 8	11 4
4	13 2	12 9	11 5	11 8

Ans 98

2 The coefficient c of a Pitot tube is 97, and the readings of the tubes A and B, taken at a certain point in the stream, are given in the accompanying table Determine the velocity of the stream

Ans 3 08 ft per sec

n	a Inches	a' Inches	b Inches	b' Inches
1	11 0	10 4	9 5	9 1
2	11 5	11 0	9 1	8 6
3	11 6	11 1	9 8	9 2
4	11 4	10 9	9 6	9 0

THE DISCHARGE TABLE AND RECORD GAUGE

59. Discharge Table.—With the rise and fall of the water surface in a stream, there is a corresponding increase or decrease in the discharge If the water of the stream is to be utilized for water-power, water supply, or any other purpose, it is usually necessary to determine the discharge of the stream at the highest and lowest stages of the water, and also its average discharge If a series of discharge measurements have been made in a given cross-section of a stream at different stages of the water, the results should be tabulated for reference, with a record of the mean velocity, volume, and gauge reading for each measurement

Fig 20 shows the cross-section of a canal with five stages of water For convenience, the heights of the different stages are assumed to vary by intervals of exactly 2 feet, which would seldom, if ever, be the case in actual observations The bottom width of the canal is assumed to be 10 feet, and the slope of each bank to be 45°, or 1 horizontal to 1 vertical This causes the width to increase 4 feet for each 2 feet increase of depth The areas of the cross-section of the water at the successive stages are therefore as follows.

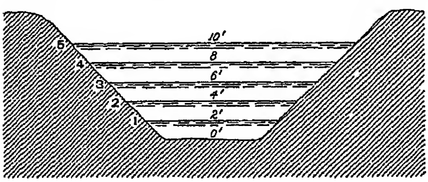

Fig 20

For a depth of 2 feet, $F_1 = \frac{10 + 14}{2} \times 2 = 24$ square feet

For a depth of 4 feet, $F_2 = \frac{10 + 18}{2} \times 4 = 56$ square feet.

For a depth of 6 feet, $F_3 = \frac{10 + 22}{2} \times 6 = 96$ square feet

For a depth of 8 feet, $F_4 = \frac{10 + 26}{2} \times 8 = 144$ square feet

For a depth of 10 feet, $F_5 = \frac{10 + 30}{2} \times 10 = 200$ square feet

DISCHARGE TABLE FOR CANAL SECTION

Number of Observation	Depth of Water Feet	Sectional Area Square Feet	Mean Velocity Feet per Second	Discharge Cubic Feet per Second
1	2	24	1 00	24 00
2	4	56	1 469	82 264
3	6	96	1 813	174 048
4	8	144	2 095	301 680
5	10	200	2 345	469 00

These areas, as thus calculated for the various depths, are shown in the third column of the accompanying table, which is called a **discharge table** The mean velocities for the various depths are given in the fourth column

From the area of the cross-section and the mean velocity for each stage of the water, the discharge is calculated, the result being written in the fifth column of the table Such a table is useful for determining the discharge at any depth within the limits of the observations The recording gauge, which will be described later, is largely used in connection with a discharge table

60. The Discharge Curve —The results entered in the discharge table, as described in the preceding article, can be plotted on cross-section paper with the depths of water, or gauge heights, as ordinates and the discharges as abscissas

This is illustrated in Fig 21 If a curve is drawn through these points as plotted, it will represent the discharge of the stream at different heights Such a curve is called the **discharge curve** for the stream in which the observations are taken From this curve, the discharge of the stream at any stage within the limits of the observations can be ascertained

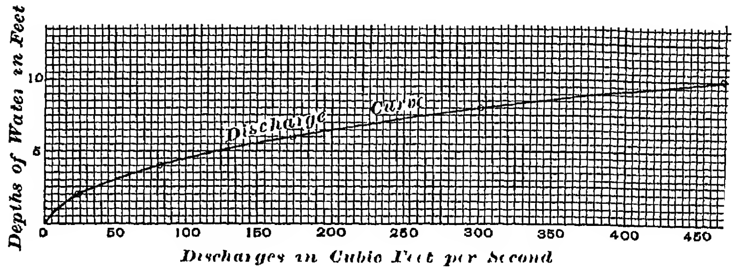

FIG 21

by merely determining the depth of water from the gaug reading and finding that point on the discharge curve whos ordinate represents the depth of the water The abscissa t the curve at the same point will represent the discharge (the stream at the given stage

The increase or decrease in the discharge per unit of ri or fall is a variable quantity depending on the velocity ar the sectional area of the stream It will be seen, by refe ence to the table in Art **59,** that the velocity increases decreases as the depth increases or decreases, but not pı portionately Since this is true of any stream, it may stated that there is no method for accurately determini the discharge of a stream at different stages except witl the limits of observed values

61. The Recording Gauge —In the practical and leg questions constantly confronting the hydraulic engineer, su as the flow of water through orifices, over weirs, the pov determination of pumping engines and motors, and the fl in streams and rivers, the gauging of the water at frequ intervals is of great importance For this purpose, an ing ious and accurate device, called a **recording gauge,** is v

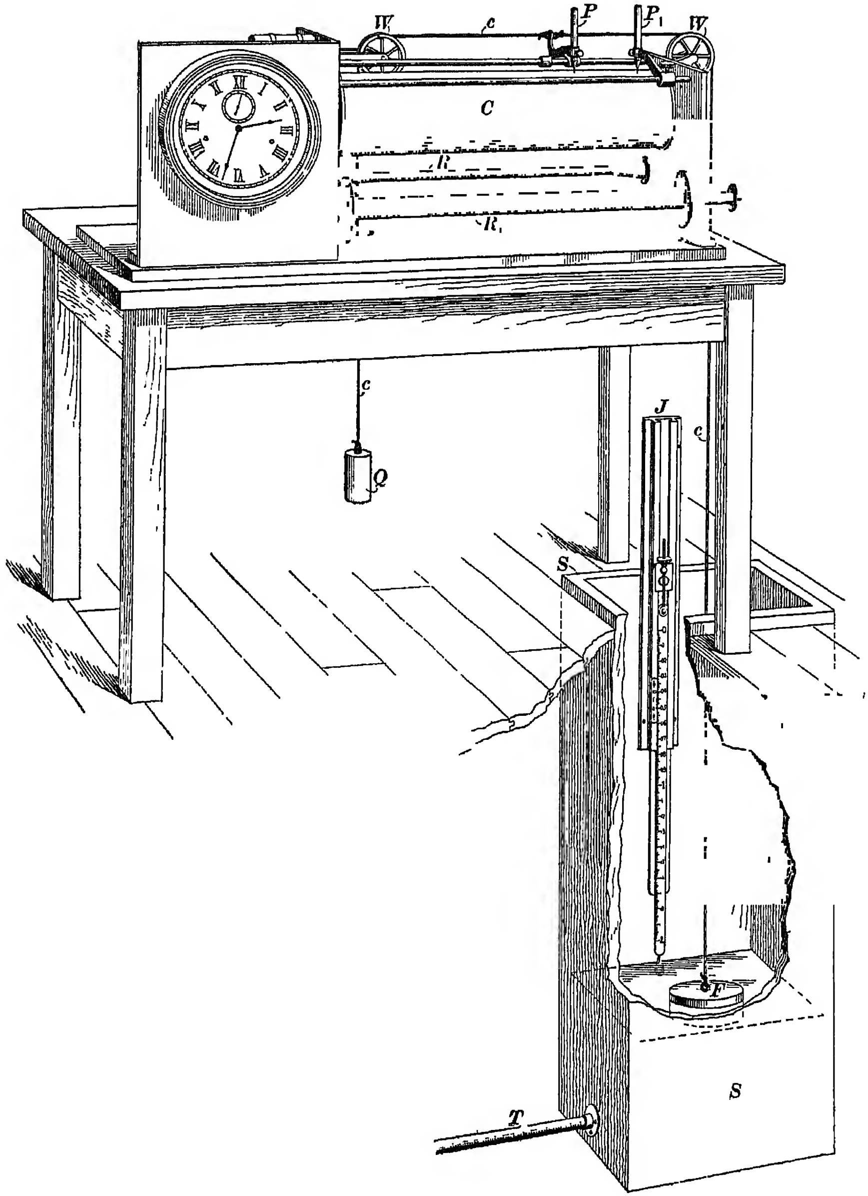

Fig 22

extensively used. This instrument is represented in Fig. 22. It is usually placed on a table in a small house or shed built for the purpose. A roll of paper, usually 100 feet long and 2 feet wide, is wound on a roller R. The loose end of the roll is passed over a drum or cylinder C and fastened to another roller R_1. The cylinder and rollers are moved at a uniform rate by clockwork so that the paper, as it unwinds from the roller R, winds on the roller R_1. A pipe T leads the water from the body of water to be measured to a still box SS, located directly under the instrument. Through a hole in the floor of the room, a float F is let down on the water in the still box. This float is fastened to a chain or rope cc, which passes over two wheels W, W, and carries a weight Q at its other extremity. The float F and weight Q are so adjusted that, when the water sinks in the box SS, the float sinks with it and pulls up the weight, and when the water rises, the float rises with it, and the weight sinks. To the chain or rope is fastened, between the wheels W, W, a pencil P that rests on the cylinder C and travels in one direction or the other, according as the float F rises or sinks. It will be seen that all fluctuations of the water level in the still box are recorded on the revolving roll; thus, if the water level falls 2 inches, the float F will drop 2 inches, the chain cc will move on the wheels 2 inches, and the pencil point P will move to the right 2 inches.

Let it be assumed that the recording gauge is used in connection with a weir, although it may be used for determining the variations of flow for any stream for which a discharge table has been constructed. A hook gauge J, set at zero, is securely fastened to a side of the still box or float box and, by means of an engineers' level, the point of the hook is set at exactly the same elevation as the crest of the weir. A reading is then taken of the water level in the float box, by means of the hook gauge. Assume this reading to be .5 foot, or 6 inches; this shows that 6 inches of water is flowing over the crest of the weir. A distance of 6 inches is then measured along the cylinder C to the right of the position occupied by the pencil P at the time

of the observation, and another pencil point P_1 is fixed or clamped to an independent rod that will hold it in position As the cylinder revolves, the point P_1 describes a line, and it is obvious that the distance of P from this line at any time indicates the height of the water above the weir, or the head h over the weir

In Fig 23 is shown a portion of a sheet taken from a record roll The vertical ordinates represent the elevations of the water at each of the 24 hours of a day The irregular line traced by the float pencil P enables the observer to determine the exact stage of water at any given period by

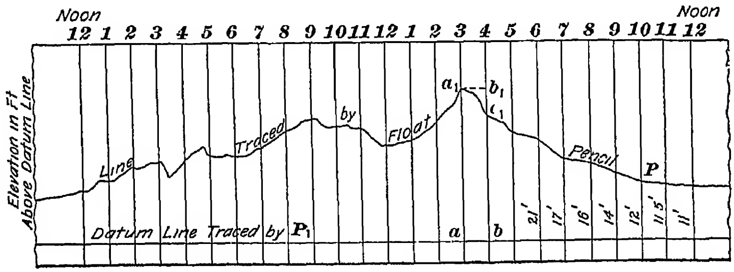

FIG 23

using the corresponding ordinate to the curved line For example, if the height on the weir had remained stationary between 3 and 4 o'clock in the morning this record was taken, the float would have remained stationary also, and, as the clock would have rolled the paper the distance $a\,b$, the float pencil would have described the line $a_1\,b_1$, but, as the stream was falling, the irregular line $a_1\,c_1$ was described instead

62. A discharge curve having been previously prepared (see Art **60**), the discharge for an hour, a day, or a month may be easily determined from the gauge If it is desired to ascertain the number of gallons that have passed over the weir from 6 o'clock in the morning to noon of the day on which this record was taken, the ordinates numbered from 6 to 12 inclusive are measured Let these ordinates be 21, 17, 16, 14, 12, 11 5, and 11, respectively,

then, the mean of the heights at 6 and 7 o'clock is taken, or $\frac{21 + 17}{2} = 19$. Consulting the discharge curve that has been constructed from the weir formula, where the curve gives the discharge in gallons per hour, or in million gallons per 24 hours, the average discharge is observed for this 1-hour period. Proceeding, similarly, for each succeeding hourly period, an average hourly discharge is determined, and the sum of these six average hourly discharges will give the total average discharge for the 6 hours.

Another method, a little less exact, but closely approximate, is to add together the varying elevations, as scaled from the roll, divide the sum by the number of elevations used, and find from the discharge curve or table the discharge corresponding to this average head or height.

If a clock with an 8-day attachment is used, the apparatus needs attention but once a week. The paper comes in rolls long enough to last 1 month.

GENERAL REMARKS

COMPARISON OF METHODS

63. Method by the Pitot Tube.—The Pitot tube is not much used at the present time. As already stated, it is well adapted to rapid work when great accuracy is not required, but it is inconvenient to handle in varying or considerable depths, and capillary attraction makes close readings difficult.

64. Method by Floats.—For a rough determination, the surface float may be advantageously used; it is cheap and convenient, and gives fairly approximate results.

Double floats give closer results than surface floats, but are expensive when many observations are to be taken, to which must be added the uncertainty as to the position of the lower float, on account of the action of the current on the cord.

Rod floats are greatly to be preferred to other floats, as they give a much closer approximation to the mean velocity

They should be adjusted to run as near to the bottom as possible and project slightly above the water level They may be used for streams of any depth The floats used by Francis in his Lowell experiments were tin tubes 2 inches in diameter, soldered together and weighted with lead at the lower end By the addition or removal of a little lead, the rods were made to sink to nearly the desired depth One advantage of the rod float is that it is unaffected by floating grass, silt, or other substances

65. Method by a Weir —Although the weir is the standard of measuring devices, it has its limitations Expense of construction often forbids its use, and a 20-foot weir with a 2-foot head, discharging about 200 cubic feet a second, is as large as is usually built, and as large as the formulas so far proposed can be safely followed

66. Method by the Current Meter —The current meter is, at the present time, the ideal instrument for the measurement of flowing water, particularly beyond the practical uses for which the weir can be used It measures the velocity of a stream in a much simpler manner than can be done by the use of floats, it is only necessary to measure one cross-section of the stream, and both field book and other computations are less than when floats are used If the stream is small and unimportant, so that it is not necessary that a high degree of accuracy be attained in the measurements, the mean velocity can be determined by placing the instrument at different depths over the entire cross-section For very accurate work, several readings should be taken at each of several points and depths

It should be borne in mind that, on account of the diameter of its vanes, the current meter can be held no nearer the surface or bottom than 6 inches, and that a velocity of at least 1 foot per second is necessary for accurate results The meter should be rated frequently.

VARIATIONS OF VELOCITY IN A CROSS-SECTION

67. Velocities at Different Points of a Cross-Section.—The velocity of flow in a stream varies both from the sides of a bed toward the center, and, in any vertical line, from the surface to the bottom. As a rule, the maximum velocity occurs in that part of the cross-section where the depth is greatest, and at a short distance from the surface. Cases, however, have been observed in which the maximum velocity is the surface velocity. The least velocity occurs near the sides of the bed, and the average velocity, as a rule, a short distance below the middle of the deepest part of the cross-section.

68. The Mean Velocity.—From a series of careful experiments, E. C. Murphy concludes that "the distance of the thread of mean velocity below the surface increases with the depth and with the ratio of the depth to the width." This distance varies, according to Mr. Murphy, from 50 to 65 of the depth. "In a broad shallow stream," he adds, "from 3 to 12 inches in depth and having a sand or fine gravel bed, the thread of mean velocity is from 50 to 55 of the depth below the surface. In broad streams from 1 to 3 feet in depth, and having gravelly beds, the thread of mean velocity is from 55 to 60 of the depth below the surface. The single-point method of measuring the velocity, by holding the center of the meter 58 of the depth below the surface, will give good results. In ordinary streams, where the depth varies from 1 foot to 6 feet, the thread of mean velocity is about 6 below the surface." This value, obtained as a mean of 378 observed values, can be taken to represent the average depth of mean velocity under the conditions that are most likely to occur in practice.

69. The all-important factor to determine in the measurements of flowing water is *mean velocity*. From the preceding it appears (1) that a fairly approximate mean velocity of the flow of a stream may be determined from a single reading in the deepest part of the cross-section at a

depth of .60 below the surface, (2) that a closer approximation may be obtained by finding the mean of a number of readings taken at .60 of the depth, (3) that where absolute accuracy is desired, a number of readings must be taken along different verticals of the cross-section, so as to cover the entire cross-section in both directions, vertically and horizontally. Readings should be taken along each vertical line, beginning at a distance of 6 inches below the surface and ending at a distance of 6 inches above the bottom. The sum of all the readings divided by their number gives the required mean velocity.

70. Fluctuations of Velocity.—Not only does the velocity change from point to point in the same cross-section, but its value at any one point is not constant; it fluctuates between values that are sometimes noticeably different from each other. Such fluctuations are very plainly shown by a current meter held for some time at one point of the stream. It follows from this that, for very accurate determinations, several readings should be taken at each point, or, if a current meter is used, it should be placed at each point for a considerable length of time—say ½ hour or 1 hour—before taking the reading.

TABLE I

VALUES OF THE COEFFICIENT OF ROUGHNESS

Character of Channel	Value of n
Clean well-planed timber	009
Clean, smooth, glazed iron and stoneware pipes	010
Masonry smoothly plastered with cement, and for very clean smooth cast-iron pipe	011
Unplaned timber, ordinary cast-iron pipe, and selected pipe sewers, well laid and thoroughly flushed	012
Rough iron pipes and ordinary sewer pipes laid under the usual conditions	013
Dressed masonry and well-laid brickwork	015
Good rubble masonry and ordinary rough or fouled brickwork	017
Coarse rubble masonry and firm compact gravel	020
Well-made earth canals in good alinement	0225
Rivers and canals in moderately good order and perfectly free from stones and weeds	025
Rivers and canals in rather bad condition and somewhat obstructed by stones and weeds	030
Rivers and canals in bad condition, overgrown with vegetation and strewn with stones and other detritus, according to condition	035 to 050

TABLE II

CONSTANTS TO BE USED IN THRUPP'S FORMULA

Character of Bed	m	x	y
Wrought-iron pipes	208 80	650	556
Riveted sheet-iron pipes	176 24	677	548
New cast-iron pipes	187 00	670	541
	148 20	630	500
Lead pipes	191 42	620	571
Pure cement	250 00	670	575
	155 55	610	513
Brickwork (smooth)	129 10	610	500
Brickwork (rough)	113 06	625	500
Unplaned plank	118 33	615	500
Small gravel in cement	84 67	660	500
Large gravel in cement	70 67	705	500
Hammer-dressed masonry	89 53	660	500
Earth (no vegetation)	65 10	720	500
Rough stony earth	46 64	780	500

TABLE III

COEFFICIENTS OF DISCHARGE FOR WEIRS WITH END CONTRACTIONS

Effective Head, in Feet	Length of Weir, in Feet						
	66	1	2	3	5	10	19
1	632	639	646	652	653	655	656
15	619	625	634	638	640	641	642
20	611	618	626	630	631	633	634
25	605	612	621	624	626	628	629
30	601	608	616	619	621	624	625
40	595	601	609	613	615	618	620
50	590	596	605	608	611	615	617
60	587	593	601	605	608	613	615
70		590	598	603	606	612	614
80			595	600	604	611	613
90			592	598	603	609	612
1 00			590	595	601	608	611
1 2			585	591	597	605	610
1 4			580	587	594	602	609
1 6				582	591	600	607

NOTE —The head given is the effective head, $H + \frac{4}{3} h$ When the velocity of approach is small, h is neglected

TABLE IV

DISCHARGES FOR GIVEN DEPTHS OVER EACH LINEAR FOOT OF WEIR WITH END CONTRACTIONS SUPPRESSED

To be Used in Connection With Francis's Formula

Head From Still Water in Feet = H	$H^{\frac{3}{2}}$	Cubic Feet per Second for 1 Foot Length of Crest	Head From Still Water in Feet = H	$H^{\frac{3}{2}}$	Cubic Feet per Second for 1 Foot Length of Crest	Head From Still Water in Feet = H	$H^{\frac{3}{2}}$	Cubic Feet per Second for 1 Foot Length of Crest
04	0080	0261	46	3120	1 0386	1 2	1 3145	4 3904
05	0112	0365	48	3326	1 1072	1 3	1 4822	4 9506
06	0147	0480	50	3536	1 1771	1 4	1 6565	5 5327
07	0185	0604	52	3750	1 2483	1 5	1 8371	6 1341
08	0226	0738	54	3968	1 3209	1 6	2 0239	6 7576
09	0270	0881	56	4191	1 3951	1 7	2 2165	7 3987
10	0316	1032	58	4417	1 4724	1 8	2 4150	8 0611
11	0365	1195	60	4648	1 5475	1 9	2 6190	8 7421
12	0416	1361	62	4882	1 6286	2 0	2 8284	9 4413
13	0469	1536	64	5120	1 7080	2 1	3 0432	10 1581
14	0524	1718	66	5362	1 7888	2 2	3 2631	10 8924

15	0581	1906	68	5607	1 8705	2 3	3 4881	11 6280
16	0640	2102	70	5857	1 9540	2 4	3 7181	12 3960
17	0701	2303	72	6109	2 0380	2 5	3 9528	13 1788
18	0764	2510	74	6366	2 1237	2 6	4 1924	13 9773
19	0828	2721	76	6626	2 2104	2 7	4 4366	14 7915
20	0894	2938	78	6889	2 2996	2 8	4 6853	15 6208
22	1032	3407	80	7155	2 3883	2 9	4 9385	16 8486
24	1176	3882	82	7426	2 4788	3 0	5 1962	17 3239
26	1326	4377	84	7699	2 5699	3 1	5 4581	18 1809
28	1482	4892	86	7975	2 6620	3 2	5 7243	19 0676
30	1643	5445	88	8255	2 7557	3 3	5 9948	19 9687
32	1790	5999	90	8538	2 8500	3 4	6 2693	20 8830
34	1983	6572	92	8824	2 9455	3 5	6 5479	21 8110
36	2160	7158	94	9114	3 0432	3 6	6 8305	22 7525
38	2342	7761	96	9406	3 1407	3 7	7 1171	23 7071
40	2530	8384	98	9702	3 2395	3 8	7 4076	24 5710
42	2722	9020	1 00	1 0000	3 3390	3 9	7 7019	25 5472
44	2919	9672	1 10	1 1537	3 8522	4 0	8 0000	26 5360

TABLE V

COEFFICIENTS OF DISCHARGE FOR WEIRS WITHOUT END CONTRACTIONS

Effective Head, in Feet	Length of Weir, in Feet						
	19	10	7	5	4	3	2
10	657	658	658	659			
15	643	644	645	645	647	649	652
20	635	637	637	638	641	642	645
25	630	632	633	634	636	638	641
30	626	628	629	631	633	636	639
40	621	623	625	628	630	633	636
50	619	621	624	627	630	633	637
60	618	620	623	627	630	634	638
70	618	620	624	628	631	635	640
80	618	621	625	629	633	637	643
90	619	622	627	631	635	639	645
1 00	619	624	628	633	637	641	648
1 2	620	626	632	636	641	646	
1 4	622	629	634	640	644		
1 6	623	631	637	642	647		

NOTE —The head given is the effective head, $H + 1\ 4\ h$ When the velocity of approach is small, h may be neglected

WATERWHEELS

(PART 1)

INTRODUCTION

ENERGY, WORK, EFFICIENCY, HEAD

1. Energy and Work of Water.—Let a weight W of water be at rest at a height h above any plane of reference. Then (see *Kinematics and Kinetics*, and *Hydraulics*, Part 1), with respect to that plane, the water possesses, by virtue of its position, an amount of potential energy equal to Wh. If the water falls freely through the height h, without doing any work, its potential energy is all transformed into kinetic energy. The latter energy is numerically equal to the potential energy Wh, and may be expressed either by this product or by the product $\frac{Wv^2}{2g}$, denoting by v the final velocity of the water, and by g the acceleration of gravity (= 32.16 feet per second). In general, if a weight W of water is moving with the velocity v, its kinetic energy is $\frac{Wv^2}{2g}$, and the water can, by losing all its velocity, perform work (although not necessarily useful work) equal to this kinetic energy.

If, at any instant, the weight W of water has a velocity v_0 and is at a distance h above a plane of reference, the total energy E of the water, with respect to that plane of

reference, is equal to the sum of the potential and the kinetic energy, or

$$E = Wh + \frac{Wv_0^2}{2g} = W\left(h + \frac{v_0^2}{2g}\right) \qquad (1)$$

It is here assumed that the water is not under pressure, as otherwise the pressure energy must be taken into account (see *Hydraulics*, Part 1)

If, while falling through the distance h, the water is made to do work on a machine, such as a water motor, the total work U done, including all work that is not useful, is equal to the difference between the original total energy E and the kinetic energy left in the water This lost energy is equal to $\frac{Wv_1^2}{2g}$, denoting by v_1 the velocity of the water after the latter has fallen through the distance h Therefore,

$$U = E - \frac{Wv_1^2}{2g},$$

or, replacing the value of E from formula **1**,

$$U = W\left(h - \frac{v_1^2 - v_0^2}{2g}\right) \qquad (2)$$

If v_0 is greater than v_1, the formula may be written in the more convenient form

$$U = W\left(h + \frac{v_0^2 - v_1^2}{2g}\right) \qquad (3)$$

EXAMPLE —A mass of water weighing 62 5 pounds enters a motor with a velocity of 20 feet per second, and leaves the motor at a point 16 feet below the point of entrance with a velocity of 8 feet per second Required the work done by the water on the motor, all losses, such as those due to friction, being included

SOLUTION —Here W = 62 5 lb , h = 16 ft , v_0 = 20 ft per sec , and v_1 = 8 ft per sec Therefore, by formula **3**,

$$U = 62\,5 \times \left(16 + \frac{20^2 - 8^2}{2 \times 32\,16}\right) = 1{,}326\,5 \text{ ft -lb} \quad \text{Ans}$$

2. Power of Water.—As explained in *Kinematics and Kinetics*, 1 horsepower is equivalent to 550 foot-pounds of work performed in 1 second If a stream of water falls continuously through a height of h feet, discharging W pounds per second, its potential energy, per second, is Wh foot-

pounds As this is the work that the water can perform, per second, in falling through the distance h, the horsepower H of the water is given by the equation

$$H = \frac{W h}{550} \qquad (a)$$

Let Q be the discharge of the stream, in cubic feet per second, and w the weight, in pounds, of 1 cubic foot of water Then, $W = w\,Q$, and equation (a) becomes

$$H = \frac{w\,Q\,h}{550} = \frac{02\,w\,Q\,h}{11} \qquad (1)$$

In nearly all practical computations, it is customary to take w as 62 5 pounds This value will be used here, unless another value is expressly stated Replacing w by 62 5 in formula **1**, that formula becomes

$$H = \frac{1\,250\,Q\,h}{11} = 1136\,Q\,h \qquad (2)$$

The following form, obtained by multiplying the two terms of the fraction $\frac{1\,250}{11}$ by 8, is often more convenient

$$H = \frac{10\,Q\,h}{88} \qquad (3)$$

3. Sometimes, the discharge is given in gallons per minute Let G be the discharge so expressed Then, since 1 cubic foot = 7 48 gallons, and the discharge in cubic feet per minute is 60 Q, we have,

$$G = 60\,Q \times 7\,48,$$

whence

$$Q = \frac{G}{7\,48 \times 60}$$

The substitution of this value in formula **3** of the preceding article gives, after reduction,

$$H = 0002532\,G\,h$$

Example 1 —What is the theoretical horsepower of a stream discharging 12 cubic feet per second through a height of 125 feet?

Solution —Here, $Q = 12$, $h = 125$, and formula **3**, Art **2**, gives

$$H = \frac{10 \times 12 \times 125}{88} = 170\,5 \text{ H P} \quad \text{Ans}$$

Example 2 —What is the theoretical horsepower of a stream discharging 5,400 gallons per minute through a height of 120 feet?

SOLUTION.—Here, $G = 5{,}400$, $h = 120$, and the formula in this article gives

$$H = .0002532 \times 5{,}400 \times 120 = 164.1 \text{ H. P.} \quad \text{Ans.}$$

4. Head.—In connection with a water motor, the following definitions are convenient.

The **total head** is the difference in elevation between the surface of the water in the source of supply and the surface of the water as it leaves the motor or its accessories.

The **effective head** is that part of the total head of which the motor actually makes use. It is equal to the total head minus the head lost in friction and otherwise in the headrace, and includes whatever pressure and velocity heads the water may have on entering the motor.

The stream of water leaving a motor or its accessories is called the **tailrace**, a name applied also to the channel or conduit by which the water is carried away. Sometimes, it is not convenient or advisable to place a motor so that its lowest point will be at the level of the tailrace. In any case, the total and effective heads are measured to the surface of the tailrace.

The conduit by which the water is brought directly to the motor is called the **headrace, flume,** or **penstock.**

5. Efficiency.—By the **efficiency** of a water motor is ordinarily meant the ratio of the energy that the motor can transmit or deliver to other machinery, in a certain time, to the energy actually supplied to the motor in the same time. The latter energy is the energy due to the effective head of the water. This efficiency is often called the **net efficiency** and the **commercial efficiency.**

The **gross efficiency,** or **total efficiency,** of a water motor is the ratio of the energy that the motor can transmit or deliver, in a certain time, to the total energy due to the total head of the water acting on the motor during the same time. This is properly the efficiency of the whole plant, rather than of the motor.

Efficiency is customarily designated by the Greek letter η (*eta*, pronounced "ay'ta").

Let h = total head on a motor, in feet;
h_1 = effective head, in feet,
Q = water supplied to the motor in cubic feet per second,
H = horsepower developed by motor,
η = net efficiency of motor,
η' = gross efficiency

The total horsepower of the water, due to the head h, is $\frac{10\,Q\,h}{88}$ (Art 3), and the horsepower due to the head h_1 is $\frac{10\,Q\,h_1}{88}$ Therefore,

$$\eta = \frac{H}{\frac{10\,Q\,h_1}{88}} = \frac{8.8\,H}{Q\,h_1} \qquad (1)$$

$$\eta' = \frac{H}{\frac{10\,Q\,h}{88}} = \frac{8.8\,H}{Q\,h} \qquad (2)$$

It will be observed that

$$\eta' = \eta \times \frac{h_1}{h} \qquad (3)$$

When the efficiency, the discharge, and the head are given, the horsepower is obtained by solving formula 1 for H, which gives

$$H = \frac{\eta\,Q\,h_1}{8.8} = \frac{10\,\eta\,Q\,h_1}{88} \qquad (4)$$

It is customary to express efficiency as a certain per cent of the power or energy supplied to the motor Thus, if η = .81, the efficiency is expressed as 81 per cent This mode of expression should be borne in mind in all applications of the foregoing formulas

6. The total energy of the water Q entering the motor is $w\,Q\,h_1$, denoting, as usual, by w the weight of 1 cubic foot of water Let the water leave the motor with a velocity of v feet per second Then, the energy carried away by the water is $\frac{w\,Q}{2g} \times v^2$, so that the energy spent on the motor is

$$w\,Q\,h_1 - \frac{w\,Q}{2g} \times v^2 = w\,Q\left(h_1 - \frac{v^2}{2g}\right)$$

A great deal of this energy is lost in friction, eddies, etc., and, besides, there is usually some water lost by leakage. Assuming the ideal condition in which none of these losses would take place, the ideal efficiency of the motor would be

$$\frac{w\,Q\left(h_1 - \dfrac{v^2}{2g}\right)}{w\,Q\,h_1} = 1 - \frac{v^2}{2g\,h_1}$$

No motor can have an efficiency greater than, nor even equal to, this value. This ideal efficiency, as well as the actual efficiency, is greater the smaller the velocity v (an otherwise evident fact, since, the less the velocity v, the less is the energy carried away by the water.)

EXAMPLE 1.—The effective head on a water motor being 25 feet, the water supplied, 50 cubic feet per second, and the power developed by the motor, 95 H. P., what is the efficiency of the motor?

SOLUTION.—Here $H = 95$, $Q = 50$, and $h_1 = 25$. Therefore, by formula **1**, Art. **5**,

$$\eta = \frac{8.8 \times 95}{50 \times 25} = .669 = 66.9 \text{ per cent. Ans.}$$

EXAMPLE 2.—The efficiency of a motor being 72 per cent. ($\eta = .72$), and the effective head 20 feet, what must be the supply of water in order that the motor may develop 80 horsepower?

SOLUTION.—Formula **1**, Art. **5**, gives

$$Q = \frac{8.8\,H}{\eta\,h_1}$$

Substituting given values,

$$Q = \frac{8.8 \times 80}{.72 \times 20} = 48.89 \text{ cu. ft. per sec. Ans.}$$

EXAMPLE 3.—A turbine having an efficiency of 75 per cent. is used to operate a pump whose efficiency is 65 per cent. The effective head being 10 feet, and the supply of water 125 cubic feet per second, required (*a*) the horsepower developed by the turbine, (*b*) the horsepower developed by the pump, (*c*) the number of gallons of water per minute that the pump can raise to a height of 100 feet.

SOLUTION.—(*a*) To apply formula **4**, Art. **5**, we have $\eta = .75$, $Q = 125$, and $h_1 = 10$. Therefore,

$$H = \frac{10 \times .75 \times 125 \times 10}{88} = 106.53 \text{ H. P. Ans.}$$

(*b*) Since the horsepower developed by the turbine is 106.53, of which the pump utilizes only .65, the power H_1 of the pump is

$$.65 \times 106.53 = 69.24 \text{ H. P. Ans.}$$

(*c*) Since the power developed by water falling through a certain height is the same as the power required to raise the water to the same height, the formula in Art **3**, solved for G, may be used Replacing H by H_1, or 69 24, and h_1 by 100, we have

$$G = \frac{H_1}{0002532\ h_1} = \frac{69\ 24}{0002532 \times 100} = 2{,}735 \text{ gal} \quad \text{Ans}$$

EXAMPLES FOR PRACTICE

1 Water enters a motor at the rate of 50 cubic feet per second with a velocity of 15 feet per second, and leaves it with a velocity of 8 feet per second Assuming that the water falls through a distance of 30 feet, find the total horsepower delivered by the water to the motor
Ans 184 68 H P

2 What is the efficiency of a motor that develops 270 horsepower with a supply of 80 cubic feet per second and an effective head of 45 feet?
Ans 66, or 66 per cent

3 What is the theoretical horsepower of a stream that discharges 25,000 gallons per minute through a height of 100 feet?
Ans 633 H P

4 The total head on a motor is 20 feet, of which 10 per cent is lost in overcoming resistances in the headrace If the net efficiency of the motor is 72 per cent , what is the gross efficiency?
Ans 64 8 per cent

WATER SUPPLY FOR POWER

7. The amount of water power available at a given site is proportional both to the head and to the quantity of water supplied in a given time If the yield of the stream from which the supply is drawn is variable, gaugings should be made to determine the proper basis of development If it is necessary that the power shall be constant in amount, the development must be limited to the least flow that is constantly available, or else auxiliary power in some form, as steam engines, must be used in periods of deficiency

Some industries require permanent power, while others, such as wood-pulp grinding, can utilize irregular power

It often pays to develop a stream beyond the limit of necessary permanent power, and to sell the surplus at a reduced rate

Where there is storage or pondage available, so that a large volume of water can be impounded during periods of surplus and drawn off at will during periods of deficiency, the permanent power of a stream can be increased beyond the limit of the minimum flow of the stream If the pond above a dam that supplies a water motor is utilized as a storage reservoir, only a slight depth at the surface can as a rule be utilized, because in drawing out the impounded water the head is reduced

8. Pondage —Storage of water for power is usually called **pondage**, and is most commonly employed in connection with motors that run in the daytime only, the power

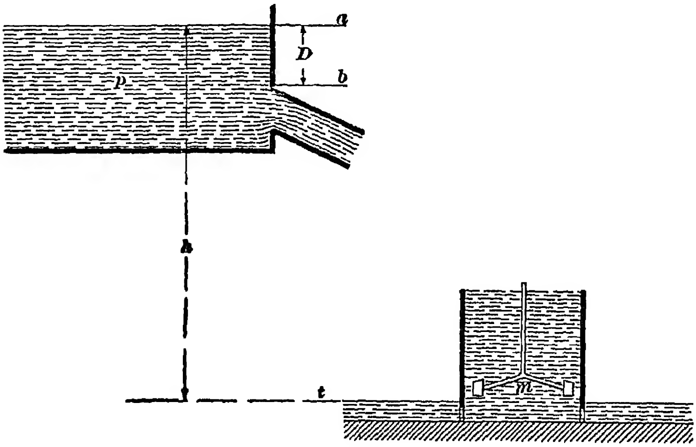

Fig 1

stored while the motors are idle being utilized during the working hours

Referring to Fig 1, where p is a pond and m shows the location of the motors, let

N = number of hours that the motors run per day,

Q = water supplied by the stream to the pond, in cubic feet per second,

A = average area, in square feet, of horizontal cross-section of pond,

D = depth, in feet, that pond is drawn down during working hours,

D_1 = most economical depth of draught, or depletion, in feet,

h = maximum head, or head before draught begins, in feet,

H = average horsepower obtainable from the *water* (not from the *motors*)

Before the draught begins, the weight of the available water is $w A D$ The potential energy of this water, with reference to the level of the tailrace t, is equal to the work required to lift the weight $w A D$ from the level t to the position $a b$ In the latter position, the center of gravity of the water is at a distance $\frac{D}{2}$ above b, or $h - \frac{D}{2}$ above t Therefore, the work required to raise the water from t to $a b$ is (see *Kinematics and Kinetics*)

$$w A D \times \left(h - \frac{D}{2}\right)$$

This is the same as the work that $A D$ cubic feet of water can do in falling through the distance $h - \frac{D}{2}$ As the motors use this water in N hours, or 3,600 N seconds, the average quantity used per second is $\frac{A D}{3{,}600 N}$ cubic feet The horsepower represented by this water is therefore (Art 2),

$$\frac{10 \times \frac{A D}{3{,}600 N} \times \left(h - \frac{D}{2}\right)}{88} = \frac{10 A D\left(h - \frac{D}{2}\right)}{88 \times 3{,}600 N}$$

To this must be added the power due to the inflowing water It can be easily shown by means of the principle of kinetics stated above, that the power of this water is the same as if the quantity Q fell through the distance $h - \frac{D}{2}$ The horsepower due to this water is, then (see Art 2),

$$\frac{10 Q\left(h - \frac{D}{2}\right)}{88}$$

For the power H we have, therefore,

$$H = \frac{10 A D\left(h - \frac{D}{2}\right)}{88 \times 3{,}600 N} + \frac{10 Q\left(h - \frac{D}{2}\right)}{88}$$

$$= \frac{10\left(h - \frac{D}{2}\right)}{88} \times \left(\frac{A D}{3{,}600 N} + Q\right) \qquad (1)$$

If the average quantity of water that the motors use per second is denoted by Q_1, then

$$Q_1 = \frac{A D}{3{,}600 N} + Q,$$

and formula **1** may be written

$$H = \frac{10\left(h - \frac{D}{2}\right)Q_1}{88} \qquad (2)$$

Since $h - \frac{D}{2} = \frac{1}{2} \times [h + (h - D)]$, and h and $h - D$ are respectively, the greatest and the least head available (see Fig 1), formula **2** shows that the average power obtainable from the water is the same as the power due to the total available water and a head that is a mean between the greatest and the smallest head used

9. It appears from formula **1** of Art **8** that the greater the depth D to which the water is drawn, the greater will be the available volume $\frac{A D}{3{,}600 N}$, and, therefore, Q_1 But, as an increase in D causes a corresponding decrease in the average head $h - \frac{D}{2}$, the horsepower H does not always increase by increasing D It can be shown that D_1, the value of D for which H is greatest, is given by the following formula.

$$D_1 = h - \frac{1{,}800 N Q}{A} \qquad (1)$$

It is obvious from this formula that, if h is equal to or less than $\frac{1{,}800 N Q}{A}$, no storage is necessary

If the value of D_1 is to be used, the value of h cannot be assumed arbitrarily, as it may happen that the flow of the

stream during the $(24 - N)$ hours that the motors are idle is not sufficient to make up the depletion D_1 The proper value of h is determined as follows Assuming that the inflow during $(24 - N)$ hours is just sufficient to make up for the depletion D_1, we must have

$$(24 - N) \times 3{,}600\, Q = D_1 A,$$

or, replacing the value of D_1 from formula **1**,

$$3{,}600\,(25 - N)\,Q = h\,A - 1{,}800\,N Q,$$

whence $$h = \frac{1{,}800\,(48 - N)\,Q}{A} \qquad (2)$$

In all cases, the flow $3{,}600\,(24 - N)\,Q$ must be at least equal to $D\,A$, and, therefore, D must not be greater than

$$\frac{3{,}600\,(24 - N)\,Q}{A}$$

In applying the foregoing formulas, losses due to evaporation should be deducted from Q

It is assumed in the formulas that the capacity of the motors is sufficient to utilize all the flow resulting from the depletion D_1

EXAMPLE 1 —What should be the mean rate of draught through the motors in order to obtain the greatest amount of power during 10 hours a day at a mill where the head h_1 with a full pond is 20 feet, the area of the pond being 5 acres, and the mean inflow Q being 200 cubic feet per second?

SOLUTION —Here $A = 5 \times 43{,}560 = 217{,}800$ sq ft Formula **1**, Art **9**, gives

$$D_1 = 20 - \frac{1{,}800 \times 10 \times 200}{217{,}800} = 3\ 47 \text{ ft}$$

Therefore (Art **8**),

$$Q_1 = \frac{217{,}800\left(20 - \dfrac{1{,}800 \times 10 \times 200}{217{,}800}\right)}{3{,}600 \times 10} + 200 = 221 \text{ cu ft per sec} \quad \text{Ans}$$

EXAMPLE 2 —In the preceding example, what will be the average horsepower developed by the motors, if their efficiency is 75 per cent ?

SOLUTION —By formula **2**, Art **8**,

$$H = \frac{10\left(20 - \dfrac{3\ 47}{2}\right) \times 221}{88}$$

As the efficiency is 75, the power developed by the motors is

$$\frac{10\left(20 - \dfrac{3\ 47}{2}\right) \times 221 \times\ 75}{88} = 344 \text{ H P} \quad \text{Ans}$$

10. Distributed Flow —If the capacity of the waterwheels exceeds the minimum flow of a variable stream, the total amount of hydraulic power that can be developed in any year may be considered as made up of two parts (1) the power available when the plant operates at full capacity, and (2) the power available during periods of deficient water supply If the mean flow of the stream during any period of N days is Q cubic feet per second, the same amount of water, if distributed uniformly throughout the year, would yield a constant supply of $\frac{NQ}{365}$ cubic feet per second This quantity is called the **distributed flow** resulting from the mean flow for the given period The **average available supply** is the sum of the distributed flows corresponding to the full capacity of the plant during periods of sufficient supply and the distributed flow during the period of deficiency

Let Q_1 = capacity of the plant, in cubic feet per second,
Q_2 = mean flow, in cubic feet per second, on days of deficient supply,
Q_a = average available supply, in cubic feet per second,
N = number of days of full supply in the year

Then,

$$Q_a = \frac{NQ_1}{365} + \frac{(365 - N)Q_2}{365} = Q_2 + \frac{N(Q_1 - Q_2)}{365}$$

EXAMPLE 1 —In a plant having turbines whose capacity is 300 cubic feet per second, what is the average available supply in a year when the flow of the stream exceeds 300 cubic feet per second on 200 days, and averages 250 cubic feet per second during the remainder of the year?

SOLUTION —Here $Q_1 = 300$, $Q_2 = 250$, and $N = 200$ Substituting these values in the above formula,

$$Q_a = 250 + \frac{200\,(300 - 250)}{365} = 277\ 4 \text{ cu ft per sec} \quad \text{Ans}$$

11. Flow of a Stream —When a stream is to be used for power, both its minimum and its maximum flow should be determined, as well as the average available flow for the proposed development A gauging record covering several

years and including wet, dry, and ordinary years is desirable Gaugings, if not previously made, should usually be started as soon as the investigation is begun Incorrect estimates of the yield of a stream, owing to inadequate records, are a frequent source of commercial failure of water powers

12. Survey of Site—A complete survey of the site should be made in order to determine the best location and form of construction for the dam, raceways, power house, and other structures Such a survey usually includes taking the topography of the dam site and reservoir flow, mapping the lands required, and making borings and soundings to determine the character of the foundations for structures

13. Estimates of Cost.—An estimate of the cost of construction and development should be made, usually for each of several possible plans In estimating cost, facilities for transporting materials, and the location of available timber, stone, sand, and other materials should be considered, together with the cost of the necessary land and water rights

The value of the power and the returns that may be expected should be estimated The net return represents the difference between the gross return and the sum of the operating charges, which include interest on investment, insurance, taxes, cost of repairs and renewals, attendance and management, supplies, and incidentals

It is evident that, after all these expenses are met, there must still be a considerable margin to cover the cost of promotion and financing, and to insure against accidents, breakdowns, or failure to produce owing to unforeseen causes, such as bad management, financial depression, or the shifting of the business center

14. Use to be Made of Power—The power, when developed, may be sold to existing industries, which as a rule have steam-power plants already, or power at low rates may be offered as an inducement for the location of new industries, or the power may be utilized by its owners or developers In most cases, an assured profitable market for at least a part of the power is a prime requisite

ACTION OF A JET

ENERGY OF A JET

15. Let a, Fig 2 (a), be a vessel that is supplied with water in such a way that the head on the orifice o in one side of the vessel is constant Let v be the velocity in feet per

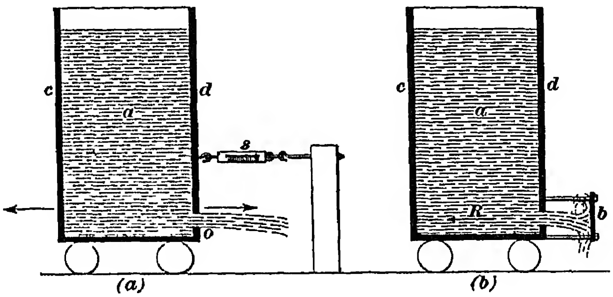

FIG 2

second of the water issuing from the orifice, W, the weight, in pounds, of water flowing out in 1 second, and K, the kinetic energy of this weight of water Then (see *Kinematics and Kinetics*),

$$K = \frac{Wv^2}{2g} \qquad (a)$$

If h is the head on the orifice, and c is the coefficient of velocity, then (see *Hydraulics*),

$$v = c\sqrt{2gh}$$

and, therefore, $$\frac{v^2}{2g} = c^2 h$$

This value substituted in equation (a) gives

$$K = c^2 W h \qquad (b)$$

If A is the area of the jet, in square feet, and w is the weight, in pounds, of a cubic foot of water, then,

$$W = w A v = w A c\sqrt{2gh}$$

Substituting the first of these values of W in equation (a) and the second in equation (b), the following formulas are obtained

$$K = \frac{w A v^3}{2 g} \qquad (1)$$

$$K = w A h c^3 \sqrt{2 g h} \qquad (2)$$

The weight w will, as usual, be taken as 62 5 pounds

EXAMPLE --What is the kinetic energy per second of a jet whose area is 1 square foot, if the head on the orifice is 25 feet, and the coefficient of velocity is 98?

SOLUTION —Here $A = 1$, $w = 62\ 5$, $h = 25$, $c = 98$, and $2g = 64\ 32$ Therefore, by formula 2,
$K = 62\ 5 \times 10 \times 25 \times 98^3 \sqrt{64\ 32 \times 25} = 5{,}897$ ft -lb per sec Ans

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PRESSURE OF A JET ON A FIXED SURFACE

16. General Formula.—Let a jet j, Fig 3, moving with a velocity v, impinge on a fixed surface $a\,b$ making an angle M with the direction of the jet When the jet strikes the surface at a, its direction is changed, it will be assumed that the surface is perfectly smooth, and the effect of eddies and other resistances will be neglected The water will therefore move along the vane with its velocity v undiminished Let the area of the orifice through which the jet issues be denoted by A

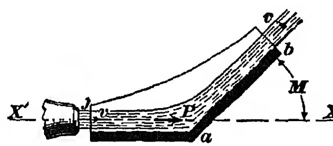

FIG 3

As shown in *Fundamental Principles of Mechanics*, the force F necessary to change, in a time t, the velocity of a body from v_0 to v_1 is given by the following equation, W being the weight of the body

$$F = \frac{W(v_0 - v_1)}{g\,t} \qquad (a)$$

At a, the velocity of the water is changed in direction This change is due to the pressure of the surface $a\,b$ on the water, which is equal and opposite to the pressure exerted by the water on the surface Let P be the magnitude of this pressure If the velocity along $a\,b$ is resolved into two components, one parallel and one perpendicular to the direction $X'X$ of the jet, the value of the former component is

$v \cos M$ Therefore, at the point a, the velocity in the direction $X'X$ is changed from v to $v \cos M$ Let t be the time in which this change takes place During this time, the weight of water passing through a is $w A v \times t$ Replacing, in equation (a), F by P, W by $w A v \times t$, v_0 by v, and v_1 by $v \cos M$, we get

$$P = \frac{w A v \times t (v - v \cos M)}{g t},$$

or, transforming,

$$P = \frac{w A v^2}{g} (1 - \cos M) \qquad (1)$$

Since $A v$ is the volume Q (cubic feet) of water delivered by the jet in 1 second, the value of P may be written in this other form

$$P = \frac{w Q v}{g} (1 - \cos M) \qquad (2)$$

This formula will give the pressure of a jet in the direction of the original motion of the jet The pressure is independent of the form of the surface for a given angle of deflection, but the jet must be confined at the sides to prevent its spreading, except in the case of a flat plate at right angles to the jet

17. Pressure on a Fixed Flat Vane at Right Angles to the Jet—If the angle M is 90°, as shown in Fig 4, then $\cos M = 0$, and formulas **1** and **2** of the preceding article become, respectively,

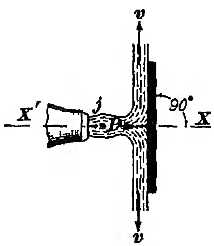

FIG 4

$$P = \frac{w A v^2}{g} \qquad (1)$$

$$P = \frac{w Q v}{g} \qquad (2)$$

The velocity v may be produced by a head h equal to $\frac{v^2}{2g}$ Therefore, $v^2 = 2 g h$, and

$$P = 2 w A h \qquad (3)$$

Now, $w A h$ is the weight of a column of water whose cross-section is equal to the area of the jet and whose height equals h It therefore follows that, with a coefficient of

velocity equal to unity, the pressure exerted on a flat surface by a jet issuing under a head h is twice the hydrostatic pressure that would be produced by the same head

18. Pressure on a Fixed Hemispherical Vane —In the case of a jet impinging on a hemispherical vane, as shown in Fig 5, the direction of motion of the jet is entirely reversed, or turned through an angle of 180° Since cos 180° = − 1, 1 − cos M = 2, and formulas **1** and **2** of Art **16** become, respectively,

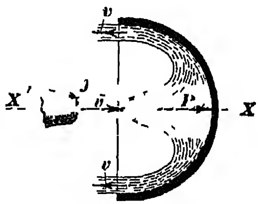

FIG 5

$$P = \frac{2\,w\,A\,v^2}{g} \qquad (1)$$

$$P = \frac{2\,w\,Q\,v}{g} \qquad (2)$$

In this case, the pressure equals four times that due to the velocity head h

EXAMPLE 1 —To find the pressure that can be exerted on a fixed flat surface by a jet 1 inch square issuing from an orifice under a head of 25 feet, the direction of the jet being perpendicular to the surface, and the coefficient of velocity being 97

SOLUTION —The equation $v = 97\sqrt{2gh}$ gives $\frac{v^2}{g} = 97^2 \times 2h = 97^2 \times 2 \times 25$ We have also, $w = 62\,5$ lb, $A = 1$ sq in $= \frac{1}{144}$ sq ft Substituting these values in formula **1**, Art **17**,

$$P = 62\,5 \times \frac{1}{144} \times 97^2 \times 2 \times 25 = 20\,4 \text{ lb} \quad \text{Ans}$$

EXAMPLE 2 —What pressure can the jet considered in example 1 exert on a vane inclined at 60° to the direction of the jet?

SOLUTION —The value of $\frac{w\,A\,v^2}{g}$ was found in the preceding example to be 20 4 lb Therefore, by formula **1**, Art **16**,

$$P = 20\,4(1 - \cos 60°) = 20\,4(1 - \tfrac{1}{2}) = 10\,2 \text{ lb} \quad \text{Ans}$$

REACTION OF A JET

19. Definition and Value of the Reaction —When the orifice o, Fig 2 (a), is closed, the pressures on the sides c and d of the vessel are equal and opposite, and there is no motion of the vessel If the orifice is opened, and the stream

is allowed to impinge on a plate *b*, as shown in Fig 2 (*b*), fastened to the vessel, and at right angles to the direction of the stream, the vessel will still remain at rest For this condition, the pressure P acting on b is $2\,w\,A\,h$, as shown in Art **17.** In order that there may be no motion, a force R, equal and opposite to P, must react on the vessel When the plate b is removed, the force P is no longer exerted on the vessel, the force R becomes unbalanced, and tends to move the vessel backwards This force R is called the **reaction of the jet**

20. Experimental Verification.—The effect of the reaction and pressure of a jet may be shown by experiment as follows Let the vessel be placed on rollers, as shown at *a*, Fig 2, in such a way that a very slight pressure will produce motion When the water issues from the orifice, the vessel will begin to move in the opposite direction If the vessel is prevented from moving by a spring *s*, this spring will show a pull equal to $\frac{w\,A\,v^2}{g}$, or $2\,w\,A\,h$, which is the value of the reaction R

PRESSURE AND WORK OF A JET ON MOVING VANES

21. Formulas for Pressure.—If the surface or vane *a b*, Fig 3, moves with a velocity v_1 in the same direction as the jet, the condition is the same as if the vane were at rest and the jet were moving with a velocity $v - v_1$ The water will strike the vane with a relative velocity equal to $v - v_1$, the quantity Q' of water striking the vane per second will be $A(v - v_1)$ cubic feet, and the pressure on the vane in the direction of motion may be found by substituting $(v - v_1)$ for v in the foregoing formulas That is (Art **16**),

$$P = \frac{w\,A\,(v - v_1)^2}{g}(1 - \cos M)$$

$$= \frac{w\,Q'\,(v - v_1)}{g}(1 - \cos M) \qquad (1)$$

For a plane surface at right angles to the jet (Art **17**),

$$P = \frac{w\,A\,(v - v_1)^2}{g} = \frac{w\,Q'\,(v - v_1)}{g} \qquad (2)$$

For a hemispherical vane (Art **18**),

$$P = \frac{2wA(v-v_1)^2}{g} = \frac{2wQ'(v-v_1)}{g} \qquad \textbf{(3)}$$

22. Work Done by the Jet on the Vane.—Since the vane moves in the direction of P through a distance of v_1 feet every second, the work U, per second, done on the vane by the jet is equal to Pv_1, or, replacing the value of P from formula **1**, Art **21**,

$$U = \frac{wAv_1(v-v_1)^2}{g}(1-\cos M)$$
$$= \frac{wQ'v_1(v-v_1)}{g}(1-\cos M)$$

23. Work of a Jet on a Series of Vanes—When a jet is made to impinge successively on a series of surfaces or vanes that come into the path of the jet one after another, as in the case of a waterwheel, all the water in the stream can be made to perform work on the vanes, for all the amount intercepted between each two vanes flows over, and does work on, the front vane while the water is impinging on the other vane Under such circumstances, the quantity of water striking the *system* per second is the quantity Q, or Av, issuing from the jet per second, and the work done per second is obtained by replacing in the preceding article Q' by Q This gives

$$U = \frac{wAvv_1(v-v_1)}{g}(1-\cos M)$$
$$= \frac{wQv_1(v-v_1)}{g}(1-\cos M) \qquad \textbf{(1)}$$

It can be shown by the use of advanced mathematics hat, other things being equal, the work U is a maximum when $v_1 = \frac{v}{2}$ Substituting this value in formula **1**, and denoting the maximum work by U_m, we have

$$U_m = \frac{wAv^3}{4g}(1-\cos M) = \frac{wQv^2}{4g}(1-\cos M) \qquad \textbf{(2)}$$

24. For flat vanes at right angles to the jet, in which the water leaves the vanes at right angles to its initial direction, $M = 90°$, $\cos M = 0$, and, therefore

$$U_m = \frac{w A v^3}{4g} = \frac{w Q v^2}{4g}$$

This represents one-half of the total energy of the water (see formula **1**, Art **15**) Therefore, a motor with vanes that deflect the jet through 90° cannot yield an efficiency of more than $\frac{1}{2}$, or 50 per cent

25. If the vanes are hemispherical cups, in which the direction of the water is reversed, as in Fig 5, $1 - \cos M = 1 - \cos 180° = 2$, and formula **2** of Art **23** becomes

$$U_m = \frac{w A v^3}{2g} = \frac{w Q v^2}{2g}$$

This is the total energy of the water (see Art **15**), and so, in this case, the efficiency of the cup is theoretically equal to 1 In practice, owing to several resistances and other conditions not taken into account in the derivation of the formula, so high an efficiency is not obtainable A very high efficiency, however, is obtained from impulse waterwheels, which are made with hemispherical vanes working on the principles just explained

EXAMPLE 1 —What is the maximum work done per second by a jet 3 inches square impinging with a velocity of 50 feet per second on a waterwheel having flat vanes placed at right angles to the jet?

SOLUTION —By the formula in Art **24**,

$$U = \frac{62\ 5 \times (\frac{3}{12})^2 \times 50^3}{4 \times 32\ 16} = 3{,}796 \text{ ft -lb per sec} \quad \text{Ans}$$

EXAMPLE 2 —What is the work done by a jet issuing from an orifice 3 inches in diameter under a head of 100 feet, and impinging on a waterwheel with hemispherical vanes moving at a speed of 30 feet per second, the coefficient of velocity being 97?

SOLUTION —Here $v = 97\sqrt{2 \times 32\ 16 \times 100} = 77\ 8$ ft per sec, $A = 7854 \times (\frac{3}{12})^2 = 0491$ sq ft, $v_1 = 30$ ft per sec, and $M = 180°$ Substituting these values in formula **1** of Art **23**,

$$U = \frac{62\ 5 \times 0491 \times 77\ 8 \times 30\ (77\ 8 - 30)(1 - \cos 180°)}{32\ 16}$$
$$= 21{,}291 \text{ ft -lb per sec} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1 What is the kinetic energy per second of a jet of water whose area is 15 square foot, if the head on the orifice is 50 feet, and the coefficient of velocity is 98? Ans 25,020 ft -lb per sec

2 The cross-section of a jet of water is 2 square inches, the jet moves with a velocity of 70 feet per second, and impinges on a fixed plane surface at right angles to its direction of motion What pressure does the jet exert? Ans 132 3 lb

3 If a jet of water 10 square inches in cross-section, moving with a velocity of 80 feet, impinges on a hemispherical cup that is moving with a velocity of 40 feet per second, what is the pressure exerted? Ans 431 9 lb

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26. Revolving Vanes.—In Fig 6 is shown a curved vane $a\,a'$ rotating about an axis o A jet impinges at a with an absolute velocity v in the direction shown, that is, making an angle M with the tangent $a\,b$ at a The absolute linear velocities of the vane at a and a' are denoted by v_1 and v_1', respectively The relative velocities of the water, with respect to the vane, are u and u' at a and a', respectively, making the angles L and L' with the tangents at those points The absolute velocity with which the water leaves the vane at a' is denoted by v', and the angle that it makes with the tangent $a'\,b'$ is denoted by M' The direction of u' is the same as the direction of the face of the vane at a' It is required to find the work done by the water in passing along the vane $a\,a'$ The data to be used are the velocity v, the weight W delivered to the vane per second, the radii r and r', the velocity v_1, and the angles M and L' The velocities v_1 and v_1' are to each other as the radii r and r', and, therefore,

$$v_1' = \frac{r'}{r} \times v_1 \qquad (1)$$

It is evident that the velocity v is the resultant of the velocity v_1 of the vane and the velocity u that the water has with respect to the vane This is plainly shown by the parallelogram of velocities $a\,b\,c\,d$ Similarly, v' is the resultant of v_1' and u'

The work U done on the vane, in foot-pounds per second, is given by the formula

$$U = \frac{W}{g}(v\,v_1 \cos M - v'\,v_1' \cos M') \qquad (2)$$

The derivation of this formula is rather complicated, and will here be dispensed with

In order to apply the formula, it is necessary to determine v' and M'

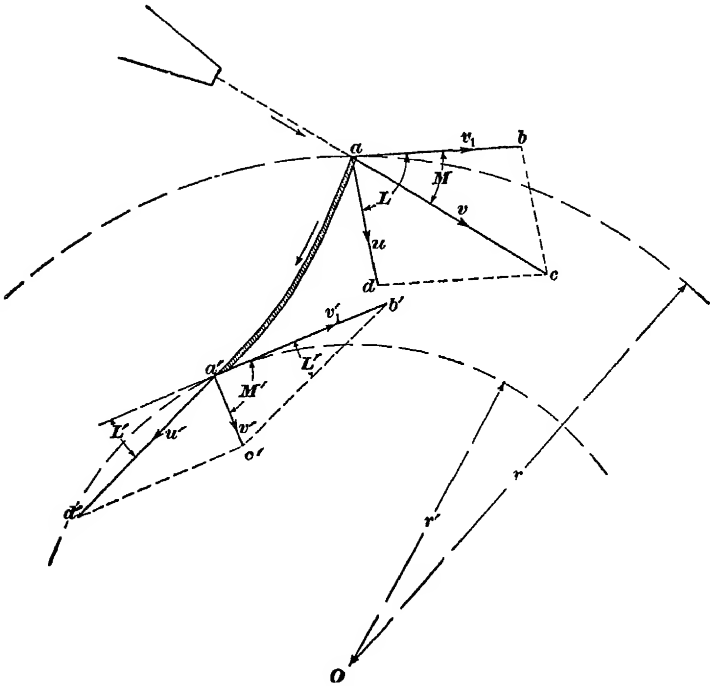

Fig 6

The relative velocity $u (= b\,c)$ is computed from the triangle $a\,b\,c$ by the formula

$$u = \sqrt{v^2 + v_1^2 - 2\,v\,v_1 \cos M} \qquad (3)$$

The relative velocity u' is computed by the following formula, which need not be derived here

$$u' = \sqrt{v^2 + v_1'^2 - 2\,v\,v_1 \cos M} \qquad (4)$$

The triangle $a'\,b'\,c'$ gives

$$v' = \sqrt{u'^2 + v_1'^2 - 2\,u'\,v_1' \cos L'} \qquad (5)$$

Also, $$\sin M' = \frac{u' \sin L'}{v'} \qquad (6)$$

If the vanes are closely spaced around the circumference of a wheel, one vane after another will come into the path of the jet, and the water intercepted between each pair of successive vanes will do work in passing over the forward vane The performance of work will thus be made a continuous process, and the amount of work done per second may be found by making W equal to the weight wQ of water flowing from the jet in 1 second

In the application of these formulas to waterwheels, it is desirable to adjust the velocity v_1 of the vane to that of the jet in such a way that the work done by the water in moving the vane shall be a maximum It is necessary to take into account the loss of velocity of the jet in passing over the surfaces of the vanes and, in some forms of waterwheel, the effect of centrifugal force It is also necessary to design the wheel so that the water will enter the vanes without shock or impact, and leave them with the least possible velocity Finally, the proportions must be such that the construction of the wheel will be simple and practicable In order to accomplish these results, the formulas must be modified by the introduction of constants determined by experiment.

INTERNAL, OR VORTEX, MOTION IN WATER

27. In smooth flowing water the paths of adjacent particles may remain parallel through considerable distances A line of particles moving in the same path is called a **filament.** If the filaments strike a firm object, they are distorted and deflected, the deflected currents often take a rotary motion in which all the particles forming a definite mass of water revolve about a common axis, as in an eddy Such a system of rotating particles is called a **vortex.** When once formed, a vortex may travel a long distance in a stream, and is not, as a rule, broken up until its energy has been expended in friction between its particles and those of the surrounding water

If it is assumed that the particles in a vortex all rotate with a velocity v, the kinetic energy of the vortex is $\frac{W v^2}{2g}$, where W is the weight of the water contained in the vortex. The energy contained in a vortex cannot be converted into useful work in a waterwheel. Under suitable conditions, a considerable percentage of the energy of a stream of water may be converted into vortex motion with a corresponding reduction in the useful energy. This frequently occurs in the action of waterwheels, and in this way the energy carried away in the tailrace may be more than double the energy represented by the mean linear velocity of the current.

Vortex motion can be easily observed by watching the motion of a small drop of ink let fall from the point of a fine pen into a tumbler of water, or by noticing the swirls that boil up and spread over the surface of a tailrace when turbines are running.

ORDINARY VERTICAL WATERWHEELS

CLASSES OF WATERWHEELS

28. In general, a **waterwheel** is a motor or machine whose principal part is a rotating wheel operated by the action of water. The wheel proper, which is mounted on a shaft that revolves with it, is usually called the **runner.** If the shaft is horizontal, so that the runner revolves in a vertical plane, the motor is called a **vertical waterwheel;** if the shaft is vertical, so that the runner revolves in a horizontal plane, the motor is called a **horizontal waterwheel.** It should be observed, however, that the terms *vertical turbine* and *horizontal turbine* are at present very commonly used to indicate, respectively, a turbine in which the *shaft* is vertical, and one in which the *shaft* is horizontal.

29. Waterwheels are further divided into three main classes. The first class comprises overshot, breast, and undershot wheels, presently to be described. There being

no satisfactory name to distinguish this class, wheels belonging to it will here be called **ordinary vertical waterwheels** The other two classes are **impulse wheels,** which are moved by a jet of water impinging on vanes distributed over the circumference of the runner, and **turbines,** in which the buckets around the circumference of the runner are all simultaneously filled by water that continually flows into them through conduits called **chutes** or **guides**

Generally, the term "waterwheel" is understood to refer either to ordinary vertical or to impulse wheels It is to be remarked, however, that many writers include impulse wheels in the turbine class The wheels here defined as turbines and impulse wheels are called by these writers **reaction turbines** and **impulse turbines,** respectively

30. Ordinary vertical waterwheels have been almost entirely superseded by turbines and impulse wheels At the present time they are rarely used except in new countries where turbines are not obtainable A few examples of old-time wooden vertical wheels may still be seen in the Catskill mountain region and elsewhere An overshot wheel 72½ feet in diameter that was built about half a century ago is still in use in the Isle of Man

The water supply for vertical waterwheels is usually drawn from small, steady-flowing streams Spring-fed streams are preferable because of their freedom from ice and freshets Pioneer mills were usually located where such streams border rivers or highways The sites thus chosen often determined the location of important cities

The principal objections to ordinary vertical waterwheels are their large size and weight, which make them unwieldy, their slow motion, which necessitates expensive and cumbrous gearing to transmit the proper speed to the machinery they operate, the reduction of their efficiency and often the absolute impossibility of operating them, caused by the formation of ice in the buckets, and, finally, their lack of adaptability to variations in the head of water During periods of high water, the water rises in the tailrace (see

Fig 7), and, if the wheel is low enough, its buckets dip into the **back water**, which thus offers a great resistance to the motion of the wheel the wheel is then said to be "drowned," and to "wallow" in the tailrace This difficulty may be obviated by placing the wheel so that it will always be above high water, but, as a rule, the loss caused by the resulting reduction of the total head is greater than the loss caused by the resistance of back water when the wheel is allowed to wallow Formerly, these wheels were often mounted on floats or pontoons designed to keep the axle of the wheel at a constant height above the surface of the tailwater.

OVERSHOT WHEELS

31. General Features —Referring to Fig 7, an **overshot waterwheel** consists of an axle *a a* mounted on suita-

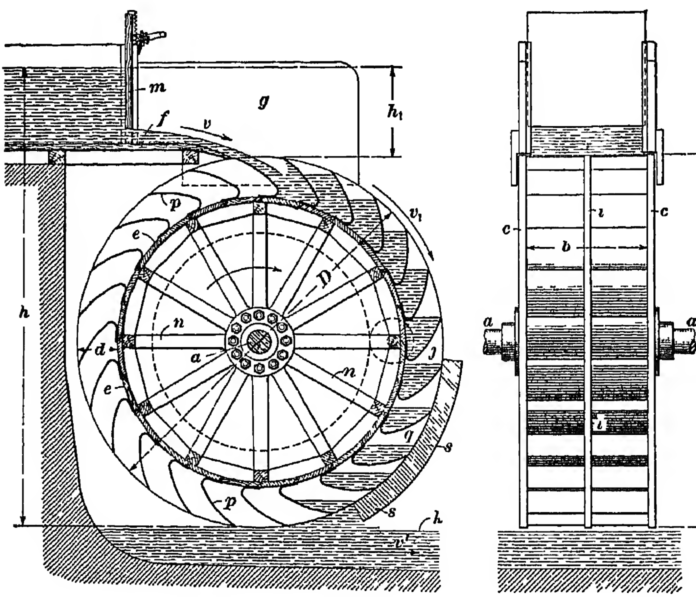

Fig 7

ble journals, at least two circular **crowns** or **shroudings** *c*, *c*, connected to the axle by radial arms *n*, *n*, and a series of

curved partitions p, p, called **vanes** or **floats**, extending between the crowns. The **sole** ee forms a cylindrical surface to which the inner edges of the vanes are attached. Intermediate crowns i, i are used in very wide wheels to give added support to the vanes. The vanes divide the space enclosed by the crowns and sole into compartments called **buckets**. Sometimes the water is admitted below the top of the wheel by a sluice passing over the wheel; such wheels are called **pitch-back waterwheels.** If the sluice is lower down, about midway between the top and the bottom of the wheel, the wheel is called a **middleshot wheel.**

Sometimes an **apron**, or **curb**, cc, Fig. 11, is added, conforming closely to the circumference of the wheel. The object of the apron is to prevent the escape of the water from the buckets before reaching the bottom of the wheel. A wheel supplied with such an apron is called a **breast wheel.** Aprons are more commonly added to middleshot than to overshot wheels. For an apron to be effective, there must be little clearance between the wheel and apron; this may cause rubbing and a loss of power by friction.

32. Action of the Water. Fig. 7 shows two views of an overshot wheel with curved iron buckets. The water is brought to the top of the wheel by a trough or sluice f, which may be curved toward the wheel, and should be so placed that the water will enter the first, second, or third bucket from the vertical center line of the wheel. The supply of water to the wheel is regulated by a gate m, which is generally operated by hand, but may be operated by an automatic governor. The thickness of the sheet of water in the trough should not exceed 6 or 8 inches.

In an overshot wheel, the water acts mainly by its weight, the water in each bucket doing work while it descends from the top to the bottom of the wheel. As, however, the water enters the buckets with some velocity, a small part of the work is due to impact. Since, even under the most favorable circumstances, only one-half of the energy due to the velocity of the entering water can be utilized by impact, the head that

produces the velocity of the entry is made small, and the greater part of the fall is taken up by the diameter of the wheel

33. Practical Values —The first point that should be considered in the design of an overshot wheel is the velocity v_1 of the circumference This varies with the diameter of the wheel, and ranges from $2\frac{1}{2}$ feet per second for the smallest diameters to 10 feet per second for the largest Wheels of this class are well adapted to heads varying between about 8 and 75 feet, and to discharges of from 3 to 25 cubic feet per second They give their best efficiency with heads between 10 and 20 feet

The diameter of the wheel is fixed by the total fall h and the head h_1 necessary to produce the required velocity of entry v of the water into the bucket (see Fig 7) The velocity of entry is always greater than the velocity of the circumference of the wheel, varying between $1\frac{1}{2} v_1$ and $2 v_1$

Owing to the frictional losses in the sluice and gate, the head h_1 required to produce the velocity v is about 1 1 times the velocity head $\frac{v^2}{2g}$, that is,

$$h_1 = 1\,1 \times \frac{v^2}{2g} \qquad (1)$$

The diameter D of the outside of the wheel is made to correspond to the difference $h - h_1$ and the clearance required between the wheel and trough

The number of revolutions per minute N is fixed by the diameter D and the circumferential velocity v_1, and is given by the formula

$$N = \frac{60\,v_1}{\pi D} = \frac{19\,1\,v_1}{D} \qquad (2)$$

The number of buckets Z is given by the formula

$$Z = 2\,5\,D \text{ to } 3\,D$$

The depth d of the buckets is made between 10 and 15 inches

The breadth b (feet) of the buckets is made between $3 \times \frac{Q}{d\,v_1}$ and $4 \times \frac{Q}{d\,v_1}$, where Q is the supply of water in cubic

feet per second, and d is the depth of the buckets expressed in *feet*

34. Buckets.—The form of buckets should be such that the water will enter freely and with little shock, and will be retained as long as possible In Fig 7, if the wheel is not provided with an apron, water will begin to spill from the buckets at some point j, at the bottom of the wheel, it will all have been discharged The effect is the same as if the water were all discharged at a point q about midway between j and the tailrace level k, the head $q\,k$ below the mean point of discharge q is lost This loss may be prevented by the addition of an apron $s\,s$ The depth d of the buckets should be small, so that the water will fall the shortest possible distance in entering them The breadth of the buckets is usually made such that they are only partly filled, in this way, the discharge begins lower down on the wheel and the loss of head is decreased With a given discharge, the quantity in each bucket will be decreased as the speed is increased If, however, the speed is great, the head required to give the entering jet its velocity will be large, and, in addition, the centrifugal force will tend to throw the water out of the buckets

35. Fig 8 shows a good method of laying out wood or iron buckets for an overshot waterwheel The description given applies especially to iron vanes, as shown at (*a*) wooden vanes may be made to approximate this form more or less closely, as shown at (*b*) In Fig 8 (*a*), $CEFG$ is a section of the crown of the wheel, and A is the mouth of the sluice Let d be the depth of the crown ring First draw the center line AB of the sheet of entering water This curve will be a parabola, and may be constructed as explained in *Rudiments of Analytic Geometry* With the axis of the wheel as a center and a radius $R = \frac{1}{2}D$, draw the arc CE, cutting the parabola in a, so that the distance e is equal to one-half the thickness of the sheet of entering water, plus the thickness of the trough, plus the clearance between the crown of the wheel and the trough From the

same center, draw the arc FG with the radius $R - d$, this gives the surface of the sole of the wheel From the point b, where this arc cuts the parabola AB, draw the straight line Ab, and mark the point a' where Ab cuts the arc CE With b as a center and a radius equal to d, draw the arc mc, cutting FG in c, and draw cf, which is a prolongation of the radius of the arc FG Draw the outline $a'f$ of the bucket

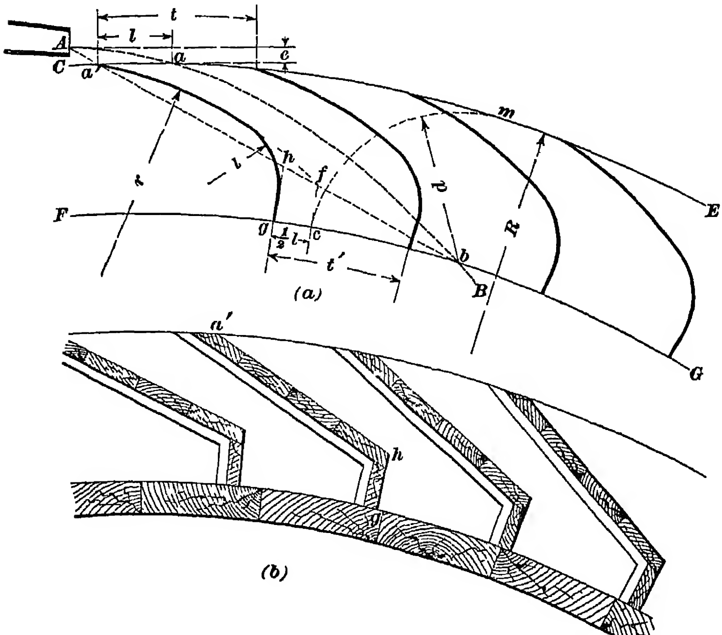

FIG 8

with the radius $r = a'b$ The center of this arc may be found by erecting a perpendicular at the middle of $a'f$ and intersecting this perpendicular with a radius $a'b$ from a' or f as a center Lay off $cg = \frac{1}{2}l$, l being the distance between a and a', draw gh parallel to cf, and, finally, join the curve $a'f$ and the line gh with an arc whose radius is equal to l This gives the outline for a bucket The pitch t is found by dividing the circumference of the wheel by the number of buckets The pitch t' of the buckets at the sole of the wheel

is found by dividing the circumference of the sole by the number of buckets

It will be noticed that, in the wooden construction shown in Fig 8 (*b*), the points *a'*, *h*, and *g* correspond to the points *a'*, *h*, and *g* in (*a*) The parts *g h* and *h a'* in (*b*) are made straight In all cases, the outer edges of the buckets should be sharpened so that they will offer as little resistance as possible to the entrance of the water

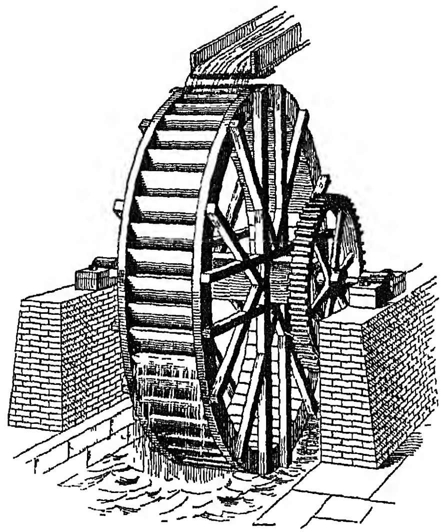

Fig 9

Fig 9 shows an overshot wheel made mostly of wood, and Fig 10 one made entirely of iron The power may be taken from the axle, as shown in Fig 9, or it may be taken from gearing on the rim of the wheel, as shown in Fig 10

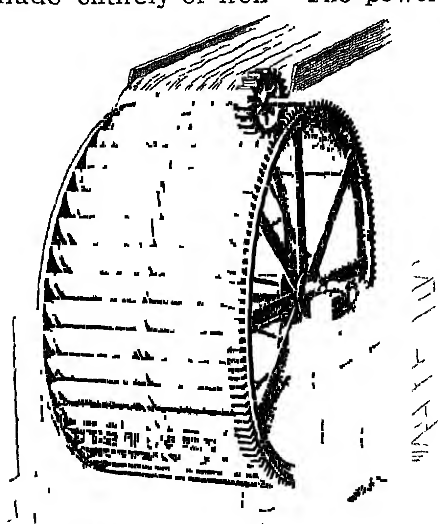

Fig 10

36. Efficiency. The efficiency of the overshot waterwheel is high, ranging from 65 to 85 per cent in well-constructed wheels When the supply of water is small, as during a drought, the buckets are only partly filled, hence, the loss from the water leaving the

buckets too early is reduced, and the efficiency of the wheel is increased.

EXAMPLE.—To compute the principal dimensions of an overshot waterwheel to utilize 10 cubic feet of water per second with a total head of 25 feet.

SOLUTION.—If the circumferential velocity v_1 of the wheel is made 8 ft. per sec., and the velocity of entry is made equal to $2\,v_1$, or 16 ft. per sec., the head h_1 required to produce the velocity of entry is (Art. 33) $1.1 \times \frac{16^2}{64.32} = 4.38$ ft. Since this corresponds to the maximum value of v for the assumed velocity v_1, a value of h_1 somewhat less, say 4 ft., may be used for the head at entrance, and the diameter D of the wheel may be taken equal to $h - h_1 = 25 - 4 = 21$ ft.

The number of buckets may be taken as $3\,D$, or 63; this makes the pitch $\frac{21\pi}{63} = 1.05$ ft.

Making the depth d of the buckets 12 in., or 1 ft., the breadth b of the wheel may be made equal to $3 \times \frac{Q}{d\,v_1} = \frac{3 \times 10}{8 \times 1} = 3.75$ ft. In order that the water will enter the buckets freely, the width of the trough should be a little less than the breadth of the wheel—say 3.5 ft. for this case. The number of revolutions of this wheel, with the assumed velocity v_1, is $\frac{19.1 \times 8}{21} = 7.28$ per min., nearly.

BREAST WHEELS

37. General Features.—A middleshot breast wheel is shown in Figs. 11 and 12. As in an overshot wheel, the water enters the buckets by impulse, but does the greater part of its work by gravity. These wheels are used for falls of from 4 to 16 feet, and discharges of from 5 to 80 cubic feet per second. The water may be admitted by a sluice or orifice, as shown in Fig. 12, or it may enter over a weir, as shown at a in Fig. 11. The weir

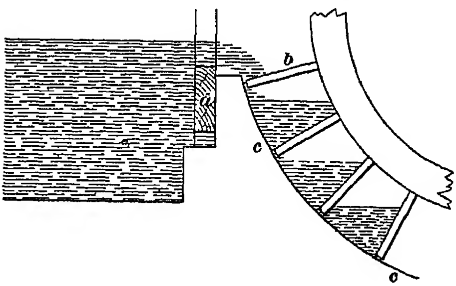

FIG. 11

board *a* can be raised or lowered to regulate the supply. The weir is the more efficient form of inlet, because a larger portion of the head is utilized by gravity and less by impact than with the form of gate shown in Fig. 12. If the water enters at the top of the apron, and not at some higher point, flat floats, as shown in Fig. 11, may be used. Curved vanes give better entry and exit conditions, and are more efficient than flat floats. The buckets of a breast wheel should be ventilated, to provide

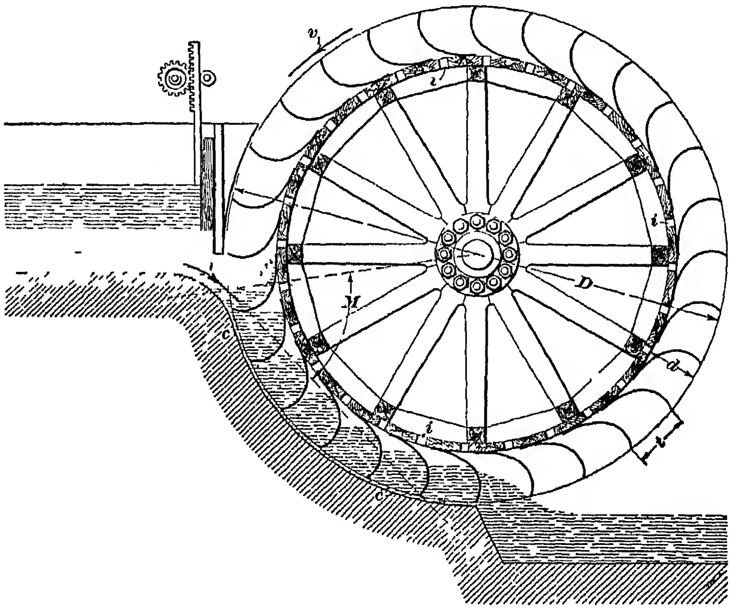

FIG. 12

for the exit of air while the bucket is filling. Holes through the sole for this purpose are shown at *i*, *i*, Fig. 12. The buckets of a wheel provided with a breast can be more completely filled than those of a wheel without an apron, so that the capacity of the wheel, for a given width, is greater. The breast or curb *c c*, Figs. 11 and 12, is made either of wood or of masonry, the latter being lined with a smooth coating of cement to make it fit closely to the wheel. A wooden breast is likely to swell and rub against the crown of the wheel. A

clearing space of between $\frac{3}{8}$ and $\frac{3}{4}$ inch should be left between the wheel and the breast

38. Buckets.—Curved vanes for breast wheels may be laid out according to the following method, which applies especially to iron floats

Draw the center line AB, Fig 13 (a), of the path of the entering water in the manner explained for overshot wheels, then, draw the center line mn of the floats, so that it is nearly tangent to AB, and draw arcs of the outer and

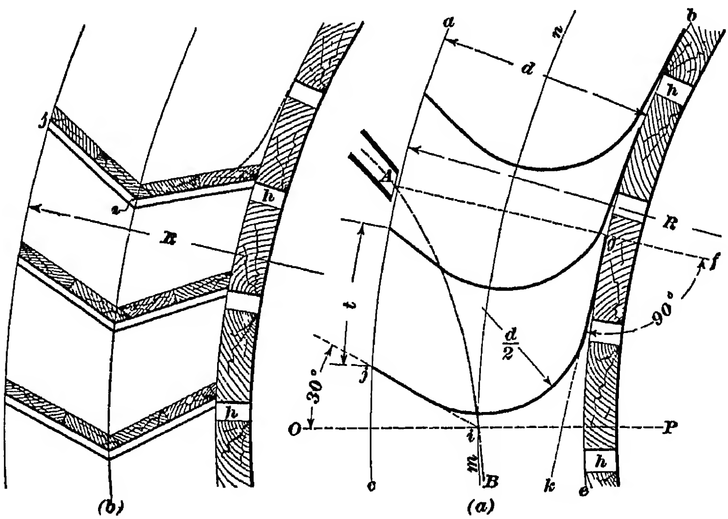

FIG 13

inner edges of the floats, as ac and be From A, draw the radial line Af, and, from the point of intersection g of this line and the inner edge be of the floats, draw a line gk tangent to be Through the point i where AB cuts mn, draw a radial line OP, and also a line ij at an angle of 30° with OP, then, join the line ij and the tangent gk by an arc whose radius is $\frac{1}{2}d$ As in Art **35**, t is the pitch of the floats

Wooden floats may be made to approximate this form, as shown in Fig 13 (b)

39. Practical Values.—In breast wheels, the circumferential velocity v_1 may usually be made between 3 and 6 feet per second, the best value being about 4 25 feet per second As for overshots, the best value of the velocity of the water at entrance lies between 1 5 v_1 and 2 v_1 The depth d of the floats is made between 10 and 15 inches, the diameter D of the wheel is made about twice the total head, the pitch t may be made equal to d, or a little smaller, and the breadth of wheel may be made between $\frac{1\,5\,Q}{d\,v_1}$ and $\frac{2\,Q}{d\,v_1}$, the letters having the same meanings as in Art **33.** The angle M, Fig 12, that the direction of the entering water makes with the radius at the point of entrance should be about 15° (it may vary between 10° and 25°)

The efficiency of middleshot wheels is less than that of overshot wheels it usually varies between 65 and 70 per cent , though, in exceptionally well-made wheels, it may run as high as 80 per cent

UNDERSHOT WHEELS

40. Confined Undershot Wheels.—A confined undershot waterwheel with radial floats is shown in Fig 14 The wheel hangs in a channel but little wider than the wheel, so that practically all the water strikes the vanes The water may be admitted to the wheel by a sluice gate a, or, if the head is small, the gate is omitted Whether with or without a sluice, the wheel operates wholly by the impact of the water against the floats According to Art **24,** the maximum theoretical efficiency that such a wheel can give is 50 per cent Practically, however, the efficiency of these wheels seldom exceeds 40 per cent , and ordinarily it lies between 25 and 35 per cent The depth of the floats for the best effect should be at least three times the depth of the approaching stream There should be as little clearance as possible between the wheel and the sides and bottom of the race For the best efficiency, the velocity of the circumference of the wheel should be about 4 of the velocity of the current Wheels of this kind are usually made between

10 and 25 feet in diameter The pitch of the vanes is made between 12 and 16 inches The depth of the water on the up-stream side of the wheel should be about 4 or 5 inches.

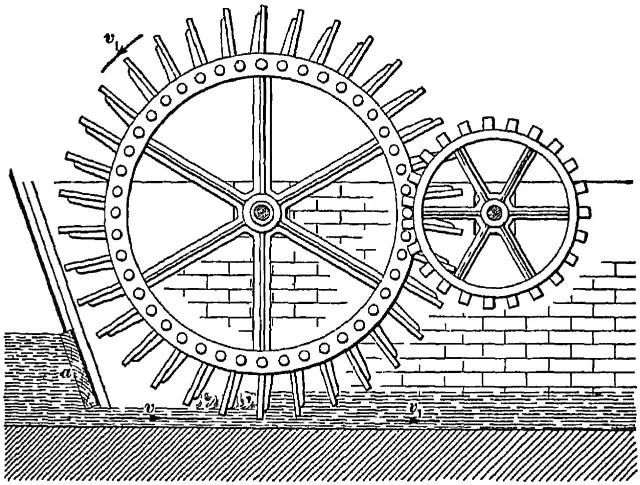

FIG 14

41. Paddle, or Current, Wheels —A **paddle wheel, or current wheel,** Fig 15, is an undershot wheel that usually has a small number of relatively large floats and is operated in a river or channel in which the water is not confined in the direction of the axis of the wheel There may also be considerable depth of water under the wheel The velocity of the circumference of a paddle wheel should be about 4 that of the current These wheels are cheap and convenient, but their efficiency is very low—usually between 25 and 30 per cent

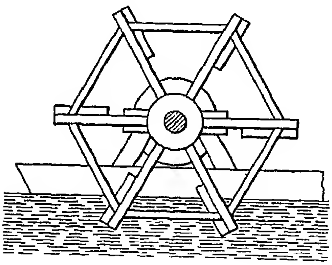

FIG 15

In order to obtain the best effect from paddle wheels, the number of floats should be great enough to allow two of them to be immersed continually

The efficiency of an undershot wheel is increased by using a breast or curb like that described for breast wheels

42. Power of Undershot Wheels —In the undershot wheels just described, the water acts by impact, and the work it performs is due to its kinetic energy The velocity v_1, Fig 14, of the water below the wheel is equal to the velocity of the wheel

Let A = area that a vane exposes to the current,
Q = volume of water acting on the wheel per second,
v = velocity of approaching current,
η = efficiency of wheel,
H = horsepower of wheel

Then, the units being the foot, second, and pound, the available energy of the water acting on the wheel is $\frac{w A v^3}{2g}$ foot-pounds per second (Art **15**) This is equivalent to

$$\frac{w A v^3}{2g \times 550} \text{ horsepower}$$

Therefore,
$$H = \frac{\eta w A v^3}{2g \times 550},$$

or, substituting 62 5 for w and 32 16 for g, and reducing,

$$H = 00177 \eta A v^3 \qquad (1)$$

Also, since $A v = Q$,

$$H = 00177 \eta Q v^2 \qquad (2)$$

Example —What horsepower can be obtained from a confined undershot whose efficiency is 30 per cent , the flow of the stream being 28 cubic feet per second, and the velocity 20 feet per second? The losses through clearance are neglected

Solution —Here $\eta = 3$, $Q = 28$, and $v = 20$ Therefore, by formula **2**,

$$H = 00177 \times 3 \times 28 \times 20^2 = 6 \text{ H P} \quad \text{Ans}$$

TRANSMISSION OF POWER

43. The power developed by a waterwheel is usually transmitted by means of gearing or belting In the case of gearing, the waterwheel itself is sometimes provided with teeth that engage with the teeth of another wheel through which the machinery is operated, in other cases, a toothed

wheel is mounted on the shaft of the waterwheel When belting is used, it is passed either around the waterwheel itself or around a pulley mounted on the shaft Whatever the method of transmission may be, the work of the waterwheel is expended in overcoming certain resistances

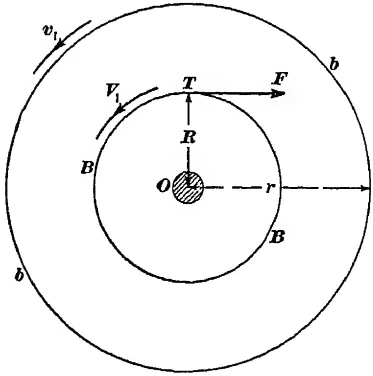

Fig 16

In Fig 16, let the resistance F (as the pull of a belt, the pressure of a gear-wheel, or the pull of a rope by which water is raised) act at the point T of the transmitting wheel $B B$, the waterwheel itself being represented by $b b$ Let v_1 and V_1 be the velocity of $b b$ and $B B$, respectively, and r and R their respective radii Then, since the two wheels have the same angular velocity,

$$\frac{V_1}{v_1} = \frac{R}{r},$$

and, therefore, $$V_1 = \frac{R}{r} v_1 \qquad (1)$$

The work of F is $F V_1$ foot-pounds per second, or $\frac{F V_1}{550}$ horsepower If, then, the horsepower of the waterwheel is denoted by H, we have

$$H = \frac{F V_1}{550}$$

or, replacing the value of V_1 from formula **1**,

$$H = \frac{F R v_1}{550\, r} \qquad (2)$$

If the number of revolutions per minute is denoted by N, then $v_1 = \frac{2 \pi r \times N}{60} = \frac{\pi r N}{30}$, and formula **2** becomes, after reducing,

$$H = 0001904\, N F R \qquad (3)$$

When H, F, and R are given, the required number of revolutions is found by solving formula **3** for N, this gives

$$N = \frac{5252.1\,H}{FR} \qquad (4)$$

It should be understood that the work done against F is not all useful work, as F includes frictional and other prejudicial resistances If the resistance F is applied at the rim of the waterwheel, R should be replaced by r

EXAMPLE —With an available power of 15 horsepower and a wheel whose efficiency is 60 per cent , how many revolutions per minute are necessary to overcome a resistance of 250 pounds acting at a distance of 10 feet from the center of the shaft?

SOLUTION —Here $H = 15 \times .60 = 9$ H P , $F = 250$ lb , and $R = 10$ ft Therefore, by formula 4,

$$N = \frac{5,252.1 \times 9}{250 \times 10} = 18.9 \text{ rev per min} \quad \text{Ans}$$

IMPULSE WATERWHEELS

GENERAL DESCRIPTION AND THEORY

44. General Features —In an impulse waterwheel, the water is supplied to the wheel in the form of one or more free jets spouting from orifices or nozzles The jet strikes a series of rotating vanes in nearly a tangential direction Impulse waterwheels are also called **tangential wheels** and **jet wheels.** As in undershot wheels, the work done on the wheel is all due to the velocity and quantity of the water, the energy of the jet being all kinetic Impulse wheels are peculiarly adapted for use in mountainous regions, and with a small supply of water under high heads

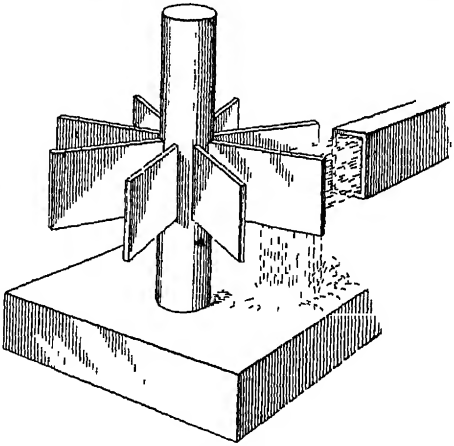

FIG 17

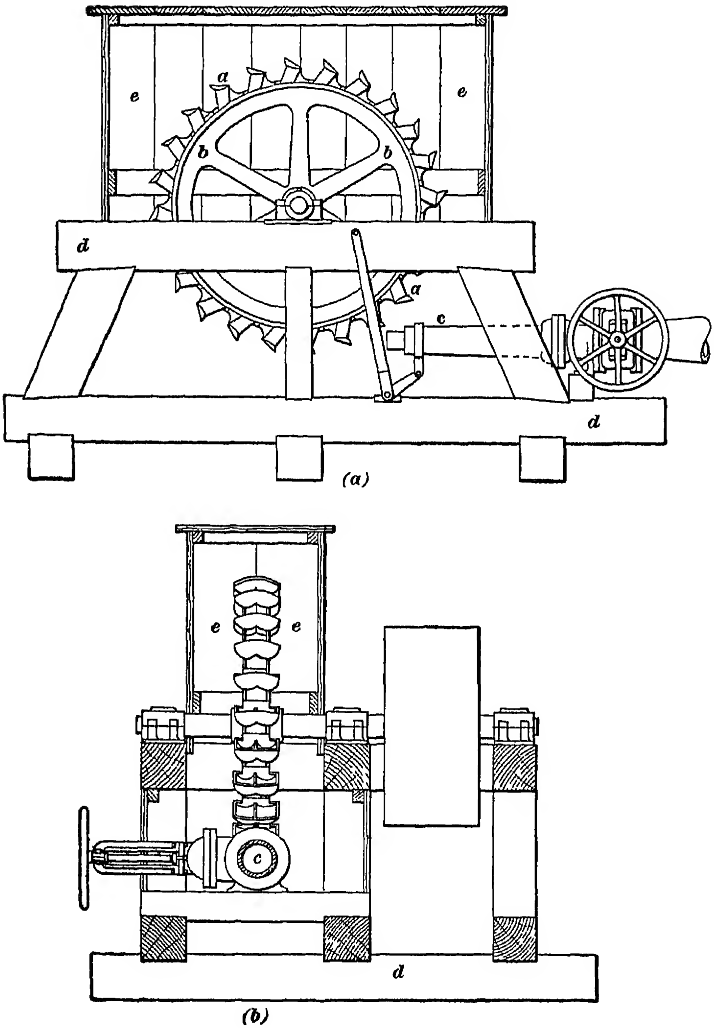

FIG 18

Fig 17 shows a primitive form of impulse wheel used for many centuries in some parts of Europe, and known as the **rouet volante.** It resembles an undershot waterwheel, but is placed on a vertical shaft and the supply stream is an unconfined jet The spent water from an impulse wheel falls away freely and is not confined in a race or carried along with the buckets as in an undershot or breast wheel

In the **hurdy-gurdy,** an early form of impulse wheel developed in the western mining regions of the United States, the vanes were attached to the rim of the wheel instead of radiating from the hub as in the rouet volante, and the runner was placed on a horizontal shaft As shown in Art **24,** a wheel having flat vanes can develop only one-half of the energy of the jet The principal improvements in impulse waterwheels have been in methods of regulation and in developing more efficient forms of buckets

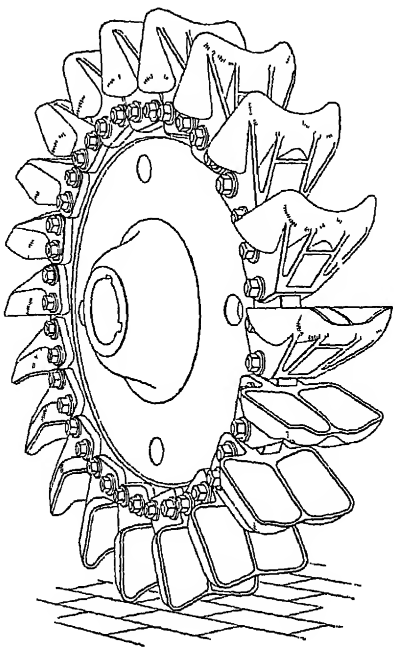

Fig 19

45. The Pelton Waterwheel —Fig 18 (*a*) shows a side elevation, and Fig 18 (*b*) an end elevation, of a **Pelton waterwheel** This wheel may be taken as the standard type of American impulse wheels The essential parts are the buckets *a*, *a*, which are mounted on the rim *b b*, the feeder nozzle *c* (sometimes two or more nozzles are used), the frame *d*, and the housing *e e*, which serves to confine the spent water The wheel is shown mounted on a timber frame such as can be built where the wheel is used Where conditions

permit, a frame and housing of cast iron or steel plate may be used. The runner of a Pelton wheel is usually made of cast iron, but for very high heads it may be made of bronze or built up from plates and rings of annealed steel, or made with a steel rim consisting of an I beam bent into a circle and connected with the hub by rods forming tension spokes. Fig. 19 is a perspective view of a Pelton wheel with double buckets.

46. Buckets for American Impulse Wheels.—Buckets are of two principal types, namely, those which discharge at the side when in full action, and those which discharge at or near the inner edge—that is, the edge toward the center

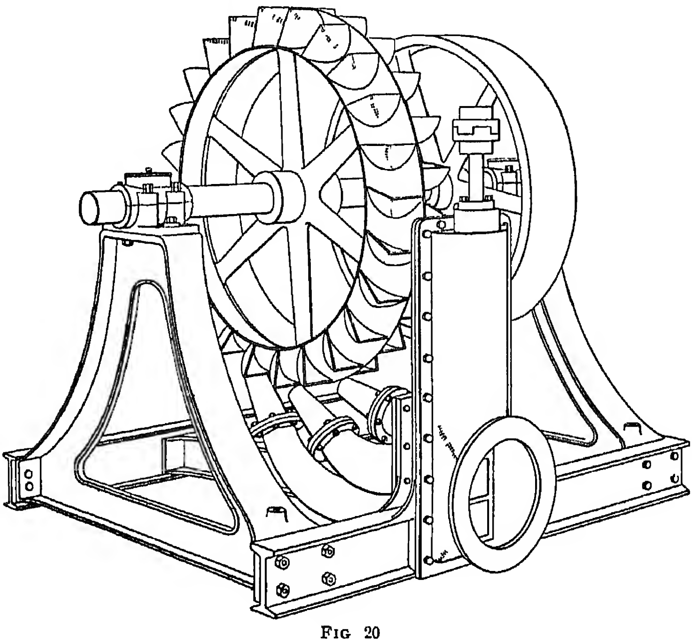

Fig. 20

of the wheel. The **Pelton**, Figs. 18 and 19, the **Cascade**, Fig. 20, and the **Cassell**, Fig. 27, are examples of side-discharge buckets. All these buckets have flat or else nearly cylindrical bottoms. In many wheels, each bucket consists of two cups, separated by a central partition that splits the

entering jet (see Fig 19) The form and the action of the bucket are frequently modified by raising or lowering the front edge and the partition In the **Knight bucket,** both the front wall and the partition are omitted, and in impulse waterwheels of Swiss design the front wall is cut very low

In the Cascade runner, Fig 20, the buckets are not joined in pairs on the rim, but are placed staggering, to insure a more continuous action of the water The **Doble bucket,** Fig 21, has ellipsoidal cups, and the lip is cut away to allow the jet to remain in full action over a longer arc of rotation of the bucket

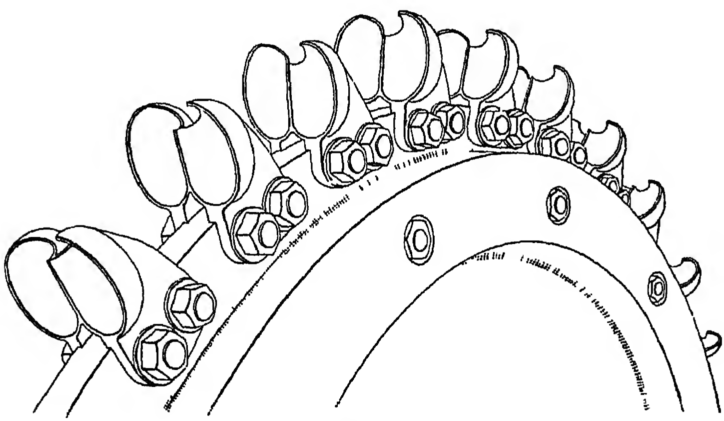

FIG 21

47. Exit Angle and Number of Buckets.—The object of the bucket is to reduce to nearly zero the final velocity of the jet relative to the earth To accomplish this, the direction of the jet on leaving the wheel should be opposite to the direction of motion of the bucket, that is, tangential to the wheel Since the bucket is in rotation and some time elapses between the entrance and the exit of the water, it follows that the direction of the jet as it leaves the bucket will not ordinarily be parallel to the entrance direction In side-discharge buckets, it is impracticable to make the exit direction exactly tangential to the direction of motion of the bucket The issuing jet is given a small velocity at right

angles to the direction of motion of the bucket, in order to carry the spent water out of the way of the succeeding buckets

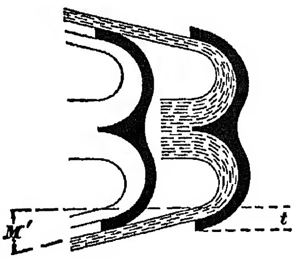

FIG 22

The angle M', Fig 22, between the direction of the water at exit and the direction of rotation is called the **exit angle.** The approximate value of this angle may be found from the formula

$$\tan M' = \frac{n\,t}{2\pi r}$$

where n = number of buckets on wheel,
r = mean radius of wheel, in feet,
t = combined thickness, in feet, of the jet at exit, and of the side wall of the bucket, as indicated in Fig 22

By the **mean radius** is meant the radius of the **mean circle** $e\,b$, Fig 23, at the mean depth of the buckets

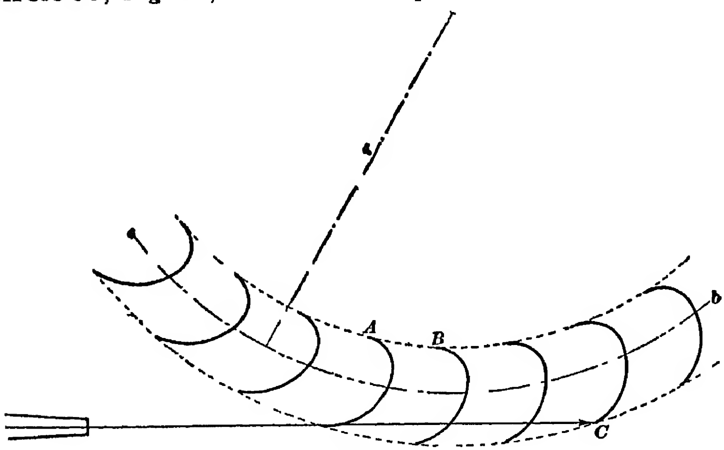

FIG 23

The smaller the number of buckets and the less the thickness of the jet, the smaller may the angle M' be made The thickness of the jet varies with the speed of the jet and with the form of bucket and amount of water In order to make this thickness as small as possible, it is desirable that

the jet should spread out in a thin, uniform stream, and that but little velocity should be lost by the friction of the jet on the bucket surface. As the bucket *A*, Fig. 23, comes into the path of the jet, it cuts off the supply of water from the bucket *B*. All the water not entering *A* must reach *B* before the latter bucket reaches the point *C*. If the pitch of the buckets is too large, some water will not be intercepted by either bucket, but will be wasted. It will be seen that the pitch of the buckets should be between the maximum and minimum limits defined by the considerations just given. The exact pitch of the buckets that will give the highest efficiency with each set of conditions can be best determined by experiment. As usually made, a 6-foot Pelton wheel has twenty-four buckets.

48. Entrance of the Jet.—In order that the jet may glide smoothly into the bucket without shock or the formation of eddies, the direction of the jet should be parallel to the front wall or edge of the vane at the entrance. The position of the jet is fixed, while that of the bucket continually changes. The position of the jet relative to the wheel should be so adjusted that the jet will be parallel to the entry edge of the bucket when the latter is in full action. Fig. 24 shows the action of a circular jet in a Pelton bucket. In (*a*), (*b*), and (*c*), the bucket is viewed in the direction of the jet *g*. In (*d*), (*e*), and (*f*) are shown side views of the jet, together with a section of the bucket on the line *l l'*. In (*a*) and (*d*), the bucket is entering and receives one-half the jet. In (*b*) and (*e*), the bucket is in full action, while in (*c*) and (*f*) it receives only the lower half of the jet, the remainder being cut off by the preceding vane. If the buckets are close together, the entire jet will be cut off after passing its position of full action, before it returns to the lip *h*. The direction of discharge is sidewise during full action, and sidewise and inwards at the entry and exit of the jet.

49. Nozzles.—The nozzles should be of such form as to convert the pressure head of the pipe into velocity with

but little loss of energy A tapering nozzle with circular cross-section meets this requirement, and also gives a form of jet that encounters a minimum frictional resistance from

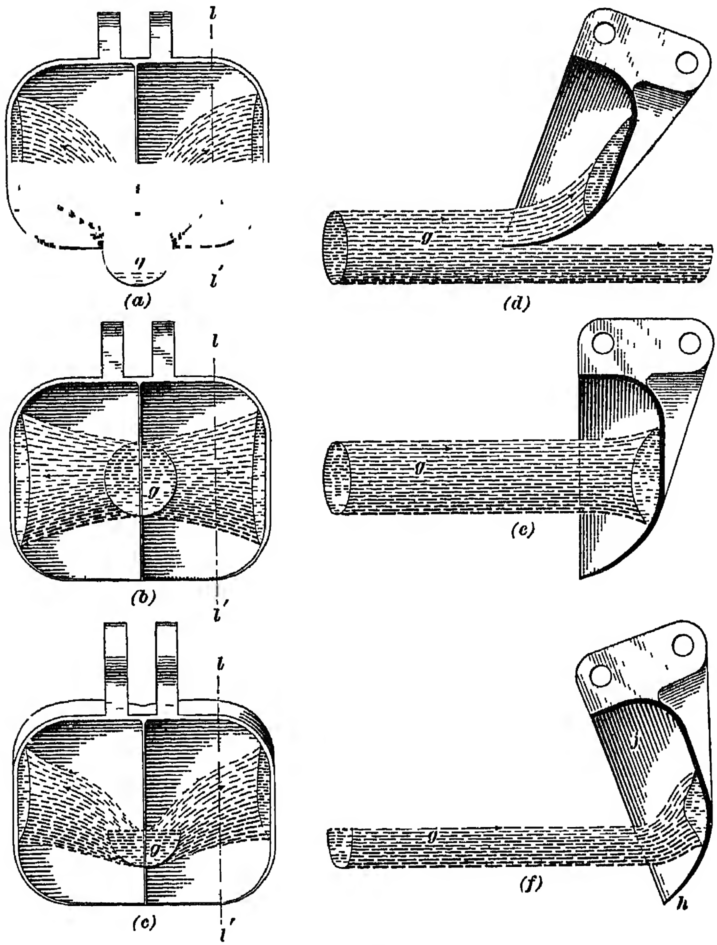

FIG 24

the air Nozzles of rectangular cross-section, however, are often used Regulation of the flow from the nozzle by means of an ordinary valve is objectionable, because the valve, when only partly opened, disturbs the smooth flow

of the jet A valve is usually placed back of the nozzle to be used only when the water is to be completely shut off

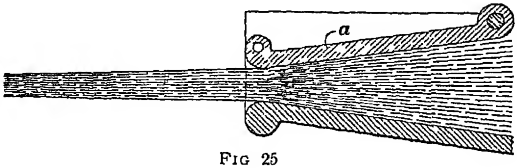

FIG 25

The size of the jet is increased or decreased by means of a swinging lip, as shown in section at *a*, Fig 25 The position of the lip may be controlled by hand or by an automatic governor Each change in the position of the lip changes both the position and the direction of the axis of the jet

When the water supply is variable, an impulse wheel is sometimes furnished with a set of nozzles or tips of different sizes When this method is used, the size of jet varies with the water supply A flat cap of metal hinged to the nozzle in such a way that it can slide over the nozzle tip forms a cut-off device that is easily operated but which changes the form of the jet and the position of its axis as the position of the cap is changed A very good way of adapting the wheel to variations in the water supply is to have several nozzles, as shown in Fig 20, some of which may be turned off when the supply is low, or when it is not necessary to run the wheel at full capacity The power of such a wheel is proportional to the number of nozzles used, and the water supply can be regulated without loss of efficiency by completely shutting off some of the nozzles

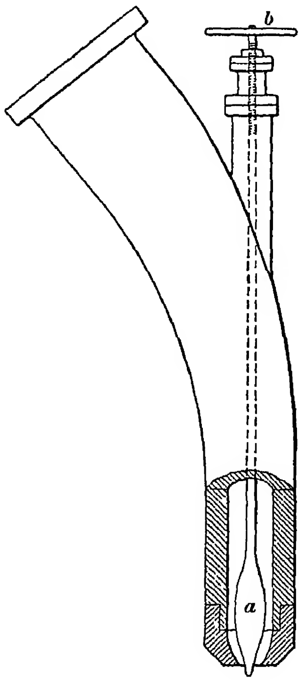

FIG 26

50. The flow through a single nozzle may be controlled by a **regulating needle,** as shown in Fig 26 The position of the conical plug *a* is controlled by the hand wheel *b* By the use of this device, a solid jet of uniform velocity is retained, the area of section of the jet varying with the size of the opening This nozzle causes but little friction loss when partly closed The diameter of a jet varies as the square root of the discharge, thus, when the discharge is reduced one-half, the jet will be $\sqrt{\frac{1}{2}}$, or 70 7 per cent , of its original diameter

51. Regulation of the Supply for Varying Loads Water hammer and other objectionable results that may follow sudden reduction in the size of an outlet orifice in a pressure pipe are described in connection with the regulation of turbines Owing to these conditions, where long closed pipe lines are used, it is impracticable to regulate the water supply to accommodate sudden changes of load by varying the size of orifice One method used for regulating the water supply for an impulse wheel in which the load changes quickly consists in deflecting the jet in such a manner that a part of it does not strike the buckets, but is wasted, when the load is decreased

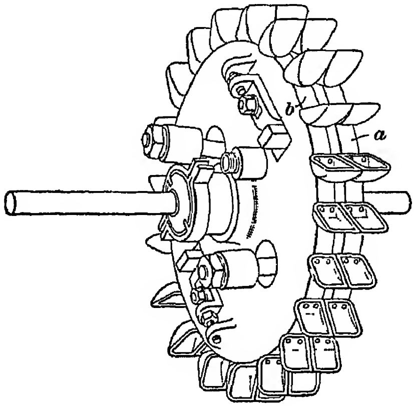

FIG 27

A **deflecting nozzle** with **ball-and-socket joint** is shown at *c*, Fig 18 The position of the nozzle is automatically controlled by a governor The quantity of water used is the same whether the wheel is operating at full or at part capacity In order to regulate the speed with sudden load changes and at the same time reduce the waste of water, a deflecting nozzle is commonly used in connection with needle or multiple nozzles, by

which the water supply is regulated for as nearly as possible the average load at all times

In the **Cassell impulse wheel**, Fig 27, the pairs of cups forming the buckets are on separate disks a, b, which may be separated or brought together automatically by a shaft governor fixed between the disks A varying volume of the jet is thus allowed to pass between the disks without striking the buckets

52. Formulas for Impulse Wheels.—In the formulas stated below,

D = mean diameter of the wheel, in feet,
h_n = net head *on nozzle*, in feet,
v = velocity of jet, in feet per second,
c = coefficient of velocity for the nozzle,
v_1 = linear velocity of mean circumference ($c\,b$, Fig 23) of wheel, in feet per second,
d = diameter of jet, in *feet*,
A = area of cross-section of jet, in square *feet*,
N = number of revolutions of wheel per minute,
Q = water supplied to the wheel, in cubic feet per second

By the **net head** *on the nozzle*, or the **nozzle head**, is meant the total head available minus the losses that occur between the source of supply and the nozzle, such losses being due to pipe friction, bends, etc The head h_n is really the head available to overcome the resistances at the nozzle itself and impart the velocity v to the jet

Waterwheel manufacturers often base their tables on *effective* heads It should be remembered that the effective head h_1 *on the wheel* is the head due to the velocity of the jet, that is, $h_1 = \frac{v^2}{2g}$ When the effective instead of the nozzle head is given, all the following formulas can be used by making $c = 1$ and putting the effective head for h_n

The velocity v is given by the usual formula

$$v = c\sqrt{2gh_n} = 8\,02\,c\sqrt{h_n} \qquad (1)$$

The coefficient c, for good nozzles, varies between 94 and 98

Theoretically (see Art 23), the best value of v_1 is $\frac{v}{2}$ Practically, however, it is found that, owing to resistances and other conditions not taken into account in the theoretical formulas, the value of v_1 for the best efficiency is about 45 v Using the value of v given by formula **1**, we have, therefore,

$$v_1 = 45\,v = 45\,c\sqrt{2\,g\,h_n} = 3\;609\,c\sqrt{h_n} \qquad (2)$$

If the head h_n and the desired number of revolutions are given, the diameter D is found from formula **2**, Art **33**, after computing the value of v_1 by formula **2** above, otherwise, thus (see Art **33**).

$$D = \frac{60\,v_1}{\pi\,N}$$

or, replacing the value of v_1 from formula **2**, and performing the operations indicated,

$$D = \frac{68\;93\,c\sqrt{h_n}}{N} \qquad (3)$$

It should be borne in mind that formulas **2** and **3** apply only when the velocity v_1 is equal to 45 v

53. Since $Q = A\,v$, we have, when Q and v are given,

$$A = \frac{Q}{v} \qquad (1)$$

Replacing v by its value from formula **1**, Art **52**,

$$A = \frac{Q}{c\sqrt{2\,g\,h_n}} = \frac{1247\,Q}{c\sqrt{h_n}} \qquad (2)$$

If the nozzle tip is circular, $A = \frac{\pi\,d^2}{4}$, and, therefore,

$$d = \sqrt{\frac{4\,A}{\pi}} = \sqrt{\frac{4\,Q}{\pi\,v}} = \sqrt{\frac{1\;2732\,Q}{v}} \qquad (3)$$

Also, writing, $c\sqrt{2\,g\,h_n}$ for v,

$$d = \sqrt{\frac{4\,Q}{\pi\,c\sqrt{2\,g\,h_n}}} = \sqrt{\frac{1588\,Q}{c\sqrt{h_n}}} \qquad (4)$$

54. Efficiency and Power of Impulse Wheels—In computing the efficiency of an impulse wheel, the energy available is the kinetic energy of the jet or jets acting on the wheel If K is the energy of the jet, in foot-pounds per

second, H the horsepower of the wheel, and η the efficiency, then $\frac{K}{550}$ represents the horsepower of the jet, and, therefore,

$$\eta = H \div \frac{K}{550} = \frac{550\,H}{K} \qquad (a)$$

Now (Art. **15**),

$$K = w\,A\,h_n\,c^3\sqrt{2\,g\,h_n} = 62.5\,A\,c^3\sqrt{2\,g\,h_n^3} \qquad (b)$$

Substituting this value in (a), and performing the numerical operations, there results

$$\eta = \frac{1.097\,H}{A\,c^3\sqrt{h_n^3}} \qquad (1)$$

The *effective* head on the wheel is

$$\frac{v^2}{2g} = \frac{\left(c\sqrt{2\,g\,h_n}\right)^2}{2g} = c^2\,h_n$$

Substituting this value in formula **1** of Art. **5**, there results

$$\eta = \frac{8.8\,H}{c^2\,Q\,h_n} \qquad (2)$$

If the nozzle is circular, $A = .7854\,d^2$, and formula **1** becomes

$$\eta = \frac{1.397\,H}{d^2\,c^3\sqrt{h_n^3}} \qquad (3)$$

If the efficiency is known, the horsepower may be found by solving one of the foregoing formulas for H, according to the data. The results are as follows:

$$H = .9114\,\eta\,A\,c^3\sqrt{h_n^3} \qquad (4)$$

$$H = \frac{10\,\eta\,c^2\,Q\,h_n}{88} \qquad (5)$$

$$H = .7158\,\eta\,d^2\,c^3\sqrt{h_n^3} \qquad (6)$$

The efficiency of impulse wheels of the Pelton type is generally very high. Wheels have been tested whose efficiency was more than 90 per cent. These, however, are exceptional cases; usually, well-made wheels, if properly installed, give an efficiency of between 75 and 85 per cent. The makers often guarantee 85 per cent., but this limiting efficiency can be obtained only by very careful installation.

EXAMPLE 1.—If the net head on the nozzle of a 5-foot Pelton wheel is 900 feet, and the coefficient of velocity of the nozzle is .95, what is

(*a*) the best circumferential velocity of the wheel? (*b*) the number of revolutions per minute?

SOLUTION —(*a*) Formula **2**, Art **52**.

$$v_1 = 3\ 609 \times\ 95\sqrt{900} = 102\ 9 \text{ ft per sec} \quad \text{Ans}$$

(*b*) Formula **2**, Art **33**

$$N = \frac{19\ 1 \times 102\ 9}{5} = 393 \text{ rev per min} \quad \text{Ans}$$

EXAMPLE 2 —What must be the diameter of an impulse wheel that is to make 400 revolutions per minute under a nozzle head of 225 feet, the coefficient of velocity of the nozzle being 98? It is assumed that $v_1 = 45\,v$

SOLUTION —Formula **3**, Art **52**:

$$D = \frac{68\ 93 \times\ 98\sqrt{225}}{400} = 2\ 533 \text{ ft} \quad \text{Ans}$$

EXAMPLE 3 —An impulse wheel is to use 5 cubic feet of water per second, with a nozzle head of 961 feet The coefficient of velocity of the nozzle is 95 (*a*) If a single nozzle is used, what must be its diameter? (*b*) If a triple nozzle (three nozzles of equal diameter) is used, what must be the diameter of each tip?

SOLUTION —(*a*) Formula **4**, Art **53**.

$$d = \sqrt{\frac{1588 \times 5}{95\sqrt{961}}} = 1642 \text{ ft} = 1\ 97 \text{ in} \quad \text{Ans}$$

(*b*) Since the combined area of the three nozzles must be the same as that of the single nozzle, we have, denoting by d_1 the diameter of each tip of the triple nozzle

$$3 \times \frac{\pi\, d_1^2}{4} = \frac{\pi\, d^2}{4},$$

whence $$d_1 = \sqrt{\frac{d^2}{3}} = \frac{d}{\sqrt{3}} = \frac{1\ 97}{3}\sqrt{3} = 1\ 14 \text{ in} \quad \text{Ans}$$

EXAMPLE 4 —A maker's catalog gives 540 35 as the horsepower of a 6-foot impulse wheel working under an *effective* head of 400 feet and using 839 20 cubic feet of water per *minute* Determine (*a*) the size of the nozzle used, (*b*) the efficiency of the wheel

SOLUTION —(*a*) Since the given head is effective head, c must be made equal to 1 (see Art **52**) Here, $Q = \frac{839\ 20}{60}$ cu ft per sec and formula **4**, Art **53**, gives

$$d = \sqrt{\frac{1588 \times \frac{839\ 20}{60}}{\sqrt{400}}} = 3332 \text{ ft} = 4 \text{ in} \quad \text{Ans}$$

(*b*) Formula **2**, Art **54**, gives, making $c = 1$,

$$\eta = \frac{8\ 8 \times 540\ 35}{\frac{839\ 20}{60} \times 400} = 85 = 85 \text{ per cent.} \quad \text{Ans}$$

55. The Girard Impulse Wheel.—A **Girard wheel**, called also a **Girard turbine** and an **impulse turbine**, is substantially an impulse wheel in which the main part of the runner consists of two equal flat rings *a b*, *a′ b′*, Fig 28, placed one above the other, the space between them being divided into buckets by curved vanes *c*, *c* The rings, or **crowns**, are properly secured to a shaft *s* The water is brought to the wheel through a pipe *A*, from the end of which it spouts on the vanes, doing work according to the principles explained in Art **26.**

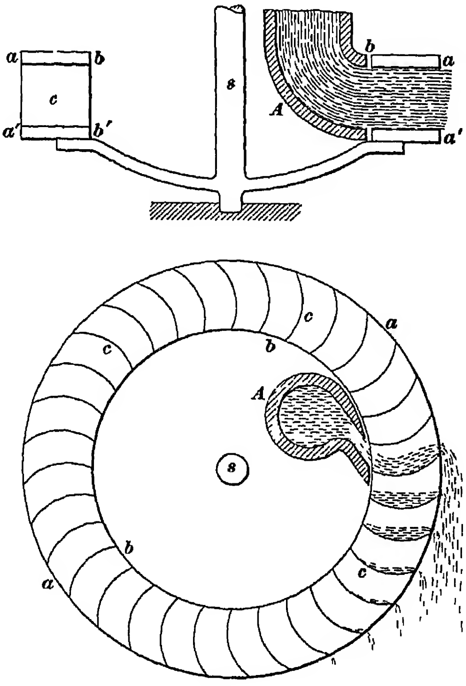

Fig 28

When, as in Fig 28, the water enters the wheel on the inside and flows outwards, the wheel is called an **outward-flow** wheel Sometimes, the water enters the wheel on the outside and flows inwards, the wheel is then called an **inward-flow** wheel

In the more elaborate forms of the Girard wheel, the supply pipe discharges into specially constructed conduits, or **guides**, from which it spouts on the vanes The guides are distributed either around the whole circumference of the crowns, so that all the vanes are acted on simultaneously, or over only a segment of the circumference, in which case only a few vanes are under its action at the same time Fig 29 shows a wheel *a b* with guides *g*, *g* While the guides are always filled, the spaces between the vanes *v*, *v*

are never completely filled, and in this a Girard wheel differs from a turbine proper, in which all the buckets are constantly full of water The holes h serve to admit air to the buckets so as to prevent the formation of a partial vacuum

The theory of the Girard wheel is based on the formulas given in Art **26.** An exposition of that theory is, however, beyond the scope of this work The quantities involved are usually so determined that the direction of the relative velocity u, Fig 6, is tangent to the vane at a, in order to avoid shock at entrance; the angles M and L' are so selected as to make $u = v_1$ and $u' = v_1'$ (the latter equation is always satisfied if the former is) The entrance angle M is usually made between 25° and 40°, and the exit angle L', between 15° and 30°

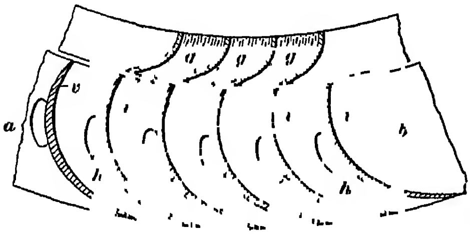

FIG 29

Theoretically, the efficiency of a good Girard wheel is very high, but in practice, it is found that, on account of the many resistances, the efficiency is seldom more than 80 per cent Even this, however, is a very good efficiency for a water motor or any other machine

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EXAMPLES FOR PRACTICE

1 Find the diameter of an impulse wheel that is to make 370 revolutions per minute under a nozzle head of 600 25 feet, it being assumed that the circumferential velocity of the wheel is 45 of the jet velocity, and that the coefficient of velocity of the nozzle is 97 Ans 4 427 ft

2 With a nozzle head of 1,024 feet and a supply of 920 cubic feet per minute, a 5-foot impulse wheel develops 1,435 horsepower If the coefficient of velocity of the nozzle is 95, determine (*a*) the diameter of the nozzle, (*b*) the efficiency of the wheel

Ans $\begin{cases} (a) & 283 \text{ ft} = 3\ 4 \text{ in} \\ (b) & 89 \text{ per cent} \end{cases}$

3 What horsepower can be obtained from an impulse wheel whose efficiency is 80 per cent, if a 3-inch nozzle and a nozzle head of 900 feet are used, the coefficient of velocity of the nozzle being 94?

Ans 803 H P

TESTING IMPULSE WHEELS

56. Apparatus Used.—In testing an impulse wheel, the power may be calculated from the electrical output of a dynamo driven by the wheel, or it may be measured directly by a friction brake The latter method is usually more accurate Fig 30 shows the apparatus for testing impulse waterwheels at the University of Michigan The wheel is contained in a case *c*, the runner shaft is attached to a brake pulley *m*, around which is wrapped a friction band *jj*, the

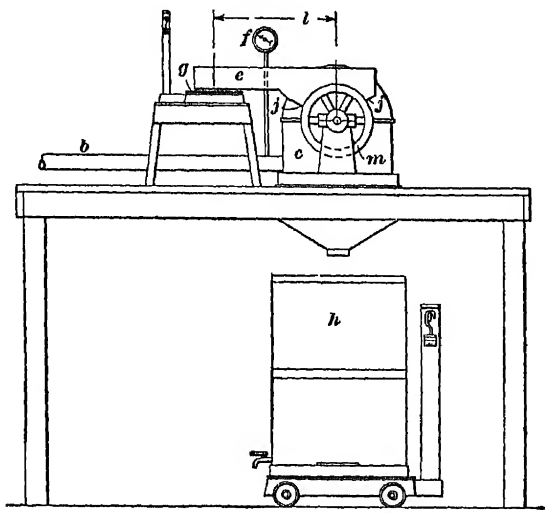

Fig 30

ends of which are attached to the brake lever *e* The brake lever bears on a platform scale *g* Water is conducted to the wheel by a pipe *b*, and a pressure gauge *f* indicates the head available immediately back of the nozzle The spent water falls into a tank *h*, and is weighed Where the quantity of water is large, a weir is generally used, instead of a weighing tank, to measure the outflow

In conducting a test, readings are taken at frequent intervals to determine the time, head, speed of the wheel, pressure on the scales *g*, and weight of water used.

57. General Formulas—In the following general formulas,

R = radius of the brake wheel m, Fig 30, in feet,
l = distance, in feet, from the center of the wheel shaft to the center of bearing of the brake arm on the scales, measured as shown in the figure,
N = number of revolutions per minute,
V = velocity of the rim of the brake wheel, in feet per second,
F = friction on the rim of the brake wheel, in pounds,
P = pressure on the scale g, in pounds,
Q = volume of water used per second, in cubic feet,
h_1 = effective head on wheel, or head due to velocity v of jet,
H = horsepower developed by the waterwheel,
η = efficiency of the waterwheel,
W = weight of water used per second, in pounds

The friction F acts tangentially to the wheel m, it being the tangential component of the forces acting on the wheel at its circumference The other component is normal or radial, and its moment about the center of the shaft is zero Since the lever e is in equilibrium under the action of forces equal and opposite to P, F, and the normal component just referred to, we have, taking moments about the center of the shaft,

$$FR = Pl,$$

whence

$$F = \frac{Pl}{R} \qquad (1)$$

The work done by the waterwheel, in foot-pounds per second, is equal to FV, and, therefore,

$$H = \frac{FV}{550} = \frac{PlV}{550R} \qquad (2)$$

or (see formula **3**, Art **43**),

$$H = 0001904\,NPl \qquad (3)$$

Since the available energy is Wh_1 pounds per second,

$$\eta = \frac{FV}{Wh_1} = \frac{PlV}{Wh_1R} \qquad (4)$$

or, since $V = \frac{2\pi R N}{60} = 1047\, N R$,

$$\eta = \frac{1047\, N P l}{W h_1} \qquad (5)$$

The head from the level of the nozzle to the water surface in the tail-pit cannot be utilized in an impulse wheel, and is commonly neglected in calculating the efficiency The wheel is usually placed close to the tail water level, and the nozzles are placed underneath the wheel in order to reduce this loss to a minimum The effective head h_1 is found from the velocity of the jet, this velocity being determined from the quantity Q and the diameter of the nozzle

EXAMPLE —From a test of an impulse wheel by means of an apparatus similar to that shown in Fig 30, the following data were obtained

Net head h_n on nozzle = 117 feet

Diameter d of nozzle = 5 inch

Weight of water used per second = 7 22 pounds

Revolutions per minute = 460

Radius of brake pulley = 5 foot

Length of brake arm = 2 0 feet

Pressure on scale platform = 6 5 pounds

Required, (a) the coefficient of velocity of the nozzle, (b) the horsepower of the wheel, (c) the efficiency of the wheel

SOLUTION —(a) Since $v = c\sqrt{2 g h_n}$, and, also,

$$v = \frac{Q}{\frac{\pi d^2}{4}} = \frac{W \div 62\ 5}{\frac{\pi d^2}{4}} = \frac{W}{62\ 5 \times 7854\, d^2} = \frac{7\ 22}{62\ 5 \times 7854 \times \left(\frac{5}{12}\right)^2}$$

we have

$$c = \frac{v}{\sqrt{2 g h_n}} = \frac{\dfrac{7\ 22}{62\ 5 \times 7854 \times \left(\frac{5}{12}\right)^2}}{\sqrt{2 g \times 117}} = 9766, \text{ or, say, } 977 \quad \text{Ans}$$

(b) Formula 3 gives

$$H = 0001904 \times 460 \times 6\ 5 \times 2 = 1\ 14 \text{ H P} \quad \text{Ans}$$

(c) To find the efficiency, it is first necessary to find the effective head h_1 Now,

$$v = c\sqrt{2 g h_n}, \text{ and } v = \sqrt{2 g h_1}$$

Therefore, $c\sqrt{2 g h_n} = \sqrt{2 g h_1}$, whence, squaring, $c^2 \times 2 g h_n = 2 g h_1$, and, solving for h_1,

$$h_1 = c^2 h_n = 977^2 \times 117$$

Formula 5 now gives

$$\eta = \frac{1047 \times 460 \times 6\ 5 \times 2}{7\ 22 \times 977^2 \times 117} = 777 = 77\ 7 \text{ per cent} \quad \text{Ans}$$

WATERWHEELS

(PART 2)

TURBINES

CLASSIFICATION AND GENERAL PRINCIPLES

1. Principal Parts of a Turbine.—The principal parts of a turbine are the *runner* and the *guides*. As in other water wheels, the **runner** is the main revolving part, or wheel proper. It is mounted on a shaft, and divided into channels, called **buckets**, by partitions, called **vanes** or **floats**. The term **guides** is applied both to the passages by which the water is brought to the runner buckets and to the partitions separating those passages. The passages themselves are often called **chutes**.

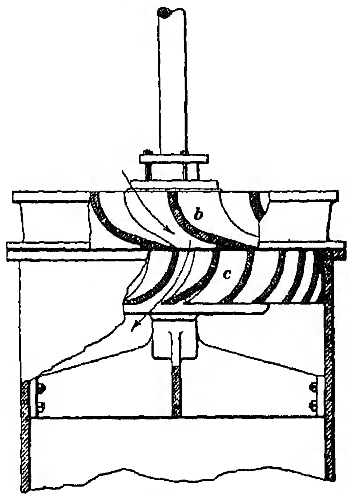

FIG. 1

2. Classes of Turbines. Turbines are best classified according to the direction of flow of the water in passing through the runner. The direction of flow is expressed with reference to the axis of the shaft.

Turbines in which the direction of flow of the water in passing through the runner is in general parallel to the axis

of the shaft are called **axial,** or **parallel-flow, turbines.** The **Jonval turbine,** shown in partial section in Fig 1, is typical of this class The direction of the water in passing through the chutes *b* and buckets *c* is shown by the arrows

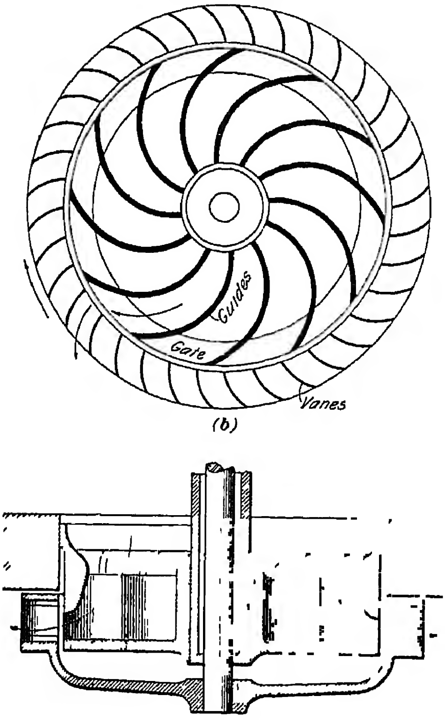

Fig 2

3. Turbines in which the general direction of flow in the buckets is perpendicular to the axis of the shaft or runner are called **radial-flow turbines.** If the flow is from inside outwards, they are called **outward-flow turbines;** if from outside inwards, **inward-flow turbines** The **Fourneyron turbine,** shown in Fig 2 [elevation (*a*), plan (*b*)], is the standard type of outward-flow

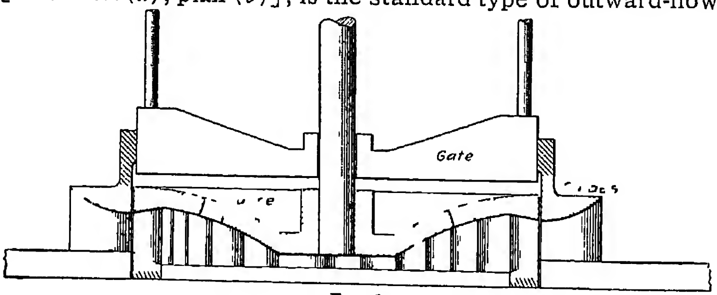

Fig 3

turbines, and the **Francis turbine,** shown in Fig 3, is the type of inward-flow turbines

4. Turbines in which the water changes its direction of flow with reference to the axis, in passing through the wheel, are called **mixed-flow turbines** Nearly all the latest American turbines belong to this class, the flow being inwards, downwards, and outwards, or tangential In this class of wheel, the buckets expand at the bottom of the runner, so that the water is discharged in a direction nearly at right angles to the axis The Leffel, McCormick, and "American" turbines are typical of this class Fig 4 shows an "American" runner

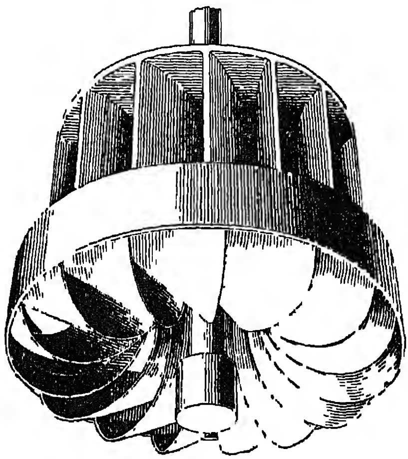

FIG 4

5. Action of Water on a Turbine—Turbines are often called **reaction waterwheels**, because the reaction or pressure exerted on the vanes opposite the outlets of the buckets (see *Waterwheels*, Part 1) is utilized to derive power from the water If the water spouted freely from the chutes to the vanes, it would have a velocity at its entrance to the runner nearly equal to that due to the head, as in the case of an impulse wheel In a turbine, however, the relation between the chutes and the buckets is such that the velocity of the water as it leaves the chutes is considerably less than the velocity due to the head it follows that there must be a back pressure from the buckets into the chutes, and an equal and opposite reaction on the vanes

If v is the velocity with which the water enters the buckets, and h' is the pressure head, or the head that produces the pressure on the vanes, then, neglecting friction, the total head h is given by the formula

$$h = h' + \frac{v^2}{2g}$$

Also, $$h' = h - \frac{v^2}{2g}$$

The energy of the water consists of two parts, namely the kinetic energy due to the velocity v, and the pressure energy, or energy due to the head h' The kinetic energy is transmitted to the runner vanes by impulse, while the pressure energy is converted into work by the action of the pressure due to the head h' through a distance represented by the path of the water in the buckets In a properly designed turbine, the water leaves the buckets with very little absolute velocity, and hence with but little energy

6. Speed for Maximum Efficiency.—In an ordinary turbine, the proportion of the head that is utilized by reaction can theoretically be varied at will without affecting the efficiency attainable, provided the turbine is properly proportioned If the theory is extended to take into account friction and the relative motion of the water in the guides and vanes, it is found that the maximum efficiency is obtained when about one-half the head is utilized in reaction and one-half in impulse Some head is lost in friction in the guides, and it is found that the peripheral velocity of the inlet ends of the vanes at the speed of maximum efficiency should usually be from 60 to 67 of the velocity due to the full head h, that is, between $60\sqrt{2gh}$ and $67\sqrt{2gh}$

7. Selection of Type of Turbine—Before undertaking the design or adoption of a turbine, the head and the quantity of water to be used are ascertained The type of turbine best adapted to the conditions may then be selected Practice as to the type of turbine to be used under given conditions varies in different countries The following represents good American practice for very low heads, say from 3 to 10 feet, Jonval or Francis turbines, generally on vertical shafts, with short draft tubes, are used, for heads of 10 to 50 feet, Francis or "American" turbines of stock patterns, mounted on vertical or horizontal shafts, for medium high heads, from 40 or 50 to 150 or 200 feet, specially designed turbines, commonly of the Francis or the

Fourneyron type (Fourneyron turbines have been successfully used for heads of over 265 feet.) For very high heads, 300 to 2,000 feet, impulse waterwheels are generally more advantageous than turbines.

FORMULAS FOR THE DESIGN OF TURBINES

8. Notation.—The methods used in designing axial-flow, radial-flow, and mixed-flow turbines are similar, and the same notation may be used to designate corresponding quantities in them. In what follows, it is necessary to use

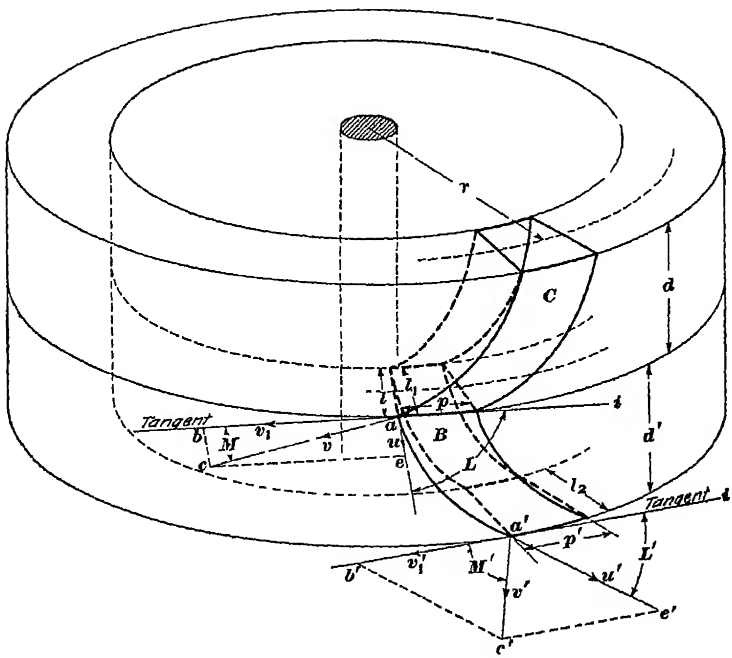

FIG. 5

both the "absolute" velocity of the water, or the velocity relative to the earth, and the velocity relative to the vanes. The former will be referred to merely as *the velocity*, and the latter as *the relative velocity*. Fig. 5 shows the outlines of

an axial-flow turbine, with one chute C and one bucket B, together with the adjacent guides, vanes, and rims Fig 6 is a similar view of an outward-flow turbine Nearly all the quantities defined below are plainly shown in these two figures and in Figs 7 and 8 Before attempting to follow out the formulas for design in which these symbols are used,

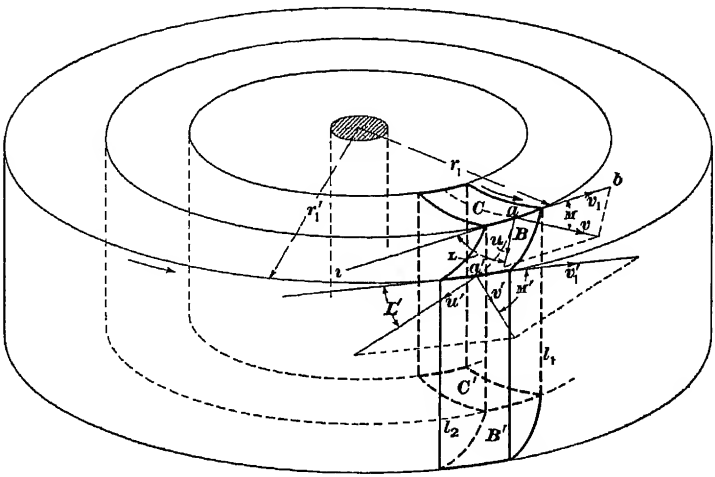

Fig 6

the student should make himself so familiar with the values shown in the diagrams as to be able to recognize the meaning of each symbol at once when it is referred to It should be understood that, when not otherwise stated, the units are the foot, the second, and the pound

Let Q = available quantity of water, in cubic feet per second,

h = head on the wheel,

r = mean radius of an axial turbine (Fig 5),

r_1 = radius of wheel at inflow, for a radial turbine (Fig 6) (a circle of this radius is called the **inflow circle**),

r_1' = mean radius of wheel at outflow, for a radial turbine (Fig 6) (a circle of this radius is called the **outflow circle**),

v = velocity of flow from the chutes,
v' = absolute velocity of water leaving the wheel buckets,
u = relative velocity of water entering the buckets,
u' = relative velocity of outflow from the buckets,
v_1 = velocity of vanes at entrance (velocity of inflow circle),
v_1' = velocity of vanes at discharge (velocity of outflow circle),
M = **guide angle**, that is, the angle that the direction of outflow from the chutes makes with a tangent bz to the mean inflow circle,
L = angle that the relative direction of inflow to the wheel makes with the same tangent,
L' = angle that the relative direction of outflow from the buckets makes with a tangent to the mean outflow circle,
M' = angle that the absolute direction of outflow from the buckets makes with the same tangent,
A = effective outflow area of guide passages,
A_1 = effective inflow area of wheel passages,
A_1' = effective outflow area of wheel passages,
N = number of revolutions per minute,
p = pitch of the guides measured on the inlet circle,
p' = pitch of the outlet ends of the vanes, measured on the outlet circle,
Z = number of guides,
Z' = number of vanes,
l = width of outlet edge of guides;
l_1 = width of inlet edge of vanes, this usually equals the width of the outlet edge of the guides,
l_2 = width of outlet edge of vanes,
x = distance between the outflow ends of two consecutive guides, measured perpendicularly to the direction of flow,
x' = distance between outflow ends of two consecutive vanes, measured perpendicularly to the direction of flow,

t = thickness of guides,
t' = thickness of vanes near their outflow ends,
s = part of the distance x that would be covered by the inflow end of one wheel bucket, to be measured in the same direction as x,
d = depth of guide rims in an axial-flow turbine,
d' = depth of runner rims in an axial-flow turbine

9. Chute and Bucket Discharge Angles.—The angles M and L', for radial turbines, are usually as follows

For outward-flow turbines, $M = 15°$ to $24°$ or more, $L' = 10°$ to $20°$ or more

For inward-flow turbines, $M = 10°$ to $25°$, $L' = 10°$ to $25°$

For axial turbines, the values of these angles depend on the discharge and head thus.

$Q \div \sqrt{h}$	M	L'
Less than 10	13° to 15°	13° to 16°
Between 10 and 80	15° to 20°	15° 30′ to 20°
Greater than 80	20° to 24°	20° to 24°

10. Ratios of Radii and Vane Lengths.—For outward-flow turbines, the ratio $\frac{r_1}{r_1'}$ is usually made between .67 and .83, for inward-flow turbines, $\frac{r_1}{r_1'}$ usually varies between 1.18 and 1.54, for mixed-flow turbines, $\frac{r_1}{r_1'}$ varies between about 1 and 1.50 or more, for axial turbines, $r_1 = r_1' = r$

In all classes of turbines, l is usually made equal to l_1. In all axial turbines, if the crowns are parallel, $l = l_1 = l_2$, however, l_2 is often made greater than l in order to secure proper outflow area. Except for mixed-flow turbines, 1 may be used as a trial value of $\frac{l}{l_2}$ *for the purpose of determining a proper value for* $\frac{A}{A_1'}$. For mixed-flow turbines, l is nearly always less than l_2, and $\frac{l}{l_2}$ may be as small as $\frac{1}{2}$ or $\frac{1}{3}$, the lengths of the outflow edges of the buckets of such turbines

being increased by their curved shape The smaller the ratio $\frac{l}{l_2}$, the smaller can be made the exit angle L'

11. Ratio of Chute and Bucket Areas.—The ratio $\frac{A}{A_1'}$ is calculated by the formula

$$\frac{A}{A_1'} = \frac{l}{l_2} \times \frac{r_1}{r_1'} \times \frac{\sin M}{\sin L'}$$

The ratio $\frac{v}{u'}$ of the velocity of outflow from the guides to the relative velocity of outflow from the buckets is equal to $\frac{A}{A_1'}$ The efficiency varies but little for a considerable change in $\frac{A}{A_1'}$, but is usually a maximum in axial-flow turbines when $\frac{A}{A_1'} =$ 75 to 1 00

The ratio $\frac{A}{A_1'}$ is often assumed, taking it between 5 and 1 5 This, when M, L', and $\frac{r}{r_1'}$ have been assumed, is equivalent to assuming a value for the ratio $\frac{l}{l_2}$

12. Velocity of Discharge Circles—The velocity v_1' of the bucket discharge circle is given by the formula

$$v_1' = m\sqrt{\frac{A}{A_1'} \times \frac{r_1'}{r_1} \times \frac{g\,h}{\cos M}} \qquad (1)$$

The value of m varies from 90 to 95 Ordinarily, it may be taken as 92 We have also

$$v_1 = \frac{r_1}{r_1'} \times v_1' \qquad (2)$$

The entrance velocity v is given by the formula

$$v = \frac{A_1'\,v_1'}{A \cos L'} \qquad (3)$$

Also,

$$u' = \frac{A}{A_1'} \times v \qquad (4)$$

The values of u', v', and L' should be so related that the absolute exit angle M' is very nearly 90°

13. Bucket Entrance Angle.—The bucket angle L may be computed from the formula

$$\tan L = \frac{v \sin M}{v_1 - v \cos M}$$

If $v \cos M$ is greater than v_1, $\tan L$ will be minus, and the angle L will be greater than 90°, and may be found by subtracting the angle corresponding to the tangent computed by the formula above from 180° The angle L usually increases as M decreases Its value varies from 40° to 120° or more It is most frequently near 90°, this being the value used by Fourneyron for his outward-flow turbines The value of L must in all cases be less than $180° - 2\,M$

14. Number of Guides and Vanes.—The practical values of Z and Z' are usually as follows

For axial turbines

If A is less than 2, $Z = 12\ 6\ r$ to $16\ 9\ r$

If A is between 2 and 16, $Z = 24$ to 28

If A is greater than 16, $Z = 6\ 3\ r$ to $7\ 5\ r$

For outward-flow turbines

$Z = 28$ to 32 for small wheels

$Z = 32$ to 38 for large wheels

For inward-flow turbines

$Z = 12\ r_1$ to $16\ 9\ r_1$ for small wheels

$Z = 6\ 3\ r_1$ to $12\ r_1$ for large wheels

For axial turbines

$Z' = Z + 1$ to $Z + 2$, ordinarily, $Z' = Z + 2$

For outward-flow turbines

$Z' = 1\ 2\ Z$ to $1\ 3\ Z$, but Z' may be less than Z

In one of the Niagara turbines, $Z = 36$ and $Z' = 32$

For inward-flow turbines

$Z' = Z$ to $7\ Z$, but Z' is sometimes greater than Z by 1 or 2

15. Thickness of Guides and Vanes.—The usual values of t and t' are $\frac{1}{2}$ to $\frac{5}{8}$ inch for cast iron, and $\frac{1}{4}$ to $\frac{3}{8}$ inch for plate iron or steel In feet,

$t = t' =$ 040 to 052 for cast iron

$t = t' =$ 020 to 031 for plate iron or steel

16. Radius and Speed.—*For axial-flow turbines*

If A is less than 2, $r = \sqrt{A}$ to $1.25\sqrt{A}$ (1)

If A is between 2 and 16, $r = 1.25\sqrt{A}$ to $1.50\sqrt{A}$ (2)

If A is greater than 16, $r = 1.50\sqrt{A}$ to $2.00\sqrt{A}$ (3)

For outward-flow turbines

$$r_1 = 1.5\sqrt{A} \text{ to } 2\sqrt{A} \text{ for ordinary heads} \tag{4}$$

$$r_1 = .90\sqrt{A} \text{ to } 1.5\sqrt{A} \text{ for very large heads} \tag{5}$$

For inward-flow turbines

$$r_1 = .75\sqrt{A} \text{ to } 2.00\sqrt{A} \tag{6}$$

If r, r_1, or r_1' is given, the number of revolutions per minute can be found by the formula

$$N = \frac{60\, v_1}{2\,\pi\, r} = \frac{9.549\, v_1}{r} = \frac{9.549\, v_1}{r_1} = \frac{9.549\, v_1'}{r_1'} \tag{7}$$

17. Pitch and Length of Guides and Vanes.—The values of p, p', x, x', l, and l_1 are computed by the following formulas

$$p = \frac{2\,\pi\, r_1}{Z} \tag{1}$$

$$p' = \frac{2\,\pi\, r_1'}{Z'} \tag{2}$$

$$x = p \sin M - t \tag{3}$$

$$x' = p' \sin L' - t' \tag{4}$$

$$l = \frac{A}{Z\,x - Z'\,s} \tag{5}$$

$$l_1 = \frac{A_1'}{Z'\,x'} \tag{6}$$

18. If D' is the diameter of the outer rim and D is the diameter of the inner rim of an axial turbine, then,

$$\left.\begin{aligned} D &= 2\,r - l \\ D' &= 2\,r + l \end{aligned}\right\} \text{at entry end} \tag{1}$$

$$\left.\begin{aligned} D &= 2\,r - l_1 \\ D' &= 2\,r + l_1 \end{aligned}\right\} \text{at discharge end} \tag{2}$$

19. Depth of Rims of Axial Turbines.—The rims need only be made deep enough to give the water the desired change of direction without shock or abruptness. As a rule, they are made equal, and within the following limits

If A is less than 2, $d = d' = \frac{r}{3}$ to $\frac{r}{2.5}$ (1)

If A is between 2 and 16, $d = d' = \frac{r}{5}$ to $\frac{r}{4}$ (2)

If A is greater than 16, $d = d' = \frac{r}{6}$ to $\frac{r}{5}$ (3)

20. Example of Design of an Axial Turbine.—To illustrate the use of the foregoing formulas, the principal quantities will be computed for an axial and two radial turbines. The first case considered will be the design of a Jonval turbine to use 112 cubic feet of water per second under a head of 16 feet, the rims to be parallel and the vanes and guides to be of cast iron with $t = t' = .041$ foot.

Here, $\frac{Q}{\sqrt{h}} = \frac{112}{\sqrt{16}} = 28$. Then (see Art. **9**), the following values will be adopted for M and L': $M = 18°$, $L' = 16°$. We have, to three significant figures,

$\sin M = .309$, $\cos M = .951$, $\sin L' = .276$, $\cos L' = .961$

According to Art. **10**, $\frac{r_1}{r_1'} = 1.0$, and $\frac{l_1}{l_2} = 1.0$. Then (Art. **11**),

$$\frac{A}{A_1'} = \frac{\sin M}{\sin L'} = \frac{.309}{.276} = 1.12$$

Formula **1**, Art. **12**, gives, making $m = .92$,

$$v_1' = .92\sqrt{1.12 \times 1.0 \times \frac{32.16 \times 16}{.951}} = 22.6 \text{ feet per second}$$

Since $r_1 = r_1' = r$, $v_1 = v_1' = 22.6$ feet per second.

Formulas **3** and **4**, Art. **12**:

$$v = \frac{22.6}{1.12 \times .961} = 21.0 \text{ feet per second}$$

$$u' = 1.12 \times 21.0 = 23.5 \text{ feet per second}$$

The area A can now be found by the formula $A = \frac{Q}{v}$, which gives

$$A = \frac{112}{21} = 5.33 \text{ square feet}$$

Also, since $\frac{A}{A_1'} = 1.12$,

$$A_1' = \frac{A}{1.12} = \frac{112}{1.12 \times 21} = \frac{100}{21} = 4.76 \text{ square feet}$$

From Art **13**,

$$\tan L = \frac{21\ 0 \times\ 309}{22\ 6 - 21\ 0 \times\ 951}, \quad L = 67^\circ\ 57'$$

The values of Z and Z' may be taken as follows (Art **14**)

$$Z = 25, \ Z' = Z + 2 = 27$$

It is desired to give the turbine as high an angular velocity as practicable The radius should, therefore, be taken as small as possible Taking it equal to $1\ 25\sqrt{A}$, as given by formula **2** of Art **16**, we have

$$r = 1\ 25\sqrt{5\ 33} = 2\ 89 \text{ feet}$$

Then (formula **7**, Art **16**),

$$N = \frac{9\ 549 \times 22\ 6}{2\ 89} = 74\ 7 \text{ revolutions per minute}$$

Formulas **1** and **2**, Art **17**, give, since here $r_1 = r_1' = r = 2\ 89$ feet, $Z = 25$, and $Z' = 27$ (π will be taken as 3 142),

$$p = \frac{2 \times 3\ 142 \times 2\ 89}{25} = \ 726 \text{ foot}$$

$$p' = \frac{2 \times 3\ 142 \times 2\ 89}{27} = \ 673 \text{ foot}$$

If the inlet ends of the vanes are rounded off, s may be taken as 01 Formulas **3** to **6**, Art **17**, give

$$x = \ 726 \times\ 309 - \ 041 = \ 183 \text{ foot}$$

$$x' = \ 673 \times\ 276 - \ 041 = \ 145 \text{ foot}$$

$$l = \frac{5\ 33}{25 \times\ 183 - 27 \times\ 01} = 1\ 24 \text{ feet}$$

$$l_1 = \frac{4\ 76}{27 \times\ 145} = 1\ 22 \text{ feet}$$

Formulas **1** and **2**, Art **18**, give

$$\left.\begin{aligned} D &= 2 \times 2\ 89 - 1\ 24 = 4\ 54 \text{ feet} \\ D' &= 2 \times 2\ 89 + 1\ 24 = 7\ 02 \text{ feet} \end{aligned}\right\} \text{at entry}$$

$$\left.\begin{aligned} D &= 2 \times 2\ 89 - 1\ 22 = 4\ 56 \text{ feet} \\ D' &= 2 \times 2\ 89 + 1\ 22 = 7\ 00 \text{ feet} \end{aligned}\right\} \text{at discharge.}$$

The depths d and d' of the rims may be made equal to

$$\frac{r}{4} = \frac{2\ 89}{4} = \ 723 \text{ foot (see Art } \mathbf{19})$$

21. Example of Design of an Outward-Flow Turbine —As the next example, a Fourneyron turbine will be

designed to operate under a head of 135 feet and develop 5,000 horsepower, a total efficiency of 80 per cent being assumed

To determine the necessary supply of water Q, we have, H being the theoretical horsepower of the water (see *Waterwheels*, Part 1),

$$Q = \frac{8\ 8\ H}{h} \qquad (a)$$

Since the efficiency is 8, we must have $8\ H = 5{,}000$, and, therefore, $H = 5{,}000 - 8 = 6{,}250$ horsepower This value in equation (a) gives

$$Q = \frac{8\ 8 \times 6{,}250}{h} = \frac{88 \times 625}{135} = 408 \text{ cubic feet per second}$$

Since $\frac{Q}{\sqrt{h}} = \frac{408}{\sqrt{135}} = 35$, the angles M and L' will be assumed as follows (see Art **9**)

$$M = 19°, \quad L' = 11°$$

Then, $\sin M = 326$, $\cos M = 946$, $\sin L' = 242$, $\cos L' = 970$

The ratio $\frac{r_1}{r_1'}$ will be taken as .83 (Art **10**). Then (see Arts **10** and **11**),

$$\frac{A}{A_1'} = 83 \times \frac{326}{242} = 1\ 12$$

Formula **1**, Art **12**, gives, taking $m = 92$,

$$v_1' = 92\sqrt{\frac{1\ 12 \times 32\ 16 \times 135}{83 \times 946}} = 72\ 4 \text{ feet per second}$$

Formulas **2**, **3**, and **4**, Art **12**, give

$$v_1 = 83 \times 72\ 4 = 60\ 1 \text{ feet per second}$$

$$v = \frac{72\ 4}{1\ 12 \times 970} = 66\ 6 \text{ feet per second}$$

$$u' = 1\ 12 \times 66\ 6 = 74\ 6 \text{ feet per second}$$

From the relations $A = \frac{Q}{v}$, $A_1' = \frac{A}{1\ 12}$ we get

$$A = \frac{408}{66\ 6} = 6\ 13 \text{ square feet}$$

$$A_1' = \frac{6\ 13}{1\ 12} = 5\ 47 \text{ square feet}$$

The formula in Art 13 gives

$$\tan L = \frac{66\ 6 \times\ 326}{60\ 1 - 66\ 6 \times\ 946}, \quad L = 97° \ 36'$$

The values used for Z and Z' in the Niagara turbine referred to in Art 14 will be adopted, that is,

$$Z = 36, \ Z' = 32$$

It will be assumed that the turbine has bronze guides and vanes, for which $x =\ 115$ foot, $x' =\ 104$ foot, and $s =\ 02$ The radius r_1 will be assumed equal to $\sqrt{H} = \sqrt{6\ 13} = 2\ 48$ feet (see Art 16) Then,

$$r_1' = \frac{r_1}{83} = \frac{2\ 48}{83} = 2\ 99 \text{ feet}$$

Formula 7, Art 16, gives

$$N = \frac{9\ 549 \times 60\ 1}{2\ 48} = 231 \text{ revolutions per minute}$$

Formulas 1, 2, 5, and 6, Art 17, give, respectively,

$$p = \frac{2 \times 3\ 142 \times 2\ 48}{36} = 433 \text{ foot}$$

$$p' = \frac{2 \times 3\ 142 \times 2\ 99}{32} = 587 \text{ foot}$$

$$l(= l_1) = \frac{6\ 13}{36 \times\ 115 - 32 \times\ 02} = 1\ 75 \text{ feet}$$

$$l_1 = \frac{5\ 47}{32 \times\ 104} = 1\ 64 \text{ feet}$$

22. Example of Design of an Inward-Flow Turbine —A Francis turbine will now be designed for a discharge of 160 cubic feet per second and a head of 12 4 feet

The angles M and L' will be selected as follows (see Art 9)

$$M = 20°, \ L' = 20°$$

Then, $\sin M = \sin L' =\ 342$, $\cos M = \cos L' =\ 940$

The ratio $\frac{r_1}{r_1'}$ will be taken as 1 3 (see Art 10), and the ratio $\frac{l}{l_1}$ as 59 (see Art 11) Then, by the formula in Art 11,

$$\frac{A}{A_1'} =\ 59 \times 1\ 3 \times \frac{342}{342} =\ 767$$

Formulas **1** to **4**, Art **12**, give

$$v_1' = 92\sqrt{\frac{767 \times 32\ 16 \times 12\ 4}{1\ 3 \times 940}} = 14\ 6 \text{ feet per second}$$

$$v_1 = 1\ 3 \times 14\ 6 = 19 \text{ feet per second}$$

$$v = \frac{14\ 6}{767 \times 940} = 20\ 2 \text{ feet per second}$$

$$u' = 767 \times 20\ 2 = 15\ 5 \text{ feet per second}$$

The relations $A = \frac{Q}{v}$ and $A_1' = \frac{A}{767}$ now give

$$A = \frac{160}{20\ 2} = 7\ 92 \text{ square feet}$$

$$A_1' = \frac{7\ 92}{767} = 10\ 33 \text{ square feet}$$

The formula in Art **13** gives

$$\tan L = \frac{20\ 2 \times 342}{19 - 20\ 2 \times 940}$$

As the denominator of this fraction is practically zero, $\tan L$ is equal to infinity (see *Plane Trigonometry*, Part 2), and, therefore, $L = 90°$

The value of r_1 will be taken equal to

$$1\ 1\sqrt{A} = 1\ 1\sqrt{7\ 92} = 3\ 1 \text{ feet,}$$

or, say, 3 feet (see Art **16**) Then,

$$r_1' = \frac{r_1}{1\ 3} = \frac{3}{1\ 3} = 2\ 31 \text{ feet}$$

Formula **7**, Art **16**, gives

$$N = \frac{9\ 549 \times 19}{3} = 60\ 5 \text{ revolutions per minute}$$

The values of Z, Z', t, and t' will be taken as follows (Arts **14** and **15**)

$$Z = 24,\ Z' = 25,\ t = t' = 02 \text{ foot}$$

The edges of the vanes will be assumed sufficiently thin to make $s = o$, practically

Formulas **1** to **6**, Art **17**, give

$$p = \frac{2 \times 3\ 142 \times 3}{24} = 785 \text{ foot}$$

$$p' = \frac{2 \times 3\ 142 \times 2\ 31}{25} = 581 \text{ foot}$$

$$x = 785 \times 342 - 02 = 248 \text{ foot}$$

$$x' = 581 \times 342 - 02 = 179 \text{ foot}$$

$$l = \frac{7\ 92}{24 \times 248} = 1\ 33 \text{ feet}$$

$$l_2 = \frac{10\ 33}{25 \times 179} = 2\ 31 \text{ feet}$$

23. Remarks.—Not all the dimensions and other quantities affecting the operation of a turbine with a given head and discharge can be deduced from theoretical considerations There are, however, certain relations between the parts, such that, if the dimensions of some parts are assumed, the dimensions of other parts that shall best correspond with those chosen can be calculated

Practice differs as to which quantities shall be assumed at the beginning of a turbine calculation The methods here given are believed to be as good as any others, but it should be borne in mind that, in all cases where empirical formulas are used, the assumptions may be modified by judgment and experience, and are often simply a matter of opinion, or even of preference

24. Power Losses and Efficiency. There is no simple formula by which the efficiency of a turbine can be computed with accuracy A close estimate may generally be made, however, by combining the different losses These are as follows, their usual values being expressed as percentages of the theoretical power of the water

1 Skin friction of the guides, 1 to 3 per cent

2 Clearance leakage, 1 to 3 per cent

3 Skin friction in buckets, 3 to 5 per cent

4 Exit velocity With the discharge below tail-water level or through a draft tube, this loss may usually be kept between 3 and 7 per cent

5 Bearing friction The power consumed in friction can be calculated by methods given in books on mechanical engineering For turbines without superincumbent machinery and with ordinary step bearings, this loss may be 1 to 5 per cent

6 Shock, eddies, and internal motion This is the most uncertain element of loss The amount of loss depends

on the clearance and on the thickness and shape of the ends of the guides and vanes, and on the form of the curved surfaces If the wheel is not run at the proper speed, or if the proper relation does not exist between the angles *M* and *L'*, the loss from shock and internal motion may be very large In such cases, the loss may result in part from the formation of eddies or the accumulation of dead water at some point along the vanes, so that the water leaves the buckets with a different angle and velocity from those intended If the design is good, this loss may usually be kept between 6 and 14 per cent

Usually, the efficiencies of turbines vary between about 70 and 80 per cent

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GUIDES AND VANES

25. Guides for Axial Turbines —The method of laying out guides for axial turbines is illustrated in Fig 7 First draw two parallel lines *A A'* and *B B'*, at a distance from each other equal to *d* On *B B'* lay off the distances *1–2*, *2–3*, etc , equal to the pitch *p*, and through the points *1*, *2*, *3*, etc , draw the lines *1–i*, *2–j*, etc , making with *B B'* an

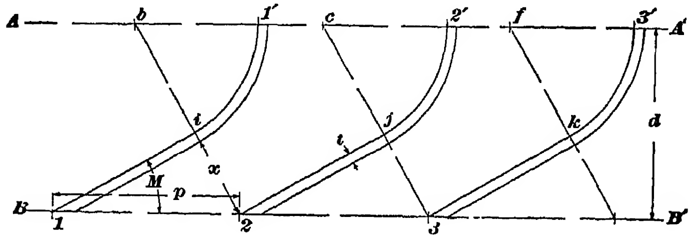

FIG 7

angle equal to the entrance angle *M* Through *2* draw a perpendicular to *1–i* and produce it until it meets *A A'* in *b*, then, with *b* as a center and *b i* as a radius, draw the arc *i–1'* This gives the form of the front of a guide, the back is made parallel to and at a distance *t* from the front The other guides are laid out in a similar manner, as plainly indicated in the figure

26. Vanes for Axial Turbines.—To lay out the wheel vanes for an axial turbine, lay off on the line BB', Fig. 8, the distances $1–2$, $2–3$, $3–4$, etc., equal to the pitch p'. Through 1, 2, 3, etc., draw lines $1–e$, $2–f$, $3–g$, etc., making with BB' an angle equal to L'. Through 2, draw a perpendicular to $1–e$, and on this perpendicular produced find by trial a point o, such that an arc drawn with the radius oe will be tangent to

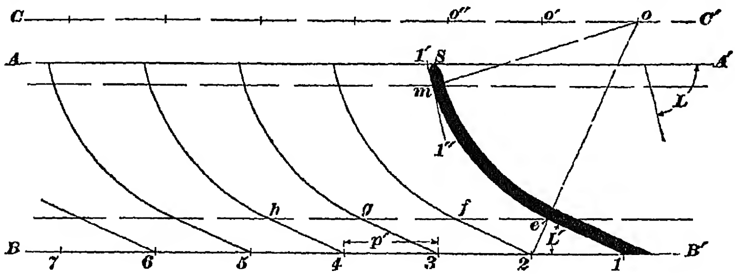

FIG. 8

a line $1'–1''$, making the angle L with AA' at a point m, a little below AA'. The line $1–e–m–1'$ gives the shape of the vane. The centers o', o'', etc. for the curved faces of succeeding vanes may be found by spacing off successively distances oo', oo'', etc., each equal to p' on a line CC' drawn through o parallel to AA'. The tops of the vanes are usually rounded off as shown at S.

27. Guides for Outward-Flow Turbines.—Referring to Fig. 9, divide the circle kk' limiting the outflow ends of the guides into as many equal parts as there are to be chutes. Draw the radius Oe to one of the points of divisions, draw also the line en, making the angle L with the tangent to kk' at e. Draw a perpendicular hg to Oe at its middle point h, and at e draw a perpendicular eg to en. The point g, where the perpendiculars eg and hg intersect, may be taken as the center from which to draw the circular arc ee' representing the convex surface of a guide. If the guides are of uniform thickness t, the concave surface may be drawn from the center g with a radius $ge–t$. The remaining guides are drawn with the radius ge and centers located on the circle gg' drawn through g with O as a center. The intersections g_1, g_2,

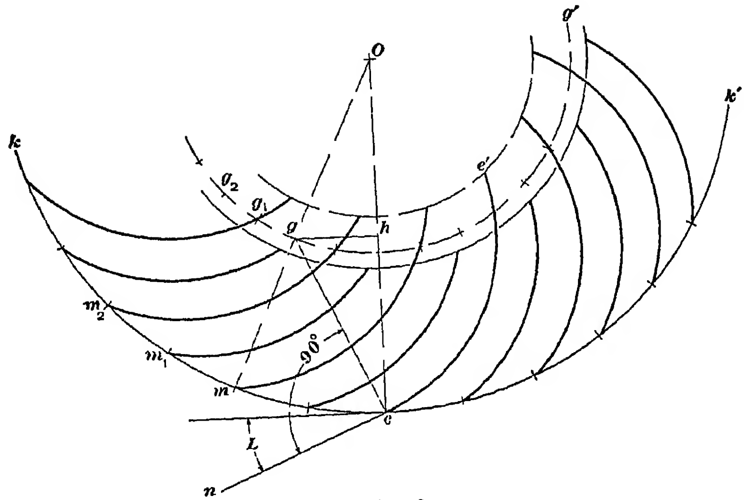
O
g'
k'
k
e'
g2
g1
g
h
m2
m1
m
90°
c
L
n

Fig 9

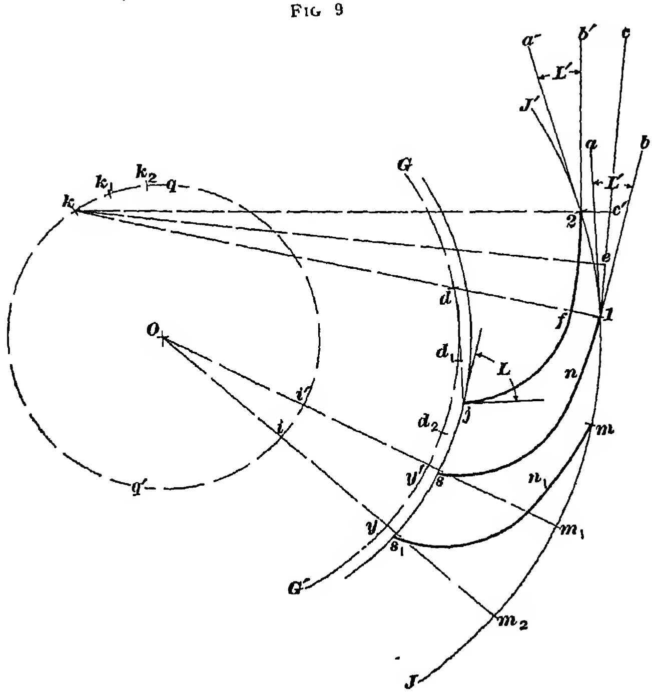
a'
b'
c
J'
L'
a
b
G
L'
k2
k1
q
k
2
c'
e
d
1
O
f
d1
L
n
i'
j
i
d2
m
y'
8
n1
q'
y
m1
81
G'
m2
J

Fig 10

etc of radial lines through m_1, m_2, etc with the arc gg' are the required centers The points m_1, m_2, etc are spaced at a distance from one another equal to the pitch of the guides

28. Vanes for Outward-Flow Turbines —Divide the outflow circle JJ', Fig 10, into a number of equal parts corresponding to the number of vanes Through two consecutive points of division, as *1* and *2*, draw the lines *1–b*, *2–b'*, making the angle L' with the tangent lines *1–a* and *2–a'* Bisect the angle *a–1–b* by the line *1–c* At *2*, draw a perpendicular to *2–b'*, intersecting *1–c* in *c'*, at the middle point *e* of *1–c'*, draw a perpendicular *ek*, to its intersection *k* with the perpendicular *c'–2* This part of the operation requires very careful and accurate work With *k* as a center and a radius equal to *k–2*, draw the arc *2–f*, meeting *1–k* at *f* If the buckets are widely spaced, the lines *kc'* and *ke* may not intersect within the drawing, in which case the portion *2–f* of the vane may be made a continuation of *b'–2*

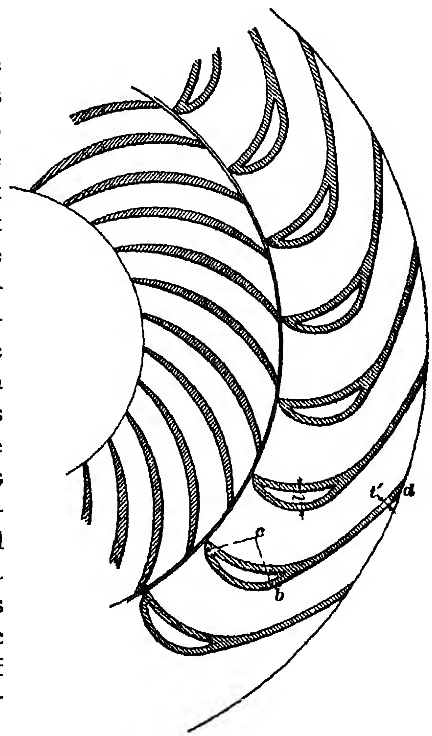

Fig 11

To draw the inner portion of the vane, choose a point *d* on *k–1* so that an arc *fj* drawn with a radius *df* will be tangent to a line making an angle equal to L with a tangent to the inflow circle at the point of intersection *j* Draw the circle qq' through *k* with *O* as a center, also, the radial

lines Om_1 and Om_2 to any two consecutive points of division on the outflow circle. Then, the centers k_1, k_2, for drawing the outer ends of the vanes $1\text{-}n$, mn_1, etc., may be found by spacing off from k distances kk_1, k_1k_2, etc., each equal to ii'. The centers d_1, d_2, etc. for drawing the inner ends ns, n_1s_1, etc. may be found on the circle GG' drawn through d with O as a center, by spacing off from d distances dd_1, d_1d_2, etc., each equal to yy'.

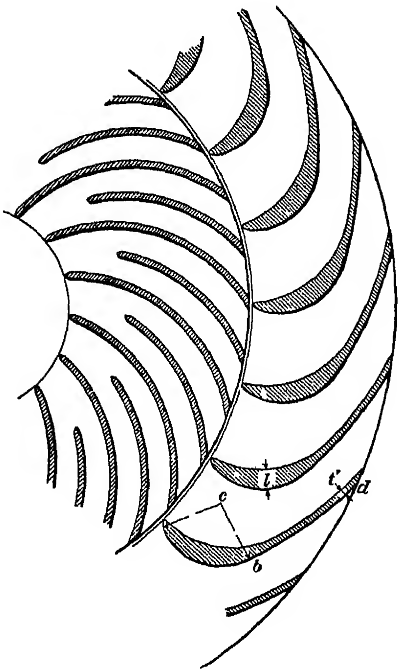

Fig. 12

29. Back Pitch or Thickening of the Vanes.—Figs. 11 and 12 show sections of Fourneyron turbines having the guides and vanes laid out by the methods given above. Both figures show the vanes drawn thicker at l so as to keep the cross-section of the bucket nearly constant. This thickening of the vanes is called **back pitch.** The center c of an arc that will give the desired form at the inlet end may be found by trial, and the part bd is so drawn as to give a smooth curved surface of the desired form and at the same time keep the thickness of the vane at least as great as t' at d. The form of the back pitch is usually so designed as to give the bucket nearly a uniformly decreasing cross-section. The vanes shown in Fig. 11 are cored out in casting to decrease the weight.

The convex surfaces of the guides in Fig. 11 are drawn with a smaller radius than the concave surfaces, in order to

increase the thickness near the center and so keep the area of the chutes nearly uniform

In Figs 9 and 12, the inner ends of the alternate guides are cut off in order to prevent the reduction in area of the inlet ends of the chutes that would result if these guides were prolonged toward the center to the full length of the other guides This construction is adapted for use with sheet-metal guides of uniform thickness

30. Guides for Inward-Flow Turbines.—Fig 13 shows a method of laying out the guides of a simple

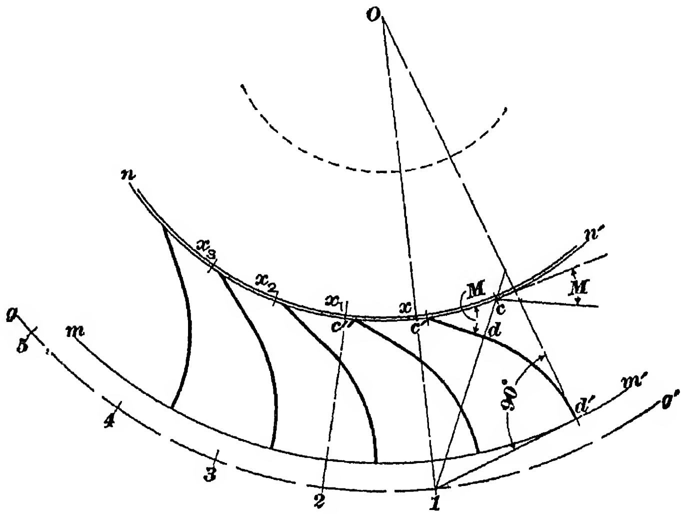

FIG 13

inward-flow turbine Draw the limiting circles $m\,m'$ and $n\,n'$. Divide the circle $n\,n'$ into as many equal parts as there are to be guides, and at each of the points of division draw a line making angle M with a tangent to $n\,n'$ From one of the points of division, as c, draw a perpendicular c–*1* to the line $c'\,d$ drawn through the next point c', and on this perpendicular choose a point *1* so that an arc drawn with *1* as a center and the radius *1*–d will meet the circumference $m\,m'$ tangent to its radius This gives the form of a guide $c'\,d\,d'$. The other guides may be drawn from centers located on the

circle gg' passing through *1* The position of the centers *2*, *3*, *4*, etc are found by drawing radial lines through the points x_1, x_2, etc spaced at distances equal to p on the circle $n n'$

31. Vanes for Inward-Flow Turbines.—Divide the outflow circle $k k'$, Fig 14, into as many parts as there are vanes, and through each of the points of division draw a line making an angle L' with the tangent Through one of the points of division, as *3*, draw a perpendicular *3–4* to the line *3′–4* through the next point, and on this perpendicular choose a point *6* so that an arc *4–5* drawn with *6* as a center and with a radius *6–4* will meet the inflow circle gg' in such a

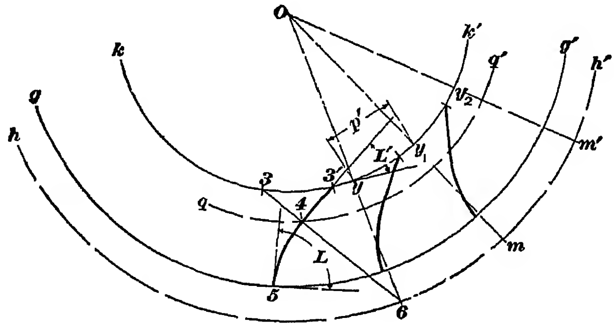

Fig 14

direction that a tangent to this arc at *5* will make the angle L with a tangent to the inflow circle The other vanes may be drawn from centers located on the circle $h h'$ passing through *6* The positions of the centers m, m', etc may be found by drawing radial lines to $h h'$ passing through the points y, y_1, y_2, etc spaced at a distance p', beginning at y on the inflow circle $k k'$ The curved portion of the vanes should terminate at the circle $q q'$ drawn through *4* from O as a center

32. Guides and Vanes of Mixed-Flow Turbines. The vanes of mixed-flow wheels are made in a great variety of forms, and each maker claims especial advantages for the peculiar form of bucket he uses The same general rules

regarding the velocities of inflow and outflow for the wheel, angles of buckets, and outflow areas of chutes and wheel buckets apply as have been given and illustrated for the simple forms of radial and axial turbines. The length of the path of the water in passing through mixed-flow wheels is usually greater and more crooked than in the case of simple axial or radial turbines, hence, in the former the loss by friction and shock is usually greater.

TURBINES BUILT FROM STOCK PATTERNS

33. In European countries, every turbine is usually designed for the special conditions for which it is intended. In the United States, every turbine builder makes a set of patterns, usually increasing uniformly in size from the smallest to the largest diameter commonly employed. These patterns are generally as nearly alike as they can be made, differing only in size, they usually embody the maker's own ideas, all peculiar features being nearly always patented. The leading features of each make of turbine have been developed from experiment or from tests, or in accordance with personal opinions, and are seldom based on definite calculations or thorough theoretical analysis.

34. A few Fourneyron turbines have been built from stock patterns, and turbines of the Jonval type are still so built to a small extent, but most of the stock-pattern turbines now on the market are of the American type, having inward and downward flow, and large ladle-shaped buckets. There is a wide variety of forms of runners, gates, and cases. As a result of the manner of development, there are still on the market many patterns of turbines that, although high results have been claimed for them, have never undergone any authentic tests, and embody features that violate the necessary conditions for good efficiency. The better class of American turbines, however, have been developed after repeated experiments to determine the best form of each part, and these turbines, *when operated under the conditions for*

which they are best adapted, usually give efficiencies as high as those obtained with turbines of special design.

Nearly all these turbines have been tested at the testing flume of the Holyoke Water-Power Company, at Holyoke, Massachusetts. The head used in testing turbines at this flume is usually about 16 or 18 feet. More turbines are in use with heads of from 10 to 20 feet than with heads either much greater or much less, and most stock-pattern turbines are specially adapted to heads of about 16 feet, for which they give their highest efficiency. The parts are made strong enough for heads up to 40 feet or more. For very large heads, special wheels, often with bronze vanes, are made; they are not designed according to theoretical principles, but represent modifications of the standard patterns.

35. As the weight and strength of a turbine must be sufficient for the greatest head to which the turbine is adapted, it follows that stock-pattern turbines are unnecessarily heavy for use under low heads. The principal advantages of American turbines made from stock patterns are their small diameter in proportion to their power and capacity, and their consequent high speed. They are cheaper than specially designed turbines of other types, both on account of their smaller size and because their construction does not require special patterns. Besides, the small size of such turbines greatly simplifies the setting, and has made possible the excellent forms of horizontal and double wheels enclosed in iron casings and connected with a draft tube. This makes it possible to place them above the tail-water in a position where inspection and repairs are easily made, and the power can be taken from them in a simple and direct manner.

36. Vent.—When turbines in scroll cases were extensively used, it was found convenient to express the discharging capacity of the turbine in terms of the size of an orifice in the side or bottom of the flume which would theoretically discharge the same amount of water as the turbine under the same head. The discharging capacity expressed in this way is called the **vent**, and is usually

given in square inches When the vent i (square inches) is given, the discharges Q and Q', in cubic feet per second and cubic feet per minute, respectively, can be computed by the formulas

$$Q = \frac{i}{144}\sqrt{2gh} = 0557\, i\sqrt{h} \qquad (1)$$

$$Q' = 60\,Q = 3\,34\, i\sqrt{h} \qquad (2)$$

where h is the head, in feet

The capacity of turbines and the water rights of mills are often expressed in terms of the vent in square inches Frequently, such rights do not specify any head, the owner being allowed to utilize the water under the greatest head available

EXAMPLE —How many theoretical horsepower is a miller entitled to whose deed calls for 1 square foot of water under a head of 16 feet?

SOLUTION —Since 1 sq ft is 144 sq in , the value of i is 144, so that $\frac{i}{144} = 1$ By formula **1**,

$$Q = \sqrt{2g \times 16} = 8\,02 \times 4 = 32\,08 \text{ cu ft per sec}$$

Then (see *Waterwheels*, Part 1),

$$\text{Power} = \frac{10\,Q\,h}{88} = 58 \text{ H P , nearly} \quad \text{Ans}$$

37. Manufacturers' Tables of Power, Speed, and Discharge —Nearly all American turbine builders publish rating tables in their catalogs, showing the discharge in cubic feet per minute, the speed in revolutions per minute, and the horsepower of every size of wheel they manufacture, for heads varying from 3 or 4 feet to 40 feet or more For turbines that have been tested at Holyoke, the quantities for different heads have usually been calculated from those determined at the speed of maximum efficiency for the head under which the wheel was tested The efficiency is usually assumed to be constant for all heads, although such is not really the case In nearly all catalogs, a constant efficiency of about 80 per cent is used as a basis in computing the horsepower

It should be borne in mind that the Holyoke tests are made under rather small heads, and that the conditions there obtaining are as a rule a great deal more favorable than those

which can be expected where the turbine is to do its actual practical work Therefore, a turbine does not usually yield the horsepower at which it is rated in the maker's catalog

38. In the following formulas, let

h = head on a turbine, in feet,
Q = discharge of turbine, or supply of water, in cubic feet per second,
N = revolutions per minute,
v = absolute velocity, in feet per second, of water issuing from chute,
A = aggregate outlet area of chutes, in square feet,
H = horsepower of turbine

Then, $$v = c\sqrt{2gh} = 8\ 02\, c\sqrt{h} \qquad (a)$$

where c is a coefficient that is practically constant for the same turbine Also, denoting the efficiency by η (see *Waterwheels*, Part, 1),

$$Q = Av = 8\ 02\, Ac\sqrt{h} \qquad (b)$$

$$H = \frac{10\,\eta\, Q h}{88} = \frac{10 \times 8\ 02\,\eta\, A c h \sqrt{h}}{88} = \frac{80\ 2\,\eta\, A c}{88} \times \sqrt{h^3} \qquad (c)$$

Formula **7**, Art **16**, gives, denoting the constant $\frac{9\ 549}{r_1'}$ by B,

$$N = B v_1'$$

or, replacing the value of v_1' from formula **3**, Art **12**,

$$N = \frac{BA \cos L'}{A_1'} \times v = \frac{BA \cos L'}{A_1'} \times 8\ 02\, c\sqrt{h} \qquad (d)$$

Let, now, H_c be the horsepower given in the maker's catalog for a wheel working under a head h_c and with a discharge Q_c, also, let N_c be the corresponding number of revolutions per minute Then, according to equations (b), (c), and (d),

$$Q_c = 8\ 02\, Ac\sqrt{h_c} \qquad (b')$$

$$H_c = \frac{80\ 2\,\eta\, A c}{88} \times \sqrt{h_c^3} \qquad (c')$$

$$N_c = \frac{BA \cos L'}{A_1'} \times 8\ 02\, c\sqrt{h_c} \qquad (d')$$

Dividing (b) by (b'),

$$\frac{Q}{Q_c} = \frac{\sqrt{h}}{\sqrt{h_c}},$$

whence
$$Q = Q_c \sqrt{\frac{h}{h_c}} \qquad (1)$$

Similarly, dividing (c) by (c'), and solving for H,

$$H = H_c \sqrt{\left(\frac{h}{h_c}\right)^3} \qquad (2)$$

Dividing (d) by (d'), and solving for N

$$N = N_c \sqrt{\frac{h}{h_c}} \qquad (3)$$

Formulas **1**, **2**, and **3** serve to compute the discharge, horsepower, and angular velocity for heads not given in the manufacturer's catalog In formula **1**, Q and Q_c may be discharges per minute Many catalogs give the discharge in cubic feet per minute, instead of per second

EXAMPLE —For a head of 56 feet, the discharge of a certain turbine is given in the manufacturer's catalog as 1,188 cubic feet per minute, the power as 100 6 horsepower, and the number of revolutions per minute as 745 What are the corresponding quantities for a head of 85 7 feet?

SOLUTION —Here h = 85 7 ft , h_c = 56 ft , Q_c = 1,188 cu ft per min , H_c = 100 6 H P , and N_c = 745 rev per min Formulas **1**, **2**, and **3** give, respectively,

$$Q = 1{,}188 \sqrt{\frac{85\ 7}{56}} = 1{,}470 \text{ cu ft per min} \quad \text{Ans}$$

$$H = 100\ 6 \sqrt{\left(\frac{85\ 7}{56}\right)^3} = 190\ 5 \text{ H P} \quad \text{Ans}$$

$$N = 745 \sqrt{\frac{85\ 7}{56}} = 921\ 6 \text{ rev per min} \quad \text{Ans}$$

39. Relation of Power, Speed, and Discharge to Size —Where the stock patterns of a turbine builder are of similar form, the depth of the buckets and the circumference of the inflow circle both vary in proportion to the diameter The inflow area is proportional to the product of these factors, and it is found that, for a given head, the capacity or discharge of most such types of turbines is proportional to the inflow area, or to the square of the diameter, the power

is proportional to the capacity, or to the square of the diameter, and the angular speed is inversely proportional to the diameter These relations can be utilized to determine the number of wheels of different sizes that would be required to furnish a given amount of power Suppose it is desired to replace, without change of power, two turbines, having a diameter D and a speed N, by a single turbine of the same pattern

Let D_1 = diameter of the single turbine,

N_1 = angular speed, in revolutions per minute, of the single turbine

Since the power of the single turbine must be twice the power of each of the two turbines of diameter D, we must have

$$D_1^2 : D^2 = 2 : 1,$$

whence $$D_1 = \sqrt{2\,D^2} = D\sqrt{2} = 1\ 41\ D \qquad (1)$$

Also, $$N_1 : N = D : D_1,$$

whence $$N_1 = \frac{D}{D_1} N \qquad (2)$$

or, since $\frac{D}{D_1} = \frac{D}{D\sqrt{2}} = 707$,

$$N_1 = 707\,N \qquad (3)$$

The listed size of a turbine should be the same as the diameter of the inflow circle of the runner, or of the circle surrounding the inflow ends of the vanes However, arbitrary size numbers differing from this are used by some builders The capacities of turbines of the same diameter, but made from the patterns of different builders, differ greatly As a rule, the later designs have the larger capacities

EXAMPLE —What should be the diameter and speed of a single turbine to replace two 18-inch turbines that are of the same pattern and make 100 revolutions per minute?

SOLUTION —From formula **1**,

$$D_1 = 1\ 41 \times 18 = 25\ 4 \text{ in}$$

If this size were used, the speed would be given by formula **3**. Usually, however, the calculated size is not a stock size, then, the nearest larger stock size should be used, and the speed computed by formula **2**. If, for example, a 27-in turbine is used, the formula gives

$$N_1 = \tfrac{18}{27} \times 100 = 66\ 7 \text{ rev per min} \quad \text{Ans}$$

40. Selection of Turbines.—Manufacturers build the greatest number of turbines of medium sizes, so that stock-pattern turbines of such sizes are both the cheapest and the most reliable. When the cost of flume and setting is considered, the total cost of a plant will in general be lower if large-sized turbines instead of a larger number of smaller turbines are used. The speed must also be considered, and it may be better to use small turbines, if the desired speed can be obtained directly by this means, than to use larger and slower-running turbines requiring jack-shafts to attain the desired speed.

In selecting a turbine, its efficiency at both full gate and part gate must usually be considered, and in addition its probable durability, freedom from obstruction, and ease of gate operation. In the absence of authentic tests, the probable full-gate efficiency may be judged from a comparison of the relation of the guide and entrance angles, the speed and the exit angle, with those given in connection with formulas for the design of turbines. The general construction of the wheel, the length, smoothness and regularity of the guide and bucket passages, and the freedom from sharp angles and abrupt changes of direction should also be considered. Authentic tests have been made of most of the reliable types of stock-pattern turbines.

ACCESSORIES

GATES

41. Classes and Requirements of Turbine Gates. The devices by which the admission of water to a turbine is regulated are called **gates**. Formerly, turbines were set in open flumes without gates or guides, but nearly all modern turbines are provided with gates and guides, and are enclosed in cases. The three principal kinds of gates used are *register gates*, *cylinder gates*, and *pivot gates*. They will be fully described presently.

Turbine gates should change the water supply with the least possible loss of efficiency There should be no lost motion either in the gates or in the operating mechanism The gates should not stick in any position, and should be so designed that they will move quickly, easily, and smoothly The total weight of the moving parts should be as light as is consistent with these conditions Without these features it will be difficult to govern the speed of the turbine There is a great difference in the power required to operate gates of different turbines, and many gates have a strong tendency to open or close, on account of unbalanced weight or water pressure, currents, or eddies A gate should be as nearly balanced in all positions as possible, so that, in moving it friction will be the only force to overcome

42. Gate Opening.—When the gate is not fully opened, the wheel is spoken of as operating at **part gate.** If ten turns of the gate stem are required to open the gate fully, an opening of six turns is spoken of as **six-tenths gate,** an opening of five turns is spoken of as **five-tenths** or **one-half gate,** and so on The terms *one-half gate*, *three-fourths gate*, etc are sometimes used to indicate that the wheel is using one-half, three-fourths, etc as much water, or that it is giving one-half, three-fourths, etc as much power as when operated at full capacity Neither of these meanings is strictly correct, because the power and water used by a wheel when operated at part gate are not generally proportional to the width of gate opening

43. Register Gates—Register gates may be of the **plate** or of the **ring type,** according as they are applied to parallel-flow or to inward-flow turbines In each class of turbines, register gates are sometimes used outside and sometimes inside of the chutes

Outside register gates, adapted to the Jonval type of wheels and to plain inward-flow turbines, were named from their similarity to a common hot-air register In a register-gate turbine the guides are thickened so that they have a width at the gate end equal to the width of the chutes The

gate consists of alternate openings and covers equal in width to the chutes. When the gates are closed, the covers lie over the chute openings. As the gates are opened, the covers slide back over the guides. In Fig. 15 is shown a section of a turbine with register gates. Half of the guides *c* are replaced by fillers *a* against which the gates *b* lie when opened. Owing to the thickening of the guides in a register-gate turbine, water can be admitted to only one-half the inlet surface of the runner.

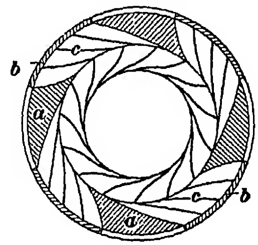

Fig. 15

A plate register gate supports the pressure due to the head directly on its surface. It is difficult to counterbalance, is likely to stick when closed, and, owing to the great friction to be overcome, usually moves hard at part gate.

Ring register gates were formerly much used on turbines of the American type. As, however, they greatly decrease the water capacity of the turbine, they have been almost entirely superseded by cylinder and pivot gates. A ring register gate is nearly self-balancing. If so constructed that it does not rub, it will open and close easily, its only bearings being on the turbine shaft.

44. Register gates are easily blocked by obstructions, which prevent the gate from closing until the water is drawn from the penstock and the obstruction removed. In both Jonval and American turbines, it is best to place the register gate outside of the guides. The chutes should be made long, so that the entering veins of water can expand and entirely fill the chutes and buckets. The water will then enter the buckets at part gate at a better and more uniform angle than if inside register gates are used. When running at part gate, the bucket of a register-gate turbine may be only partly filled, especially near the outer side.

In order to open or close the gate, it is only necessary that the covers should rotate about the axis of the turbine through a distance equal to their width. They are usually

operated by a segment of rack secured to the gate ring and meshing with a pinion on the gate stem

45. Cylinder Gates —A **cylinder gate** consists of a hollow ring, like a short section of pipe, that slides up and down around the inlet portion of the runner, and so regulates the water supply Cylinder gates are placed either outside of the guides or between the guides and runner—more frequently in the latter position They are used on Fourneyron, Francis, and American-type turbines

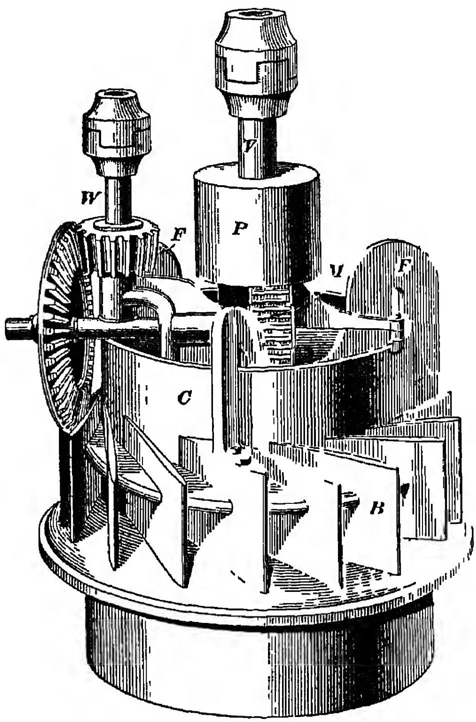

Fig 16

Fig 16 is a general view and Fig 17 a section of a Risdon turbine This is an American type of wheel with an inside cylinder gate C that works in a space between the runner A and the guides B The gate is raised and lowered by means of a rack and pinion M, operated by a hand wheel or by a governor acting through the shaft W and the bevel gearing The **U**-shaped pieces F, F support the crown plate E, and rest on the guide vanes There is a stationary cylinder P supported by the crown plate, in this cylinder is a piston $O\,O$ that serves to balance the weight of the gate by the action of the pressure of the water under it The wheel shaft V is supported by the wooden step U and the bearing K Projections D, called **garnitures**, are cast on the

cylinder C and move up and down between the guides with it Various forms of garnitures have been devised As a rule, they increase the part-gate efficiency somewhat, but they may also cause a strong downward pressure that tends to close the gate and is difficult to counterbalance

When a cylinder-gate turbine operates at part gate, the water is shut out of the upper part of the runner A partial vacuum may be formed, causing the water to rise and nearly fill the runner, or, if the gate opening is small, the turbine may act almost wholly by impulse, then becoming practically an impulse wheel

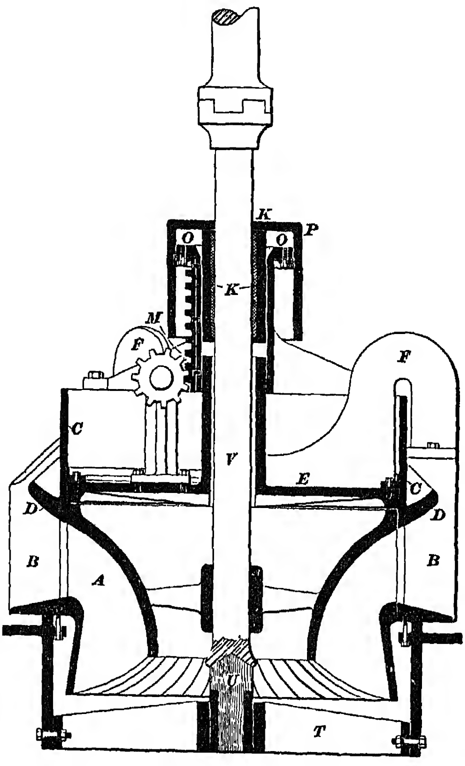

Fig 17

46. Division Plates —The runner of a cylinder-gate turbine is sometimes subdivided by partition plates into separate sets of compartments These compartments form virtually separate runners from which the water is successively shut off as the gate is closed

The Niagara Fourneyron turbine shown in Fig 21 is an outside cylinder-gate wheel divided in this manner Fig 18 shows a runner of a Jones Little Giant turbine subdivided into two parts by the division plate a, one part has about one-third and the other about two-thirds of the full capacity For

ordinary conditions, the larger division is used When the head is reduced by backwater, the water is admitted to both compartments, thus increasing the capacity Good efficiency under ordinary conditions is secured in this manner without the necessity of maintaining an extra wheel to keep up the power in periods of reduced head The device is useful in cases where variation in speed is permissible

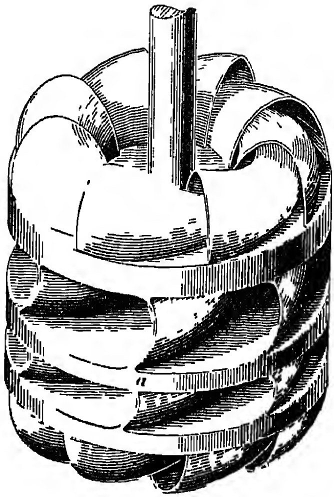

FIG 18

Where division plates are used, the surface-friction loss through the runner is greater than for a runner of the same capacity without partitions The water probably leaves the wheel at a better average angle of exit where the channels through the runner are small than if a single deep bucket is used

47. A divided runner gives nearly full efficiency through a much wider range of gate opening than an undivided runner of the same capacity If the efficiency of any one part of a divided wheel is the same as that of an undivided wheel at full gate, then a wheel with one division plate will give the same efficiency at half as at full gate, and a wheel with two division plates will give full-gate efficiency both at one-third and at two-thirds gate

Let η_1 = full-gate efficiency of either the single runner or of one part of the divided runner,

η_2 = efficiency, for a proportional discharge q, of one part of the divided runner

When the divided runner has n parts full-opened and one partly opened so as to discharge the proportional amount q, the total efficiency η is given by the formula

$$\eta = \frac{n\eta_1 + q\eta_2}{n + q}$$

EXAMPLE 1.—If the full-gate efficiency of one part of a divided turbine is 80 per cent., and the one-half-gate efficiency is 65 per cent., the efficiencies of an undivided turbine at the same proportional discharge being the same, what will be the gain in efficiency by the use of two division plates when the turbine is running at one-half gate?

SOLUTION.—The divided runner will contain three parts. At half gate, one part will operate at full discharge, another at half discharge, and the third part will be closed. Here, $n = 1$, $q = \frac{1}{2}$, $\eta_1 = 80$, $\eta_2 = 65$, and the above formula gives

$$\eta = \frac{1 \times 80 + \frac{1}{2} \times 65}{1 + \frac{1}{2}} = 75 = 75 \text{ per cent}$$

The efficiency of the undivided runner being 65 per cent., the difference, 10 per cent., is the gain in efficiency. Ans.

EXAMPLE 2.—What will be the efficiency of the divided turbine at three-fourths discharge if the efficiency of one part at 24 discharge is 40 per cent., and at three-fourths discharge is 79 per cent.?

SOLUTION.—At three-fourths discharge, two parts will run full capacity, making 67 discharge, and (75 − 67), or 08, of the full capacity of the runner will pass through the third part. The capacity of one part is one-third that of the whole runner. This third part will therefore operate at 3 × 08, or 24, of its full capacity. Here, $n = 2$, $q = 24$, $\eta_1 = 79$, $\eta_2 = 40$, and the formula gives

$$\eta = \frac{2 \times 79 + 24 \times 40}{2 + 24} = 748 = 74.8 \text{ per cent} \quad \text{Ans.}$$

48. In order to accomplish the same result as if a division plate were used, the Case National turbine is constructed as shown in Fig. 15. This is an outside register-gate wheel. The chutes, which are in groups of four, are separated by fillers covering an arc of the circumference equal to that of the group of guides. The covers *b* of the register ring slide back over the fillers, opening one after another of the chutes. The course of the water through the chutes is well regulated, but the buckets may be only partly filled at part gate, as in the case of an undivided cylinder-gate wheel.

49. Pivot, or Wicket, Gates.—In pivot-gate turbines, the gates are so arranged as to form guides also. A common way of doing this is by using gate leaves pivoted between the guide rims. Fig. 19 is a section of a Smith Success turbine showing the pivot gates. The gate leaves *a* swing about the axis *b*. The gates are shown full-opened.

If the inner ends of the leaves are swung outwards, the width of the chutes at x will decrease, and, with sufficient movement of the leaves, the chutes will be closed The gate leaves are opened and closed simultaneously by means of a circular ring connected to their outer ends and bearing a rack operated by a pinion connected to the gate stem In the Leffel and new American turbines, the pivot gates are operated by linkages connecting the gate leaves to a collar on the main shaft of the turbine The collar is rotated by a rack and pinion as above described

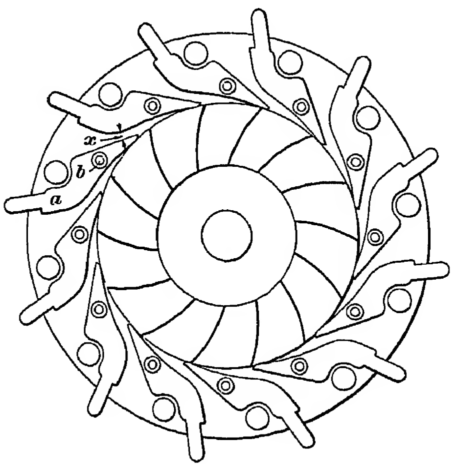

FIG 19

From Fig 19 it will be seen that the angle of outflow from the chutes relative to the runner is greatest when the gates are full-opened, and decreases as the gate opening is decreased It follows that the proper relation between the guide angle and the velocity of the entrance circle can only be obtained for one position of the gates—usually when full-opened—while at other gate openings there is interference and loss of energy in impact and eddies at entrance There are, however, several devices by which the change of entrance angle is at least partly avoided

50. Turbines with pivot gates contain more parts than those with cylinder or register gates, and are often more liable to obstruction, leakage, and breakage than other forms In order to prevent leakage and secure the best conditions of entrance of the water, the crowns as well as the top and bottom and the outflow edge should be finished and fitted

Pivot gates usually give the entering jets better form and avoid the sharp contraction common with register gates when partly closed They do not shut the water entirely out of the upper part of the buckets, as may be the case with a turbine having a cylinder gate, when operated with the gate partly closed For a given diameter and depth of wheel, the entrance area obtainable—and hence the capacity of the wheel—is larger than for a register-gate turbine, but smaller than for a cylinder-gate turbine

Pivot gates, unless properly balanced, usually have a strong tendency to open when partly closed

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BEARINGS

51. Foot-Step Bearing —For wheels on vertical shafts, the weight of the runner and shaft, and of the gearing, pulley, or dynamo armature at its upper end, is usually supported by a **foot-step bearing** at the lower end of the turbine shaft A common form of foot-step bearing is shown at *U*, Fig 17 it consists of a block of lignum vitæ whose upper end is either conical or made in the form of a segment of a sphere, and whose lower end rests on a cross-bar *T*, called the **bridge tree.** The lower end of the turbine shaft is turned cup-shaped to fit over the upper end of the foot-step bearing This end has sometimes four grooves radiating from the center, to give the water access to the bearing for the purpose of lubrication The thrust is taken on the ends of the fiber of the wood, and the block is usually adjustable vertically so that wear can be taken up As wear takes place, the runner tends to fall lower in the case than its original position If this is not corrected, the clearance may increase, causing leakage, or the runner may rub on the case

52. The pressure, or thrust, on the bearing includes the weight of the attached parts, and the hydraulic pressure caused by the water in passing through the wheel In an axial or Jonval turbine, this pressure is relatively great In a radial turbine, there may be little or no hydraulic thrust In an inward-and-downward-flow turbine, there is first an

upward pressure on the vanes due to the deflection of the water from an inward to an axial direction, and then a downward thrust due to the deflection of the water from an axial to a radially outward direction as it leaves the wheel In addition, there may be a downward pressure on the runner

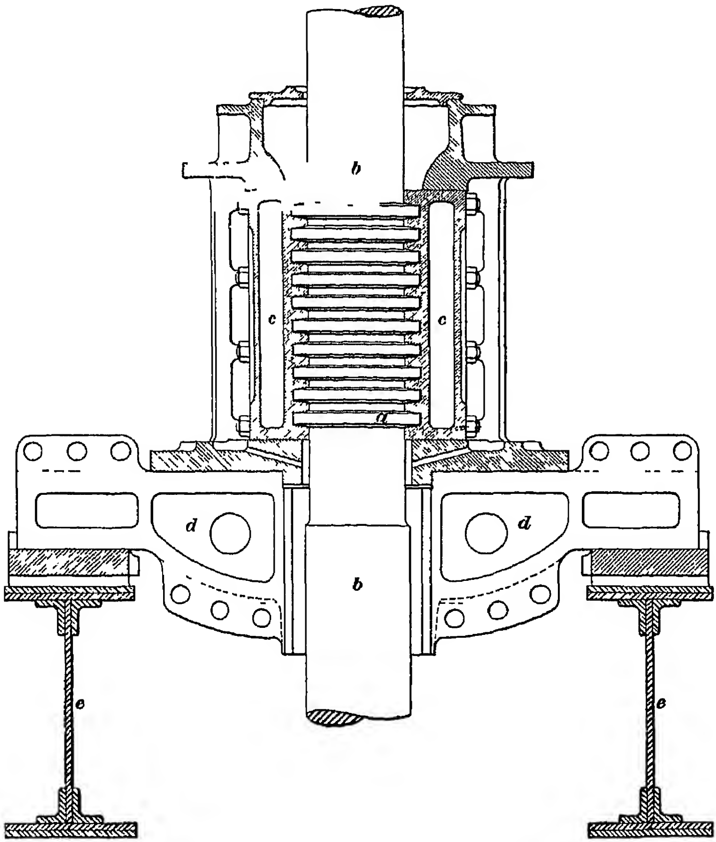

Fig 20

disk, due to water passing through the clearance between this disk and the crown The resultant of these pressures is usually a downward thrust When the wheel is mounted on a horizontal shaft, the thrust due to the weight of the parts disappears, but the hydraulic thrust remains the same,

and what have been described as upward and downward thrusts become pressures in the direction of the bottom and top of the runner, respectively

53. Wheels placed on horizontal shafts are commonly mounted in pairs discharging in opposite directions, and by this method the thrust is nearly neutralized A collar bearing should, however, be provided to take care of inequalities of pressure

A collar thrust bearing placed on the shaft above the turbine is sometimes used Such bearings have the advantage that they are readily accessible for inspection and lubrication, and that they can readily be kept free from grit contained in the water Grit is often carried into a bearing placed on the bridge tree, causing the bearing to cut and wear

Fig 20 shows a collar bearing used to take part of the weight and pressure of a 5,000-horsepower turbine at Niagara Falls, as the weight to be supported is great, a large bearing surface is necessary in order to keep the pressure per square inch on the bearing within a safe limit In order to accomplish this, and at the same time keep the diameter (and hence also the moment of friction and loss of power) as small as possible, a series of ten narrow bearing rings *a* was placed one above another, on the main driving shaft *b b* Oil or water to cool the bearing circulates in the chamber *c c* The weight of the shaft and suspended machinery is carried on the girder *d d*, which rests on the I beams *e*, *e*

54 **Water-Balanced Turbines**—Fig 21 shows a partial cross-section of the double Fourneyron turbine used in the first installation of the Niagara Falls Power Company This turbine is operated under a head of about 135 feet It is mounted in a cast-iron penstock similar to that used in early New England practice, with the exception that two wheels are used, one being placed at the top and the other at the bottom of the penstock As shown in the figure, the runners *i*, *i'* are attached to the vertical shaft *k* The chutes and buckets are subdivided into three compartments by partition plates *e e*, *e' e'* The discharge is regulated by

outside cylinder gates f, f' The gate rings for the upper and lower wheels are connected by rods, one of which is shown at j The gate rings f, f' are raised and lowered simultaneously to shut off the outflow from, or to open, the horizontal compartments one after another, as required The cylindrical penstock is shown by cross-section lines The disk, or drum, g forming the lower end of the penstock

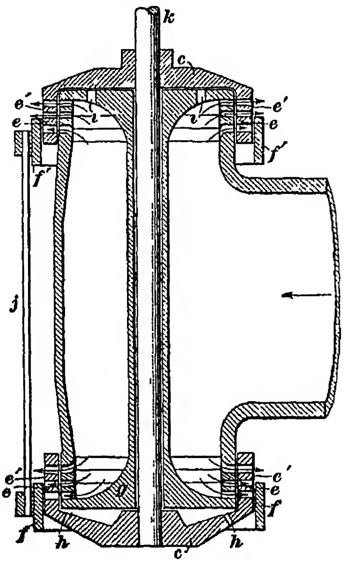

FIG 21

is made solid, and holes h, h are provided in the lower runner to let out any water that may enter between the lower drum g and the lower runner through the clearance spaces Holes i, i are provided in the upper penstock drum to allow water under full pressure of the head to pass through and act vertically against the upper runner c' In this way, the vertical pressure of the great column of water is neutralized, and a means is provided to counterbalance the weight of the long, vertical shaft and the armature of the dynamo at its upper end These turbines discharge 430 cubic feet per second, make 250 revolutions per minute, and are rated at 5,000 horsepower.

Where a single vertical runner is used, a piston is sometimes placed on the shaft revolving in a cylinder placed either above or below the runner Water under flume pressure is admitted underneath the piston The upward pressure of the water supports the weight of the rotating parts

DRAFT TUBES

55. Draft Tubes of Constant Diameter.—Let D, Fig. 22, be a turbine in a tight vertical penstock A, which connects the reservoir B with the tailrace C. This vertical penstock is called a **draft tube**. The total head, or difference in elevation between the surface of the water in the reservoir and the surface of the tailrace, is denoted by h, as shown. This head is made up of the head h_1 above the turbine, and the head h_2 between the turbine and the level of the tailrace. The pressure of the atmosphere, acting on the surface of the water in the reservoir, and also on the surface of the tailrace water, is equivalent to a head of about 34 feet; this head will be denoted by h_a. Now if the turbine is entirely closed, so that no water can pass through it, the pressure on the top is evidently equal to the pressure due to the head h_1 plus the atmospheric pressure, and the upward pressure on the under side is equal to the pressure of the atmosphere minus the pressure due to the head h_2.

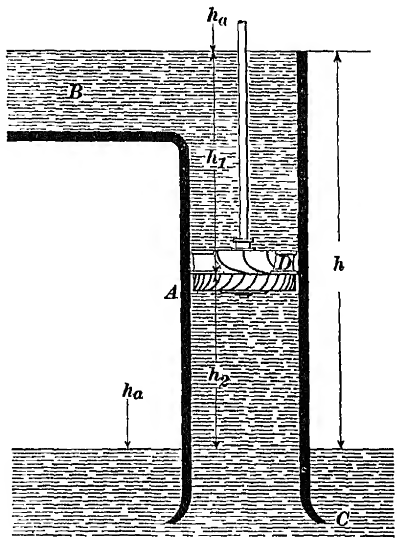

FIG. 22

The pressure that tends to produce flow through the wheel is, according to the principles of hydromechanics, the difference between the pressure on the two sides of the turbine; hence, if for the pressures are substituted their equivalent heads, the head that tends to produce the flow is

$$(h_1 + h_a) - (h_a - h_2) = h_1 + h_2 = h$$

56. Turbines are sometimes placed below the surface of the tail water, as shown in Fig. 23, in which case they are

said to work "drowned" Here, the effective head is still the difference h in level between the surface of the water in the reservoir and the surface of the tail-water, as will be made clear from the following With the notation shown in the figure, the total head on the top of the turbine is $h_1 + h_a$, and the head on the under side of the turbine is $h_a + h_2$ Therefore, the resultant head is

$$(h_1 + h_a) - (h_a + h_2) = h_1 - h_2 = h$$

57. It will be observed that, by the use of a draft tube, the turbine can be placed far above the tailrace, without any loss of head, and this makes the wheel more easily accessible for inspection and repairs

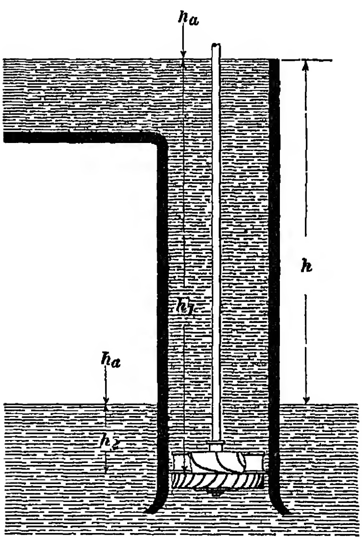

Fig 23

The theoretical limit of the distance h_2, Fig 22, that the turbine may be placed above the surface of the tail-water is never greater than 34 feet, since that is the limit of the height of a column that will be supported by the pressure of the atmosphere The expression $h_a - h$ for the pressure head under the wheel shows that this pressure is always less than the pressure of the atmosphere, and is decreased as h_2 is made greater Owing to this reduced pressure, there is a tendency for the air to leak into the draft tube, air will also separate from the water that passes through the wheel If the tube is very long, this air may collect in the upper end, thus reducing the head h_2, and, consequently, the total effective head $h_1 + h_2$ For these reasons, turbines, unless they are very small, are seldom placed more than 15 feet above the level of the tail-water

58. The diameter of a draft tube is generally fixed by the design of the wheel Draft tubes are best made of cast iron or riveted plate, and, in all cases, must be thoroughly air-tight Wooden tubes are sometimes made, but are not to be recommended on account of the difficulty in preventing leakage The lower ends should extend at least 4 inches below the surface of the tail-water at its lowest stage, and must open into the tailrace in such a manner that the outflow will be free Any obstruction to the flow from the draft tube causes a loss of effective head, and a consequent loss of efficiency Circumstances sometimes require that draft tubes should be made curved or be placed in inclined positions, straight, vertical draft tubes are, however, preferable, because short bends or unusual lengths cause an appreciable loss of head

59. Expanding Draft Tubes —The efficiency of a turbine in which the absolute velocity of discharge from the wheel vanes is high may be increased by the use of a draft tube whose cross-section increases gradually with the distance from the wheel A tube of this kind is called an **expanding draft tube** Such tubes are usually made from steel plates in the form of a tube of uniformly increasing diameter, and are often called **conical draft tubes.** The area of the tube at the wheel should be nearly equal to the discharge area of the wheel buckets, in order to prevent a sudden change in velocity in the entering water, and its section should be gradually enlarged toward the outlet

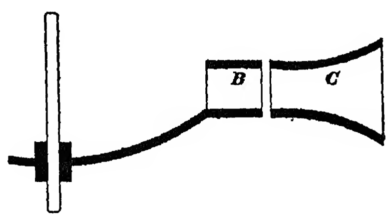

Fig 24

60. The **Boyden diffuser**, shown in Fig 24, is a device used on outward-flow turbines for the same purpose as an expanding draft tube It consists of a stationary annular casing C which surrounds the wheel, and into which the water from the wheel buckets B is discharged The area of the passages through

this casing gradually increases from the wheel outwards, as shown The result is a decrease in the velocity of the outflowing water

61. The value of a diffuser or of an expanding draft tube depends on the absolute velocity of flow from the wheel buckets If this velocity is small, the water carries very little energy with it, and there will be little gain by the use of any device intended to check the velocity of the spent water It sometimes happens, however, that, with a given diameter of wheel or a given number of revolutions, the velocity of outflow from the wheel cannot be made small, and then a diffuser or a draft tube is of much value

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GOVERNORS

62. Variations in Speed —In many classes of work, the load on a turbine is subject to constant change This is especially the case with turbines that drive electric generators It is, however, advisable to keep the speed as nearly uniform as possible This end may be attained by varying the gate opening, which can be done either by hand or by means of a device, called a **governor**, that is operated by the turbine itself, and works, therefore, automatically

Variation in the speed of a waterwheel may be due to changes either in the load or in the head, or in both Usually, changes in the load are sudden, while variations in the head are gradual The ease or difficulty of governing a waterwheel is chiefly controlled by the following factors (1) the cause of speed variations, according to whether they are due to changes in the load or in the head, or in both, (2) the magnitude and frequency of such variations, (3) the weight of the gate mechanism and the force required to move it, (4) the size and kind of flume, (5) the kinetic energy stored in the turbine runner and other revolving parts connected to the turbine shaft If the head did not vary, a certain gate opening would always give the proper speed for a certain load, but, with a varying head, the proper gate opening for a given load and speed will vary

63. Classes of Waterwheel Governors.—All waterwheel governors are equipped with centrifugal weights similar to those on a steam-engine governor, driven by a belt from the waterwheel shaft. A change in the position of these weights, resulting from a change in the speed, starts the mechanism that opens or closes the gates.

Waterwheel governors are classed as **friction-gear, ratchet-and-pawl, differential-gear, electrical,** and **hydraulic,** according to the means employed to open or close the gates. In hydraulic governors, the gate is operated either by water under a pressure due to the head in the flume, or by a piston driven by oil kept under a constant pressure by means of a power-driven pump. In most other classes of governors, the waterwheel gate is opened or closed by power from the turbine shaft.

In the simpler forms of governors, when the speed varies from the normal at which the wheel is intended to run, the movement of the centrifugal weights connects the gate mechanism with the waterwheel, and the gates begin to open or close, and continue to do so, usually at a uniform rate, until the speed returns to the normal.

64. The Snow Waterwheel Governor.—Fig. 25 shows a general view of the Snow waterwheel governor, and Fig. 26 is a diagram showing the principles of its action. The shaft b is driven from the wheel shaft by a belt on the pulley a, and drives the spur wheel c by a pinion. The shaft to which c is keyed carries a bevel wheel d and a crank e. Two pawls f, f' on the arms l are given a rocking motion by means of the crank e and the connecting-rod m. A cam, formed of two arms n, n, and operated by the governor balls, acts on the pawls as follows: When the wheel is running at the normal speed, the cam is held in its central position, as shown in the diagram, and holds both pawls away from the ratchet wheel o. If the wheel runs too slowly, the governor balls drop and move the cam to the right, thus allowing the pawl f to engage the ratchet wheel, and turn it to the left. The motion of the ratchet wheel is transmitted

to the gate shaft *i* through the bevel gears *g*, and as the ratchet turns the gate is opened, thus admitting more water to the wheel If the wheel runs too fast, the cam is moved to the left, bringing the pawl *f'* into action this turns the ratchet to the right, and partly closes the gate The spur wheel *h*

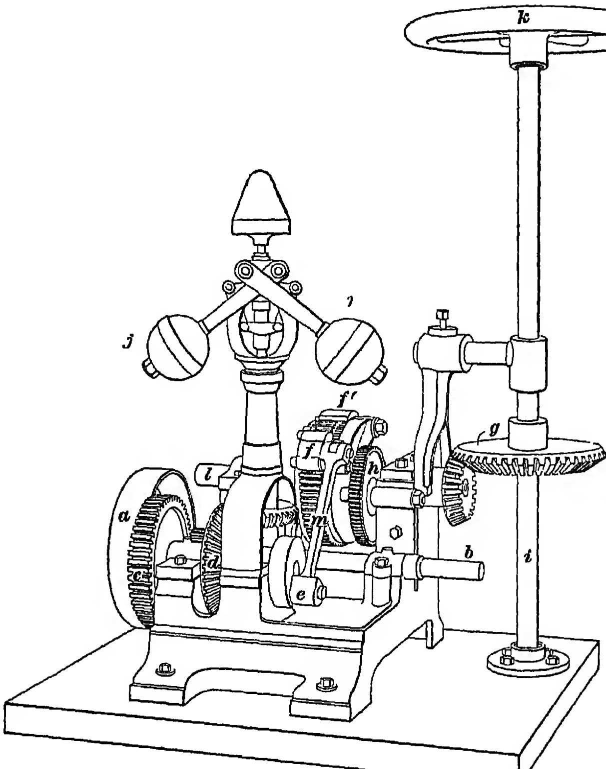

FIG 25

acts through a pinion on the ratchet shaft to operate a stop that disengages the ratchet *f* when the gate is fully opened In order to stop the wheel, the pawl *f'* is disengaged by hand, thus leaving the gate shaft free to be turned by the hand wheel *k*

65. The Replogle Governor.—Fig. 27 shows the Replogle waterwheel governor The centrifugal balls *g* are

driven by a belt from the main shaft A rise or fall in the speed of the shaft causes a corresponding rise or fall in the lever *b*, which forms part of an electric circuit When the lever *b* is in contact with the screw *d*, an electric magnet *a*, forming part of the gate-operating mechanism, becomes energized, and by its attraction throws a pawl *e* into a ratchet wheel *f*, by which the waterwheel gate is closed If, on the contrary, the speed decreases, the lever *b* comes

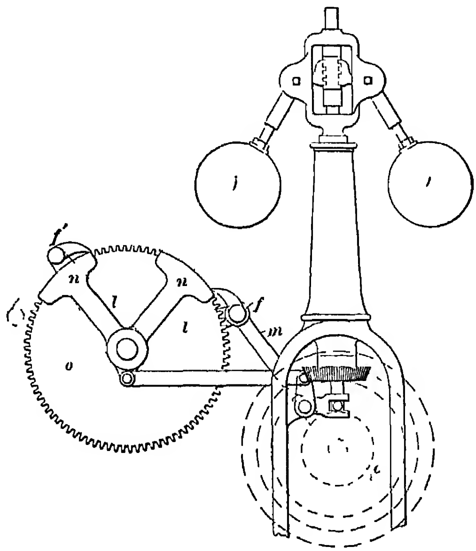

FIG 26

in contact with the screw *c*, and completes the electric circuit by which another pawl *h* is thrown into gear, which operates to open the waterwheel gate

66. The Lombard Governor—A very desirable feature of a waterwheel governor is that it should make the necessary change in the gate opening as rapidly as possible Some time is required for the wheel and the connected mechanism to adjust themselves to the change of load If, for example, the wheel has been operating at full gate, and one-quarter of the load is suddenly taken off, the speed will

increase, and will remain above the normal for some time, even if the gate has been closed by the proper amount In most governors in which the gate is operated by power from the wheel shaft, the gate movement is comparatively slow This lag in the gate regulation prolongs further the time required to regulate the speed In most simple governors, the gate will continue to close as long as the speed remains above the normal, and vice versa It follows that the gate will be closed or opened too far before the governor speed returns to normal and disconnects the gate mechanism from the driving shaft When

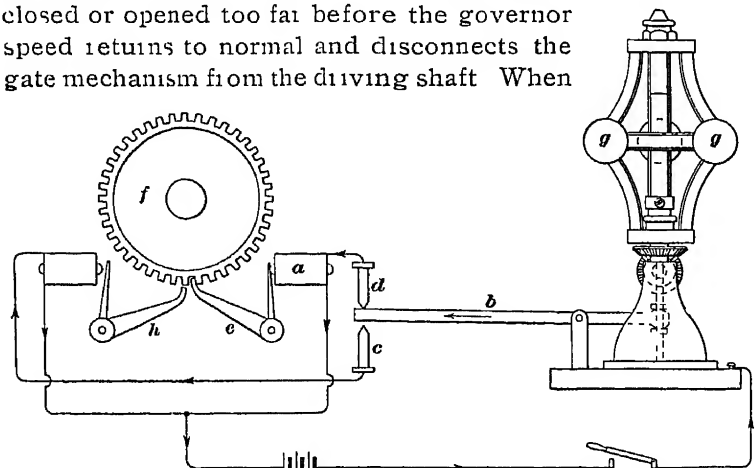

FIG 27

this is done, the speed of the governor will begin to change from the normal, and the gate will again begin to move The process will again be carried too far In order to prevent this "see-sawing" or "racing," as this is called, means are employed, first, to open or close the gate very quickly as soon as there is a change of speed, and, second, to stop the opening or closing of the gate before the speed returns to normal

Rapid gate operation is accomplished by the use of hydraulic pressure or by quick-acting clutches to open or close the gate The stopping of the gate movement in advance of the return of the speed to the normal is accomplished by what are called **compensating**, or **returning, devices.**

67. The **Lombard waterwheel governor**, which is a hydraulic governor with a compensating device to prevent racing, is shown in Figs 28 and 29, the latter being a simplified sketch of a part of the mechanism The same letters refer to similar parts in both figures The tank *a* contains oil kept under constant pressure by means of an air chamber and power pump This oil acts as a reservoir of power that is used to operate the pistons contained in the cylinders *b* and *d*, and to open or close the waterwheel gates The flyball governor *n* is driven by a belt from the wheel shaft As the speed changes, the governor balls raise or lower the valve stem *o'*, which controls the admission of oil from the reservoir *a* The piston in cylinder *b* then forces the rack *h* backwards or forwards, according as the speed is to be increased or decreased The rack *h* meshes with the floating gear *g*, which in turn meshes with the pinion *f* on the turbine gate shaft *m* The axle of the floating gear *g* is not fixed, but this gear can move backwards or forwards a short distance between the rack above and the pinion below, carrying with it the valve stem *k* The valve in the chest *c*, which is operated by this stem, controls the admission of oil under pressure to the main cylinder *d*

Consider the rack *h* as having moved forwards The gate shaft *m* and pinion *f* have not moved, and hence the floating gear *g* and valve stem *k* are moved forwards at the same time as the rack *h* The oil under pressure is admitted from the chest *a* to the main cylinder, and drives the main rack *e* forwards This rack rotates the pinion *f* and the gate shaft *m*, thereby opening or closing the gate At the same time, the pinion *f* rotates the floating gear *g*, moving it backwards, and thus restoring the valve in the chest *c* to its middle position, cutting off the admission of oil to cylinder *d*, and preventing any further motion of the gate shaft The system of cylinders and valves *b*, *c*, and *d* is called a **relay** The object of the duplicate cylinders and valves is to reduce the size of the valve that must be controlled by the centrifugal balls, and thus enable the large valve, necessary to control the pressure cylinder *d*, to be operated by the pressure caused on the valve

stem *o'* by a small change in the speed of the governor balls

The manner in which the motion of the rack *h* is stopped automatically at the same time that the wheel gate is being moved will now be considered The rack *h* is connected to the valve stem *o o'* by a system of levers and linkages similar

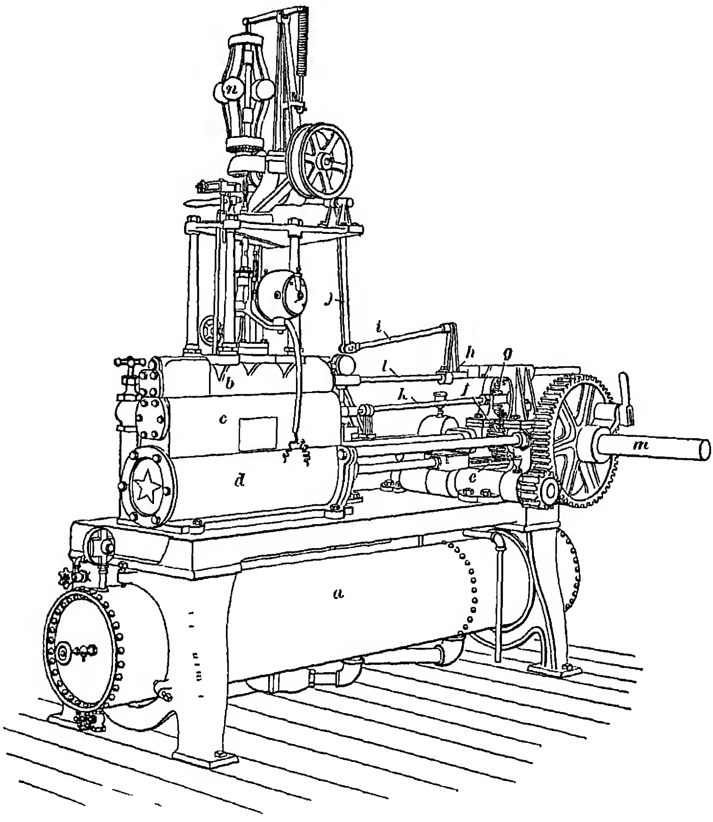

FIG 28

to *i*, *j*, *q*, *t*, *u*, Fig 29 The lever *j* is connected at one end to the rack *h* by the link *i*, and is pivoted at *p* to the frame of the governor The other end is connected to the link *q q'*, which contains the dashpot *r* The link *q q'* moves backwards and forwards in the opposite direction to the motion of the rack *h*, and it in turn rotates the nut *t*, which meshes

with a thread cut on the lower part o' of the valve stem. The rotation of this nut raises or lowers this part of the valve stem independently of the rise or fall of the upper valve stem o. If, for example, the speed changes, raising the valve stem, and forcing the rack h forwards, this will in turn move the link $q\,q'$ backwards, rotating the nut t and lowering the valve stem o', shutting off the oil supply from cylinder b, Fig. 28, and thus stopping the movement of the racks h and e.

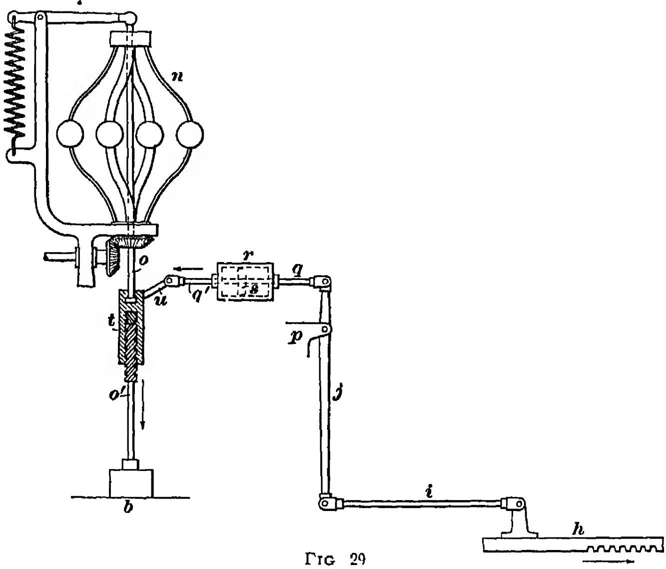

FIG. 29

The dashpot r consists of a closed cylinder filled with oil and containing a loosely fitting piston. The oil allows the piston to move slowly in either direction without great resistance, or the dashpot itself may move and the piston remain stationary. The oil is not compressible, and, owing to the small space through which it must flow past the piston when any movement takes place, it resists any sudden movement of the piston, so that, if the right-hand portion of the link $q\,q'$ is quickly moved, it will carry the dashpot and the left-hand portion q of the link with it. A sudden movement of the rack h will move both piston and dashpot, and also the valve stem o'. The dashpot will, however, permit the valve stem

to return slowly to its central position under the action of a spring

The lower valve stem o' responds directly to any rise or fall of the governor balls, and in addition it responds to any sudden movement of the rack h. The mechanism is so adjusted that a movement of the valve stem o' from the latter cause can occur only after the former movement, and is in the opposite direction. The relay, the pressure reservoir, and the compensating device working together enable the gate to be changed almost instantly to the full amount required to maintain the speed constant, and at the same time the change in the gate opening is stopped before the speed is readjusted, thus preventing racing.

The dashpot is so adjusted as to allow the valve stem o' to return to the position corresponding to the normal speed by the time the speed is readjusted, and the governor is then ready for another change. If, for example, the first gate movement is not exactly the necessary amount, a second smaller movement will take place. It is desirable to adjust the governor so that it is as nearly "dead beat" as possible, that is, so that it will make very nearly the proper change in gate opening at the first movement.

68. Regulation Where the Head Varies.—In most streams, the available head is least in times of freshet, when the discharge is greatest, this is due to the rise of the water in the tailrace. The power of a waterwheel varies as the three-halves power of the head, while the speed varies as the square root of the head. If the head is reduced by backwater, the requisite power can be maintained by the use of additional units or turbines. If, however, the same turbines are used when the head is reduced, an intermediate or auxiliary shaft, called a **jack-shaft,** may be necessary to maintain the proper speed.

If the head is subject to large variations, separate sets of turbines may be installed, one for ordinary use and one for use during periods of reduced head. The turbines should be of different designs, in order that both sets may operate

at the same speed under their respective heads One low-head and one high-head turbine may conveniently be mounted on the same shaft, but they should have separate gates The capacity of the low-head turbine should be equal to that of the high-head turbine under the least head at which the latter is to operate For heads between the minimum for the two turbines, the low-head turbine is used at part gate, while for heads exceeding the minimum for the high-head turbine, the latter is used at part gate

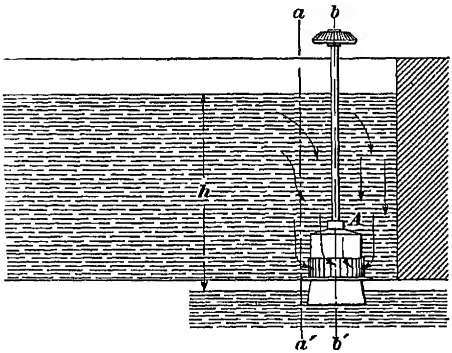

FIG 30

69. Effect of a Long Penstock on Regulation.—In Fig 30 is shown a waterwheel set in an open flume, while in Fig 31 is shown a waterwheel of the same size and capacity set in a closed penstock and supplied by a long, cylindrical flume Suppose that each wheel has a capacity

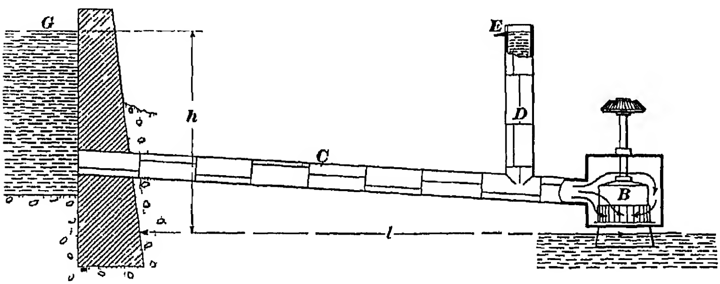

FIG 31

of Q cubic feet per second, and works under a head h If the gates of wheel A, Fig 30, are suddenly opened, the water will begin to flow through the wheel to its full capacity as soon as the force of gravity can impart the necessary

motion to the water lying immediately over and around the wheel

If the gates of wheel B, Fig 31, are suddenly opened, then, in order to supply the wheel to full capacity, the entire column of water in the conduit C must be set in motion with a velocity v equal to $\frac{Q}{A}$, denoting by A the area of cross-section of the flume C The time required to impart a given velocity to the water contained in the penstock C increases in direct proportion to the ratio of the length l of the flume C to the head h

Whenever there is any change, either an increase or decrease, in the gate opening of the turbine B, there will be a corresponding lag or lapse of time before the velocity in the flume C is changed, and hence also a lapse of time will occur before the turbine begins to operate at the proper capacity The amount of this lag will be greatest for a turbine supplied by a long, closed flume, and least for a turbine set in an open flume

It is impossible for a governor to operate the turbine gates until the load begins to change It follows that, in the case of a turbine supplied with a long pipe, a certain amount of time will elapse between a change of load and the readjustment of the turbine speed During this period of lag, the speed will tend to fall too low if the load has been increased, and to rise too high if the load on the turbine has been decreased

70. As explained in *Waterwheels*, Part 1, the kinetic energy K of the water column in the flume C, expressed in foot-pounds per second, is given by the formula

$$K = \frac{w A v^3}{2g}$$

where w is the weight of 1 cubic foot of water

If the gates of the turbine B, Fig 31, are suddenly closed in part, so that the velocity in the flume required to supply the turbine is reduced to some amount v_1, which is less than v, then, before the water can slow down to the velocity v_1, an

amount of energy K', equal to the difference in the energy contained in the water before and after the change in velocity, must be expended Hence,

$$K' = \frac{w A}{2g}(v^2 - v_1^2)$$

The energy of the water contained in the flume is expended by the exertion of pressure on the moving vanes of the turbine, when the velocity is decreased, the pressure must increase until the surplus energy is expended It follows that a sudden reduction in the gate opening of a turbine supplied by a long, closed flume will cause a temporary increase in the pressure in the flume and in the turbine itself, and a corresponding increase in the speed of the turbine

71. Water Hammer —The sudden increase in pressure following a sudden change of velocity is called **water hammer** This pressure may be prevented from producing injurious results by the use of a standpipe connected to the flume near the turbine, as shown at D in Fig 31 The standpipe should have an overflow E at about the same level as the water in the pond G

When the turbine gates are closed and the pressure begins to rise, the water level will rise in the standpipe, permitting some water to overflow and preventing the pressure from increasing to an undesirable amount A similar result may be accomplished by the use of a **pressure-relief valve** or by placing an **air chamber** in connection with the flume or penstock of the turbine The pressure-relief valve consists of a valve connected to the flume and ordinarily kept shut by means of a spring, it opens and permits some water to waste whenever the pressure rises above the proper limit Its action is similar to that of the ordinary safety valve used on steam boilers

In determining the necessary strength of long, closed pipes, allowance should be made for the pressure that may result from water hammer

THE TESTING OF TURBINES

72. The Holyoke Testing Flume.—Fig. 32 is a cross-section of the Holyoke flume, showing the wheel pit *A* with a turbine *D* in place for testing. Turbines are usually tested with the shaft in a vertical position, as shown in the figure.

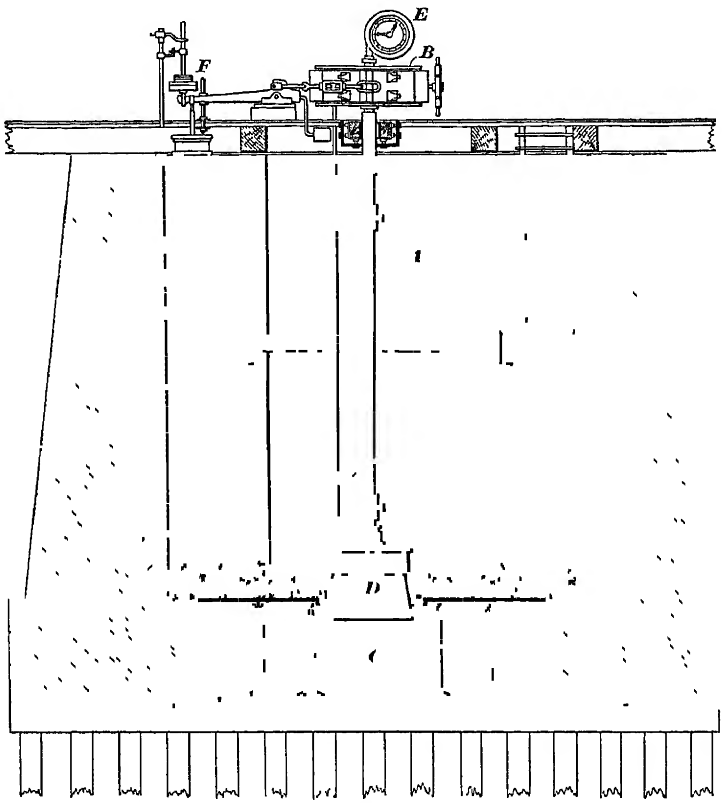

FIG. 32

The shaft is extended upwards to the floor of the testing room, and carries a brake pulley *B* at its upper end. The power is calculated from the readings of the scales *F* and the speed indicator *E*, by the method described in connection with the testing of impulse waterwheels. The spent water

is discharged into a tailrace *C*, from which it passes over a weir Hook gauges are used to determine the elevation of the water surface in the flume and in the tailrace above the weir, and from readings of these gauges the head acting on the wheel and the discharge are determined Each wheel is tested at several widths of gate opening A series of tests at a given gate opening usually comprises sets of observations at several different speeds, both above and below the speed of maximum efficiency A set of observations at a given speed and gate opening covers from 3 to 5 minutes, during the test, readings of all the gauges and of the break scales and speed indicator are taken at frequent intervals, and the means of the readings are used in performing the computations

73. Tabulation of Results —The table on page 60 shows the results of a Holyoke test of a 36-inch Hercules turbine at full gate The measured head and the revolutions per minute are given in columns 4 and 5, respectively The discharge in column 6 is calculated from the readings of the tailrace weir, and is corrected for leakage, if any occurs The brake horsepower in column 8 is calculated from the weight on the brake scales as given in column 7, and from the length of the brake lever, the diameter of the brake pulley, and the speed given in column 5 (See the testing of impulse wheels in *Waterwheels*, Part 1) The efficiency may be determined from the brake horsepower (column 8), head (column 4), and discharge (column 6), in the manner explained in *Waterwheels*, Part 1 The speed ratio given in column 11 is the ratio of the circumferential velocity v_1 of the inflow circle of the runner to the theoretical velocity $\sqrt{2gh}$ due to the head

74. The head usually varies slightly during the tests In order to compare the operation of the turbine at different speeds and gate openings, the corresponding discharge and power under a constant or standard head are determined The head corresponding to the speed of maximum full-gate efficiency is usually chosen as a standard head The

TEST OF A 36-INCH HERCULES TURBINE

1	2	3	4	5	6	7	8	9	10	11	12
Number of Test	Proportional Gate Opening	Duration of Test Minutes	Mean Head Feet	Revolutions per Minute	Discharge Cubic Feet per Second	Weight on Dynamometer Lever Pounds	Horse-power Developed	Discharge Reduced to Standard	Proportional Discharge	Speed Ratio	Efficiency Per Cent
1	1 000	4	16 82	Still	100 20	345			1 139		
7	1 000	4	16 90	123 50	89 87	193	145 30	90 14	1 019	588	84 39
6	1 000	4	16 94	129 10	89 66	185	145 59	89 82	1 016	614	84 55
5	1 000	4	16 95	135 33	89 00	177	145 02	89 13	1 008	644	85 38
4	1 000	4	16 96	140 62	83 33	170	145 72	88 43	1 000	669	85 80
3	1 000	4	16 98	144 80	87 79	163	143 88	87 84	993	688	85 14
2	1 000	4	17 05	150 00	87 12	155	143 62	86 99	984	721	85 29

discharges for the different speeds, reduced to the standard head, are shown in column 9 of the table. The proportional discharge given in column 10 is the ratio of the discharge reduced to standard head for any speed to the measured discharge at full gate for the speed of maximum efficiency.

Let Q_0 = full-gate discharge at maximum efficiency,
Q = actual discharge for any speed trial,
Q_1 = corresponding discharge at standard head,
q = proportional discharge,
h_0 = standard head,
h = actual head in the trial considered.

Then, since the discharges are proportional to the velocities, and the velocities to the square roots of the heads, we have

$$Q_1 : Q = \sqrt{h_0} : \sqrt{h},$$

whence

$$Q_1 = Q\sqrt{\frac{h_0}{h}} \qquad (1)$$

Also,

$$q = \frac{Q_1}{Q_0} = \frac{Q}{Q_0}\sqrt{\frac{h_0}{h}} \qquad (2)$$

EXAMPLE.—The maximum full-gate efficiency of a turbine is found on a test when the head is 14.01 feet and the discharge 201.0 cubic feet per second. What is the discharge at standard head and the proportional discharge for a speed trial at part gate for which the head is 15.26 feet and the discharge 153.5 cubic feet per second?

SOLUTION.—Here, $h_0 = 14.01$, $Q_0 = 201.0$, $h = 15.26$, and $Q = 153.5$. Then, by formula **1**,

$$Q_1 = 153.5\sqrt{\frac{14.01}{15.26}} = 147.1 \text{ cu. ft. per sec.} \quad \text{Ans.}$$

By formula **2**, $$q = \frac{147.1}{201.0} = .732 \quad \text{Ans.}$$

75. As will be seen from the table on page 60, the proportional discharge varied with the speed in a nearly regular manner. This is found to be the case at part gate as well as at full gate. For the ordinary range of speed variation, the discharge of a turbine at a given gate opening usually decreases as the load decreases, or as the speed increases. An overloaded turbine will as a rule use a little more water than one running at its normal load under the same head and

gate opening The maximum full-gate efficiency for this t bine is about 85 8 per cent , with a speed ratio of about The efficiency decreased but little for a variation of seve per cent in speed, but when the speed ratio was below ab 64, the efficiency decreased more rapidly The horsepow which is affected by both the discharge and the efficien varies with the speed somewhat more rapidly than efficiency The wheel gives its maximum horsepower a different speed from that at which the efficiency is a maximu

76. Tables similar to the one on page 60 can be c structed for tests in which the gate is only partly open Usually, the amount of opening is expressed as a decir fraction, and tabulated as "Proportional gate opening "

Although the horsepower of a turbine increases with gate opening, and is usually greatest at full gate, the sa is not always true of the efficiency, which often is a ma mum when the wheel runs at part gate, generally, the great efficiency occurs for a proportional opening of betw 75 and 1, although sometimes a much smaller open gives the maximum efficiency If a turbine is to be opera at part gate, it is desirable to secure as great an efficiency possible at the gate opening at which the wheel is intend to run If, as is usually the case, it is desired to maint the speed constant at various gate openings, the efficier at the constant speed at which the wheel is intended to r must be taken into account

Usually, the discharge is not directly proportional to gate opening, the proportional discharge being somew greater than the proportional gate opening This is gen ally true of American-type turbines For example, a wh generally uses more than one-half the full amount of wa when operating at one-half gate The relation between proportional gate opening and the discharge varies w the type of gate used

77. Testing Turbines in Use.—In testing turbines use, the power is either measured by a friction brake or some other form of dynamometer, or else, as in the case

turbines driving electrical generators, it may be computed from the recorded electrical output during the test. The discharge is usually measured by a weir, although floats or current meters are sometimes used. The details of the methods of conducting the test vary greatly with the conditions. Great care must be taken to insure accuracy and to prevent unmeasured losses of power or water from taking place. As a rule, the data obtained and the general methods of reducing the results are about the same as those used at Holyoke.

TURBINE INSTALLATIONS

78. Definitions.—A turbine plant usually includes a dam to impound the water, a conduit to carry the water to the turbine, a compartment for the turbines, and a draft tube or tailrace, or both, to return the water to the stream below. The word **flume** will be used here to describe the pipe or channel that leads the water from the dam or power canal to the compartment containing the waterwheel. A **penstock** is a compartment separate from the flume, and containing one or more wheels. Waterwheels are often set in open wooden flumes without the use of separate penstocks. The words *flume* and *penstock* are often used with the meanings here given interchanged. An iron or steel penstock is often called a **case**, or **casing**; but, as the waterwheel itself has a case, this use of the word should be avoided.

79. Conduits.—The simplest, and in many cases the cheapest, form of conduit consists of a canal dug along the side slope of the stream valley. In order to prevent loss by percolation through the porous soil, canals dug in earth may have their banks puddled by thoroughly mixing and compacting a wall of wet, plastic clay in the center of the dikes. Clay, when wet, is likely to slip and yield, and for this reason the entire banks should not be built of clay. A loamy mixture of clay with a firmer soil, as sand or gravel, is the best bank-forming material. Sometimes, the water is taken directly into the penstock from the pond or head-race, but

frequently a wood or iron flume is used. Open rectangular wooden flumes are cheap, and easily constructed, but, as they rot easily, they are not very durable. Power canals are sometimes lined with cement to prevent seepage and to decrease the friction and consequent loss of head.

Circular wooden flumes of stave pipe are often used. They are commonly made of cypress or of California redwood, and are cheap and very durable. They can stand very heavy pressures, and can be run up and down the irregularities of the ground surface, or buried at a sufficient depth to prevent freezing. Stave pipe has, besides, the advantage that its inner surface, being very smooth, causes little friction loss. For further particulars regarding stave pipe, see *Water Supply*, Part 2.

Riveted iron or steel pipe is used for short flumes under high pressures, or where it is desired to make short bends or connections. For light pressures, spiral riveted pipe may be used; it can be purchased ready made in suitable lengths to be put together with slip or flanged joints. All forms of iron and steel pipe should be coated outside and inside with asphaltum or coal tar to prevent rusting. The carrying capacity of such pipes decreases with time, on account of the growth of a vegetable slime on their interior surfaces, which greatly increases the loss by friction.

Conduits are often made of reinforced concrete, their interior surfaces being usually washed with neat cement to render them smoother and water-tight.

In order to prevent excessive loss of head by friction, the mean velocity in power flumes is not usually allowed to exceed 4 feet per second.

80. A drain valve should always be provided at the lower end, so that the water can be drawn out of the flume when the waterwheels are shut down. If a closed flume passes over hills, there should be an air valve at each summit, so that the imprisoned air can escape when the pipe is being filled. Care should be exercised to allow the air to enter the pipe freely in case the head-gates at the entry end

are closed when the drain valve is opened A standpipe having an open end rising above the hydraulic gradient answers this purpose If the air cannot enter the pipe, a vacuum may form in it, and the pressure of the atmosphere on the outside of the pipe may be sufficient to cause the pipe to collapse

81. Head-Gates —Head-gates that will close the inlet end of the flume or penstock should always be provided, so

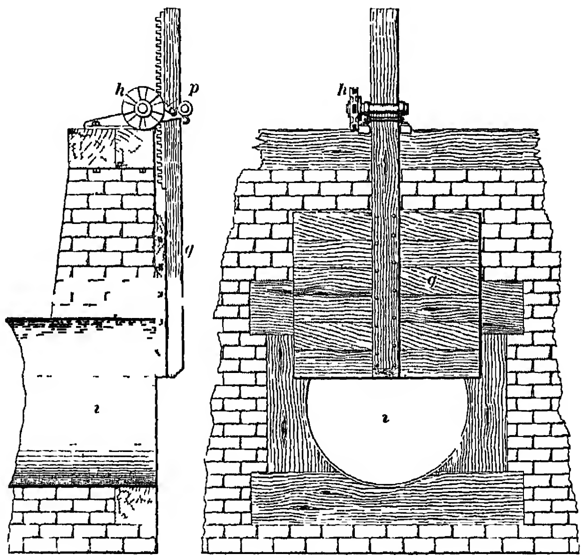

FIG 33

that the wheel, penstock, and flume may be drained for inspection and repairs Fig 33 shows a simple form of head-gate, consisting of a plank gate *g* that slides over the inlet end *i* of a flume or penstock The gate is raised or lowered by means of a rack and pinion and a lever that can be inserted in the capstan head *h* of the pinion shaft A pawl *p* serves to hold the gate from running down

Various combinations of screws, worm-gears, and trains of spur wheels are also used for operating head-gates, in place of the simple lever arrangement shown in Fig 33

An elevation (*a*) and a vertical section (*b*) of an iron **gate valve,** such as is often used with iron and steel pipes and penstocks, are shown in Fig 34 The wedge-shaped gate *a* fits closely against the valve seat *c* when the gate is closed, preventing leakage The valve seat is usually made of brass or bronze to prevent the gate from rusting and sticking When the valve is opened, the gate is drawn up into the dome *d* This dome should, if necessary, be protected against freezing The force required to start such a valve is really much greater than that required to operate it after starting A common fault with gate valves is the use of operating gears that are not strong enough When used under heavy pressures, the flume is sometimes filled through the by-pass valve *b*, in order to produce a back pressure on the valve, and thus enable the gate to be opened more easily

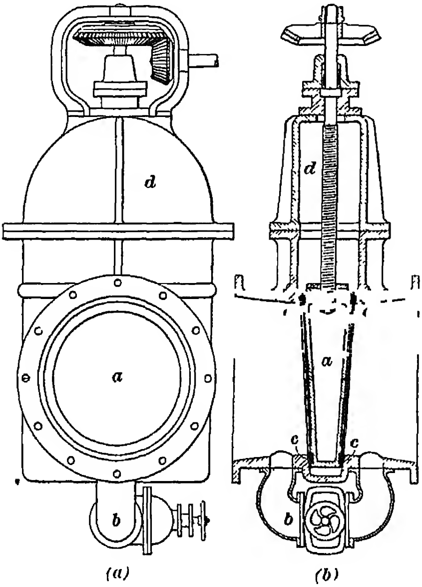

FIG 34

82. Racks and Screens, and Booms —Turbines must be protected from ice, leaves, sticks, fish, and similar substances that might clog them by catching between the wheel and guides A rack made of thin bars of iron is usually placed in the flume just above the penstock The bars in the rack must be far enough apart to allow the water to flow through freely The bars are usually separated by iron washers, and the bars and washers are held together by long bolts passing through both Fig 35 shows a single bar of

good form. The end *b* is bolted to the floor of a platform on which a workman stands to remove the trash with a short tooth rake having teeth spaced the same distance apart as the rack bars. A coarse wooden rack is sometimes used at the entrance to the race, and the finer iron rack is placed at the entrance to the penstock. A floating log or logs chained together to form a boom may also be stretched across the entrance to the raceway. This will usually prevent floating logs or cakes of ice from entering the head-race.

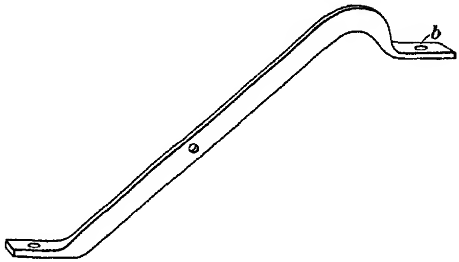

FIG. 35

83. Needle Ice and Anchor Ice.—Power canals, screen racks, and turbines are often obstructed in winter by a form of soft, spongy ice resembling snow slush, and called **needle ice**, or **frazil**. This ice does not form solid cakes, but floats suspended in the water, and not entirely at the surface. Needle ice is formed in two ways: (1) in rapids, where the water is cooled to 32° without having time to freeze solid, and (2) on stones or other dark objects in the bed of a stream, which are cooled below freezing temperature by radiation. This form is known as **anchor ice**. On warm and cloudy days, masses of anchor ice break loose and rise, often carrying stones embedded in them. The two forms of needle ice resemble each other closely, and both adhere to the object with which they come in contact. Screen racks and turbine vanes very quickly become blocked if needle ice is allowed to enter them. Needle ice does not form in deep, still ponds nor under a cover of surface ice. It is sometimes carried for some distance under the surface ice, but it will not usually give trouble where there is a deep, frozen pond, if the water is taken from the pond into a closed flume or penstock at a depth

of several feet below the surface If once allowed to accumulate in a power canal, needle ice is hard to remove, and may prevent the use of the power for some time

84. Penstocks.—Consider a wheel set in an open flume, as shown in Fig 30 All the water entering the wheel passes the section *a a'* Half of the water passes the section *b b'* through the center of the wheel There must be a considerable space at each side of the wheel as well as a suitable depth at the top, otherwise, the water must make a very abrupt turn to get into the wheel, and the water coming from opposite sides may form an eddy As a result, the wheel buckets may be only partly filled, which causes low efficiency, and in addition the head due to the draft tube may be lost owing to the suction of air through the wheel The lack of sufficient access room around the guide inlets has been the cause of many failures, especially where steel penstocks are used Such failures are often wrongly attributed to the waterwheels themselves

In the design of penstocks, a careful study of the course that the water will take in reaching the wheels should be made, in order that sufficient space may be provided This is especially true when there are several wheels in line, fed from the same flume It may happen that the wheels nearest the entry end of the penstock will receive abundant water, while those farther removed will receive a deficient supply, with the result that their apparent efficiency will be low

85. Vertical Wheels in Open Flume.—Fig 36 shows the method of setting a turbine in an open wooden flume, which also serves as a penstock In order to secure the advantage of the entire fall, the floor of the penstock must be low enough for the discharge opening of the wheel case to be always submerged in the tail-water This method of setting is cheap, and usually provides sufficient water space around the wheel Several wheels are often placed in the same flume The disadvantages of this method are the necessity of stopping all the wheels and drawing the water out of the flume in order to make repairs or inspection.

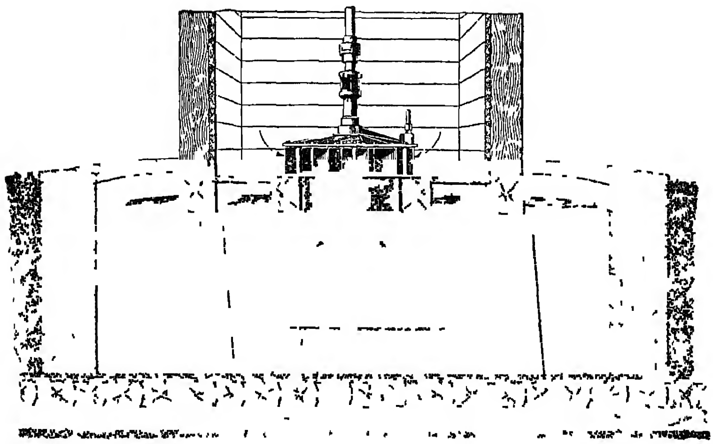

FIG 36

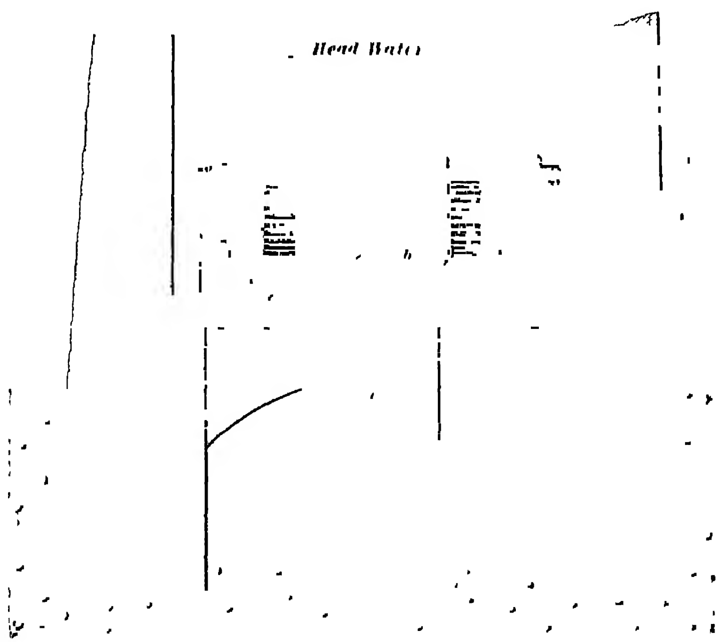

FIG 37

If it is desired to utilize the wheels together, they are usually connected by means of bevel gearing to a horizontal shaft running along the top of the flume

86. Horizontal Wheels in Open Flumes —Fig 37 shows a pair of turbines mounted on a horizontal shaft and discharging into a central draft chest *b*, from which the draft tube *c* conducts the spent water to the tailrace *a* The runners face in opposite directions, and the end thrust is therefore neutralized Two or more pairs can be set in line in the same flume, and thus a very large power can be obtained on a single high-speed shaft During low water, the gates of one pair can be closed if necessary

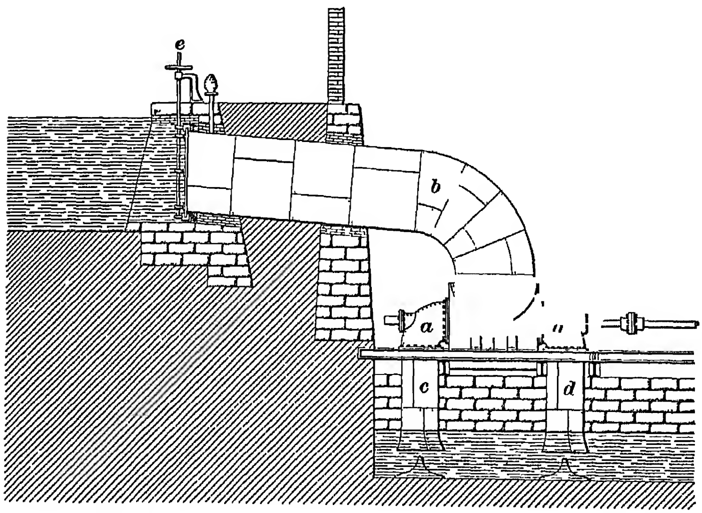

Fig 38

87. Wheels in Cylindrical Penstocks —Fig 38 shows a pair of wheels fed by a steel flume *b* and mounted in a cylindrical steel penstock *a a* The runners discharge from the ends of the penstock into the draft tubes *c* and *d* Several pairs of wheels can conveniently be placed side by side in the same power house A head-gate *e* is provided at the entrance to the flume, and a manhole is provided in the cylindrical case, so that the wheels are accessible for inspection and repair

88. Tailrace.—The spent water from turbines is discharged into a tail-pit, directly under the flume. The water may flow from the tail-pit directly into the stream or into a tailrace. A tailrace is necessary where the power plant is located at a distance from the stream; it may also be used to increase the head by carrying the water down stream past rapids or shoals to a point where the surface of the stream is lower than at the foot of the dam. In some cases, the tailrace is constructed by walling off a portion of the natural stream channel with a masonry or timber breakwater. The breakwater should extend above the high-water level of the stream, and should be nearly water-tight, in order to prevent water from flowing through or over it into the tailrace. The bottom of this tailrace will usually require excavation in order to reduce the loss of head by slope and friction, and to give the spent water a free outlet. The amount that can be economically expended in excavating a tailrace depends chiefly on the amount of head to be gained, the length of race, the character of the material to be excavated, and the value of the power. The velocity in a tailrace may be usually between 2 and 4 feet per second. In cold climates, the velocity should not be so low as to permit ice formation. If the bottom of the tail-pit is soft material, it should be floored with timber or concrete; if this is not done, the downward discharge from the turbines may stir the soil and deposit it in the tailrace, where it may form a barrier or dam, which will reduce the head on the wheels by backwater.